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Article

4-Total Prime Cordial Labeling of Some Derived Graphs

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Abstract: Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a map where $k \in \mathbb{N}$ is a variable and $k > 1$. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-Total prime cordial labeling of G if $\left|t_f(i) - t_f(j)\right| \leq 1$, $i, j \in \{1, 2, \cdots, k\}$ where $t_f(x)$ denotes the total number of vertices and the edges labelled with x. A graph with a k-total prime cordial labeling is called k-total prime cordial graph. In this paper we investigate the 4-total prime cordial labeling of some graphs like dragon, mobius ladder and corona of some graphs.

Keywords: Dragon, $P_n \odot K_2$, Corona of prism
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1. Introduction

Graphs considered here are finite, simple and undirected. A weaker version of graceful and harmonious labeling called cordial labeling was introduced by cahit [1]. Subsequently cordial related labeling have been studied by several author [2–4]. Ponraj et al. [5], have been introduced the notion of k-total prime cordial labeling and the k-total prime cordial labeling of certain graphs have been investigated. In [5–9], the 4-total prime cordial labeling behaviour of path, cycle, star, bistar, some complete graphs, comb, double comb, triangular snake, double triangular snake, ladder, friendship graph, flower graph, gear graph, Jelly fish, book, irregular triangular snake, prism, helm, dumbbell graph, sunflower graph, corona of irregular triangular snake, corona of some graphs and subdivision of some graphs. 3-total prime cordial labeling behaviour of some graphs have been investigated [9]. In this paper we investigate the 4-total prime cordial labeling of few graphs like dragon, mobius ladder and corona of some graphs.

2. Preliminary Results

Definition 1. *Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2 is the graph $G_1 \odot G_2$ obtained by taking one copy of G_1 , p_1 copies of G_2 and joining the i^{th} vertex of G_1 by an edge to every vertex in the i^{th} copy of G_2 where $1 \leq i \leq p_1$.*

Definition 2. *The cartesian product of two graphs G_1 and G_2 is the graph $G_1 \times G_2$ with vertex set $V_1 \times V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent whenever $[u_1 = v_1$ and u_2 adj $v_2]$ or $[u_2 = v_2$ and u_1 adj $v_1]$.*

Vertices	$u_1, \ldots, u_r$	$u_{r+1}, \ldots, u_{2r}$	$u_{2r+1}, \ldots, u_{3r}$	$u_{3r+1}, \ldots, u_{4r-1}$	u_{4r}
Labels	4	2	3	1	3

Table 1

Definition 3. *An n-sided prism Pr_n is a planar graph having 2 faces viz., an inner face and outer face with n sides and every other face is a 4-cycle. In other words, it is $C_n \times K_2$.*

Definition 4. *The mobius ladder M_n is the graph obtained from the ladder $P_n \times P_2$ by joining the opposite end points of the two copies of P_n.*

Theorem 1. [5] *All paths are* 4*-total prime cordial.*

For the purpose of the main section we once again recall the 4-total prime cordial labeling of the path:

For $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. The labeling pattern is given in Table 1.

For $n = 4r + 1$, $r > 1$ and $r \in \mathbb{N}$. Label the vertices u_i $(1 \leq i \leq 4r)$ as above pattern in Table 1. Assign the label 4 to the vertex u_{4r+1}.

For $n = 4r + 2$, $r > 1$ and $r \in \mathbb{N}$. The labeling pattern is given in Table 2 and Table 3

Vertices	$u_1, \ldots, u_{r+1}$	$u_{r+2}, \ldots, u_{2r+1}$	$u_{2r+2}, \ldots, u_{3r+1}$
Labels	4	2	3

Table 2

Vertices	$u_{3r+2}, \ldots, u_{4r}$	u_{4r+1}	u_{4r+2}
Labels	1	3	2

Table 3

For $n = 4r + 3$, $r > 1$ and $r \in \mathbb{N}$. Label the vertices u_i $(1 \leq i \leq 4r + 2)$ as above pattern in Table 1. Assign the label 3 to the vertex u_{4r+3}.

A 4-total prime cordial labeling of P_n, $n = 3, 4, 5, 6, 7$ is given in Table 4.

n	u_1	u_2	u_3	u_4	u_5	u_6	u_7
3	4	2	3				
4	4	2	3	4			
5	4	3	2	4	3		
6	4	3	4	2	4	3	
7	4	1	4	2	4	3	3

Table 4

Theorem 2. [5] *The cycle C_n is 4-total prime cordial iff $n \notin \{4, 6, 8\}$.*

When $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. Assign the same labeling pattern of P_{4r} u_i $(1 \leq i \leq n)$ given in Theorem 1. Next we relabel the vertex u_{4r-1} by 4.

When $n = 4r + 1$, $r > 1$ and $r \in \mathbb{N}$. The same labeling pattern of P_{4r+1} given in Theorem 1 is also a 4*-total prime cordial labeling of cycle C_{4r+1}.*

When $n = 4r + 2$, $r > 1$ and $r \in \mathbb{N}$. Assign the same labeling pattern of P_{4r+2} u_i $(1 \leq i \leq n)$ given in Theorem 1. Next we relabel the vertices u_{3r+2}, u_{4r+1}, u_{4r+2} respectively by 3, 2 *and* 1.

When $n = 4r + 3$, $r > 1$ and $r \in \mathbb{N}$. The same labeling pattern of P_{4r+3} $(1 \leq i \leq 4r)$ is given in Theorem 1. Finally we assign the labels to the vertices u_{4r+1}, u_{4r+2}, u_{4r+3} respectively by 1, 3 *and* 2.

A 4-total prime cordial labeling of C_n, $n = 3, 5, 7$ is given in Table 5.

n	u_1	u_2	u_3	u_4	u_5	u_6	u_7
3	4	2	3				
5	4	2	4	3	3		
7	4	4	2	2	3	3	1

Table 5

Remark 1. *2- total prime cordial graph is 2-total product cordial graph.*

3. k-total prime cordial labeling

Definition 5. *Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \dots, k\}$ be a function where $k \in \mathbb{N}$ is a variable and $k > 1$. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-Total prime cordial labeling of G if $\left|t_f(i) - t_f(j)\right| \leq 1$, $i, j \in \{1, 2, \cdots, k\}$ where $t_f(x)$ denotes the total number of vertices and the edges labelled with x. A graph with a k-total prime cordial labeling is called k-total prime cordial graph.*

First we investigate the 4-total prime cordial labeling behavior of the mobius ladder.

Theorem 3. *The mobius ladder M_n is 4-total prime cordial iff $n \neq 2, 3$.*

Proof. Let $V(L_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\} \cup \{u_1v_n, v_1u_n\}$. It is easy to verify that $|V(M_n)| + |E(M_n)| = 5n$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \dots, u_{r+1}$ and assign the label 2 to the vertices $u_{r+2}, u_{r+3}, \dots, u_{2r+1}$. Next we assign the label 3 to the vertices $u_{2r+2}, u_{2r+3}, \dots, u_{3r+1}$. Finally we assign the label 1 to the vertices $u_{3r+2}, u_{3r+3}, \dots, u_{4r}$. Next we consider the vertices v_i $(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_1, v_2, \dots, v_r$ and assign the label 2 to the vertices $v_{r+1}, v_{r+2}, \dots, v_{2r}$ and next we assign the label 3 to the vertices $v_{2r+1}, v_{2r+2}, \dots, v_{3r+1}$. Finally we assign the label 1 to the vertices $v_{3r+2}, v_{3r+3}, \dots, v_{4r}$. Clearly $t_f(1) = t_f(2) = t_f(3) = t_f(4) = 5r$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, $r > 1$ and $r \in \mathbb{N}$. As in case 1, assign the label to the vertices u_i $(1 \leq i \leq n-1)$, v_i $(1 \leq i \leq n-2)$. Finally we assign the labels 4, 2, 3 respectively to the vertices u_{4r+1}, v_{4r} and v_{4r+1}. Here $t_f(1) = t_f(2) = t_f(3) = 5r + 1$ and $t_f(4) = 5r + 2$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r > 1$ and $r \in \mathbb{N}$. As in case 1, assign the label to the vertices u_i $(1 \leq i \leq n-2)$, v_i $(1 \leq i \leq n-2)$. Finally we assign the labels 3, 4, 3, 2 to the vertices u_{4r+1}, u_{4r+2}, v_{4r+1} and v_{4r+2} respectively. It is easy to verify that $t_f(1) = t_f(4) = 5r + 2$ and $t_f(2) = t_f(3) = 5r + 3$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r+3$, $r > 1$ and $r \in \mathbb{N}$. Assign the label to the vertices u_i $(1 \leq i \leq n-4)$, v_i $(1 \leq i \leq n-4)$ by case 1. Finally we assign the labels 3, 4, 3, 3, 2, 4, 2, 1 respectively to the vertices u_{4r}, u_{4r+1}, u_{4r+2}, u_{4r+3}, v_{4r}, v_{4r+1}, v_{4r+2} and v_{4r+3} . Obviously $t_f(1) = t_f(2) = t_f(3) = 5r + 4$ and $t_f(4) = 5r + 3$.

Case 5. $n = 2$.

Suppose f is a 4-total prime cordial labeling of M_2 then any one of the following labeling pattern occurs:

- $t_f(1) = t_f(2) = 3$ and $t_f(3) = t_f(4) = 2$.
- $t_f(1) = t_f(3) = 3$ and $t_f(2) = t_f(4) = 2$.
- $t_f(1) = t_f(4) = 3$ and $t_f(2) = t_f(3) = 2$.
- $t_f(2) = t_f(3) = 3$ and $t_f(1) = t_f(4) = 2$.

- $t_f(2) = t_f(4) = 3$ and $t_f(1) = t_f(3) = 2$.
- $t_f(3) = t_f(4) = 3$ and $t_f(1) = t_f(2) = 2$.

To get the label 4, it should be a label of exactly two vertices this gives $t_f(4) = 3$. To get the label 3, it should be a label of exactly two vertices this gives $t_f(3) = 3$. Note that the case $t_f(4) = 2$ does not exists then $t_f(2) = 0$, a contradiction.

Case 6. $n = 3$.

Suppose f is a 4-total prime cordial labeling of M_3 then any one of the following labeling pattern occurs:

- $t_f(1) = t_f(2) = t_f(3) = 4$ and $t_f(4) = 3$.
- $t_f(1) = t_f(2) = t_f(4) = 4$ and $t_f(3) = 3$.
- $t_f(1) = t_f(3) = t_f(4) = 4$ and $t_f(2) = 3$.
- $t_f(2) = t_f(3) = t_f(4) = 4$ and $t_f(1) = 3$.

Clearly $t_f(4) \neq 4$ and $t_f(3) \neq 4$, a contradiction. Hence M_3 does not admits a 4-total prime cordial labeling.

Case 3. $n = 4, 5, 6, 7$.

A 4-total prime cordial labeling follows from Table 6.

n	4	5	6	7
u_1	4	4	4	4
u_2	4	4	4	4
u_3	2	2	2	2
u_4	3	4	3	3
u_5		2	3	3
u_6			2	4
u_7				2
v_1	3	4	4	4
v_2	3	3	4	4
v_3	2	3	2	2
v_4	4	3	3	3
v_5		3	3	3
v_6			1	1
v_7				3

Table 6

□

Next we investigate the 4-total prime cordial labeling behavior of the dragon $C_m@P_n$.

Theorem 4. *The dragon $C_m@P_n$ is 4-total prime cordial if $m, n \geq 8$.*

Proof. Let C_m be the cycle $u_1u_2 \dots u_mu_1$ and P_n be the path $v_1v_2 \dots v_n$. Let the dragon $C_m@P_n$ be obtained from C_m and P_n by identifying u_1 and v_1. Clearly $|V(C_m@P_n))| + |E(C_m@P_n)| = 2m + 2n - 2$. The proof is divided into following sixteen cases depends on the nature of m and n.

Case 1. $m \equiv 0 \pmod 4$ and $n \equiv 0 \pmod 4$.

As in case 1 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n-2)$ as in case 1 of Theorem 1. Finally we assign the label 3, 4 to the vertices v_{4r-1} and v_{4r} respectively.

Case 2. $m \equiv 0 \pmod 4$ and $n \equiv 1 \pmod 4$.

As in case 1 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 2 of Theorem 1. Next we relabel the vertex v_{4r-2} by 4.

Case 3. $m \equiv 0 \pmod 4$ and $n \equiv 2 \pmod 4$.

As in case 1 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 3 of Theorem 1.

Case 4. $m \equiv 0 \pmod 4$ and $n \equiv 3 \pmod 4$.

As in case 1 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 4 of Theorem 1. Then we relabel the vertex v_{4r-3} by 4.

Case 5. $m \equiv 1 \pmod 4$ and $n \equiv 0 \pmod 4$.

As in case 2 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 1 of Theorem 1. Finally we relabel the vertex v_{4r-1} by 4.

Case 6. $m \equiv 1 \pmod 4$ and $n \equiv 1 \pmod 4$.

As in case 2 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n-3)$ as in case 2 of Theorem 1. Next we assign the label 3, 4, 3 to the vertices v_{4r-2}, v_{4r-1} and v_{4r} respectively.

Case 7. $m \equiv 1 \pmod 4$ and $n \equiv 2 \pmod 4$.

As in case 2 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n-3)$ as in case 3 of Theorem 1. Then we assign the label 3, 2, 3 to the vertices v_{4r-2}, v_{4r-1} and v_{4r} respectively.

Case 8. $m \equiv 1 \pmod 4$ and $n \equiv 3 \pmod 4$.

As in case 2 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n-3)$ as in case 4 of Theorem 1. Finally we assign the label 2, 3, 3 to the vertices v_{4r-1}, v_{4r-1} and v_{4r} respectively.

Case 9. $m \equiv 2 \pmod 4$ and $n \equiv 0 \pmod 4$.

As in case 3 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 1 of Theorem 1. Next we relabel the vertex v_{4r} by 4.

Case 10. $m \equiv 2 \pmod 4$ and $n \equiv 1 \pmod 4$.

As in case 3 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 2 of Theorem 1. Then we relabel the vertex v_{4r-2} by 4.

Case 11. $m \equiv 2 \pmod 4$ and $n \equiv 2 \pmod 4$.

As in case 3 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 3 of Theorem 1.

Case 12. $m \equiv 2 \pmod 4$ and $n \equiv 3 \pmod 4$.

As in case 3 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 4 of Theorem 1. Finally we relabel the vertex v_{4r-3} by 4.

Case 13. $m \equiv 3 \pmod 4$ and $n \equiv 30 \pmod 4$.

As in case 4 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 1 of Theorem 1. Then we relabel the vertices v_{r+1}, v_{4r-3} by 4 and 2 respectively.

Case 14. $m \equiv 3 \pmod 4$ and $n \equiv 1 \pmod 4$.

As in case 4 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 2 of Theorem 1. Next we relabel the vertex v_{4r-1} by 4.

Case 15. $m \equiv 3 \pmod 4$ and $n \equiv 2 \pmod 4$.

As in case 4 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 3 of Theorem 1. Finally we relabel the vertex v_{4r} by 4.

Case 16. $m \equiv 3 \pmod 4$ and $n \equiv 1 \pmod 4$.

As in case 4 of Theorem 2, assign the label to the vertices u_i $(1 \leq i \leq m)$ and assign the label to the vertices v_i $(1 \leq i \leq n)$ as in case 4 of Theorem 1. Then we relabel the vertex v_{4r-3} by 4.

□

Example 1. *A* 4*-total prime cordial labeling of* $C_{12}@P_{13}$ *is given in Figure 1.*

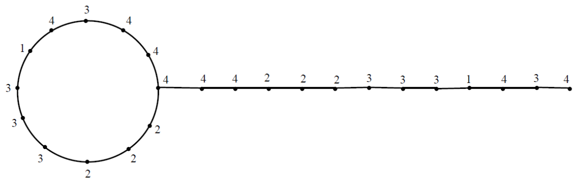

Figure 1

Now we investigate the 4-total prime cordial labeling behavior of the graph $(C_n \times P_2) \odot K_2$.

Theorem 5. *The corona of* $C_n \times P_2$*,* $(C_n \times P_2) \odot K_2$ *is* 4*-total prime cordial for all* $n \geq 3$*.*

Proof. Let $V((C_n \times P_2) \odot K_1) = \{u_i, v_i, w_i : 1 \leq i \leq n\}$ and $E((C_n \times P_2) \odot K_1) = \{u_1u_n, v_1v_n\} \cup \{u_iv_i, v_iw_i : 1 \leq i \leq n\} \cup \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\}$. It is easy to verify that $|V((C_n \times P_2) \odot K_1)| + |E((C_n \times P_2) \odot K_1)| = 7n$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \ldots, u_r$ and assign the label 2 to the vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. Next we assign the label 3 to the vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{3r+1}$ then we assign 1 to the vertices $u_{3r+2}, u_{3r+3}, \ldots, u_{4r-1}$. Finally we assign the label 4 to the vertex u_{4r}. Next we consider the vertices v_i $(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_1, v_2, \ldots, v_r$ and assign the label 2 to the vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$ and next we assign the label 3 to the vertices $v_{2r+1}, v_{2r+2}, \ldots, v_{3r}$. Finally we assign the label 1 to the vertices $v_{3r+1}, v_{3r+2}, \ldots, v_{4r}$. Now we move to the vertices w_i $(1 \leq i \leq n)$. Assign the label 4 to the vertices $w_1, w_2, \ldots, w_r$ and assign the label 2 to the vertices $w_{r+1}, w_{r+2}, \ldots, w_{2r}$ and next we assign the label 3 to the vertices $w_{2r+1}, w_{2r+2}, \ldots, w_{3r}$. Finally we assign the label 1 to the vertices $w_{3r+1}, w_{3r+2}, \ldots, w_{4r}$. Clearly $t_f(1) = t_f(2) = t_f(3) = t_f(4) = 7r$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, $r > 1$ and $r \in \mathbb{N}$. As in case 1, assign the label to the vertices u_i $(1 \leq i \leq n-2)$, v_i $(1 \leq i \leq n-2)$ and w_i $(1 \leq i \leq n-2)$. Finally we assign the labels 4, 4, 2, 1, 3, 1 respectively to the vertices u_{4r}, u_{4r+1}, v_{4r}, v_{4r+1}, w_{4r} and w_{4r+1}. Here $t_f(1) = t_f(2) = t_f(4) = 7r + 2$ and $t_f(3) = 7r + 1$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r > 1$ and $r \in \mathbb{N}$. As in case 2, assign the label to the vertices u_i $(1 \leq i \leq n-2)$, v_i $(1 \leq i \leq n-2)$ and w_i $(1 \leq i \leq n-2)$. Finally we assign the labels 4, 4, 1, 3, 2, 3 to the vertices u_{4r+1}, u_{4r+2}, v_{4r+1}, v_{4r+2}, w_{4r+1} and w_{4r+2} respectively. It is easy to verify that $t_f(1) = t_f(2) = 7r + 3$ and $t_f(3) = t_f(4) = 7r + 4$.

Case 4. $n \equiv 4 \pmod 4$.

Let $n = 4r + 3$, $r > 1$ and $r \in \mathbb{N}$. Assign the label to the vertices u_i $(1 \leq i \leq n-2)$, v_i $(1 \leq i \leq n-2)$ and w_i $(1 \leq i \leq n-2)$ by case 3. Finally we assign the labels 4, 4, 2, 3, 3, 3 respectively to the vertices u_{4r+2}, u_{4r+3}, v_{4r+2}, v_{4r+3}, w_{4r+2} and w_{4r+3}. Here $t_f(1) = t_f(2) = t_f(3) = 7r + 5$ and $t_f(4) = 7r + 6$.

Case 5. $n = 3, 4, 5, 6, 7$.

A 4-total prime cordial labeling follows from Table 7.

n	3	4	5	6	7
u_1	4	4	4	4	4
u_2	2	2	2	4	4
u_3	3	3	3	2	2
u_4		4	2	2	2
u_5			1	3	3
u_6				3	3
u_7					1
v_1	4	4	4	4	4
v_2	2	2	2	4	4
v_3	3	3	3	2	2
v_4		3	3	3	2
v_5			4	3	3
v_6				1	3
v_7					1
w_1	4	4	4	4	4
w_2	3	2	2	2	4
w_3	3	3	3	2	2
w_4		1	3	3	3
w_5			4	3	3
w_6				1	3
w_7					1

Table 7

□

Next we investigate the graph $P_n \odot K_2$.

Theorem 6. *The graph $P_n \odot K_2$ is 4-total prime cordial for all n.*

Proof. Let $V(P_n \odot K_2) = \{u_i, v_i, w_i : 1 \le i \le n\}$ and $E(P_n \odot K_2) = \{u_iv_i, u_iw_i, v_iw_i : 1 \le i \le n\} \cup \{u_iu_{i+1} : 1 \le i \le n-1\}$. Obviously $|V(P_n \odot K_2)| + |E(P_n \odot K_2)| = 7n - 1$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \ldots, u_r$ and assign the label 2 to the vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. Next we assign the label 3 to the vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{3r}$. Then we assign the label 1 to the vertices $u_{3r+1}, u_{3r+2}, \ldots, u_{4r-2}$. Finally we assign the labels 4, 3 to the vertices u_{4r-1} and u_{4r} respectively. Next we consider the vertices v_i $(1 \le i \le n)$. Assign the label 4 to the vertices $v_1, v_2, \ldots, v_r$ and assign the label 2 to the vertices $v_{r+1}, v_{r+2}, \ldots, v_{2r}$ and next we assign the label 3 to the vertices $v_{2r+1}, v_{2r+2}, \ldots, v_{3r}$. Finally we assign the label 1 to the vertices $v_{3r+1}, v_{3r+2}, \ldots, v_{4r}$. Now we move to the vertices w_i $(1 \le i \le n)$. Assign the label 4 to the vertices $w_1, w_2, \ldots, w_r$ and assign the label 2 to the vertices $w_{r+1}, w_{r+2}, \ldots, w_{2r}$ and next we assign the label 3 to the vertices $w_{2r+1}, w_{2r+2}, \ldots, w_{3r}$. Finally we assign the label 1 to the vertices $w_{3r+1}, w_{3r+2}, \ldots, w_{4r}$. Clearly $t_f(1) = 7r - 1$ and $t_f(2) = t_f(3) = t_f(4) = 7r$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, $r > 1$ and $r \in \mathbb{N}$. As in case 1, assign the label to the vertices u_i $(1 \le i \le n-1)$, v_i $(1 \le i \le n-1)$ and w_i $(1 \le i \le n-1)$. Finally we assign the labels 3, 4, 2 respectively to the vertices u_{4r+1}, v_{4r+1} and w_{4r+1}. Here $t_f(1) = t_f(4) = 7r + 1$ and $t_f(2) = t_f(3) = 7r + 2$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r+2$, $r > 1$ and $r \in \mathbb{N}$. Assign the label to the vertices u_i $(1 \le i \le n-2)$, v_i $(1 \le i \le n-2)$ and w_i $(1 \le i \le n-2)$ as in case 1. Finally we assign the labels 4, 3, 4, 3, 2, 1 to the vertices

u_{4r+1}, u_{4r+2}, v_{4r+1}, v_{4r+2}, w_{4r+1} and w_{4r+2} respectively. It is easy to verify that $t_f(1) = 7r + 4$ and $t_f(2) = t_f(3) = t_f(4) = 7r + 3$.

Case 4. $n \equiv 4 \pmod 4$.

Let $n = 4r + 3$, $r > 1$ and $r \in \mathbb{N}$. As in case 2, assign the label to the vertices u_i $(1 \le i \le n-2)$, v_i $(1 \le i \le n-2)$ and w_i $(1 \le i \le n-2)$. Finally we assign the labels 4, 3, 4, 4, 2, 3 respectively to the vertices u_{4r+2}, u_{4r+3}, v_{4r+2}, v_{4r+3}, w_{4r+2} and w_{4r+3}. Here $t_f(1) = t_f(2) = t_f(3) = t_f(4) = 7r + 5$.

Case 5. $n = 2, 3, 4, 5, 6, 7$.

A 4-total prime cordial labeling follows from Table 8.

n	2	3	4	5	6	7
u_1	4	2	4	4	4	4
u_2	3	3	2	4	4	4
u_3		3	3	2	2	2
u_4			1	3	3	2
u_5				3	3	3
u_6					1	3
u_7						4
v_1	4	4	4	4	4	4
v_2	3	4	2	2	4	4
v_3		4	3	2	2	2
v_4			3	3	3	3
v_5				1	3	3
v_6					1	4
v_7						3
w_1	2	4	4	4	4	4
w_2	1	3	2	2	2	2
w_3		2	3	3	2	2
w_4			1	3	3	3
w_5				1	1	3
w_6					1	1
w_7						1

Table 8

□

Next we investigate the graph $C_n \odot K_2$.

Corollary 1. *The graph $C_n \odot K_2$ is 4-total prime cordial for all n.*

Proof. Let $V(P_n \odot K_2) = \{u_i, v_i, w_i : 1 \le i \le n\}$ and $E(P_n \odot K_2) = \{u_1u_n, v_1v_n\} \cup \{u_iv_i, u_iw_i, v_iw_i : 1 \le i \le n\} \cup \{u_iu_{i+1} : 1 \le i \le n-1\}$. Clearly $|V(C_n \odot K_2)| + |E(C_n \odot K_2)| = 7n$.

Case 1. $n \equiv 0, 1, 3 \pmod 4$.

Let $n = 4r$,$n = 4r + 1$,$n = 4r + 3$, $r \in \mathbb{N}$. The vertex labelled in Theorem 6 is also a 4-total prime cordial of $C_n \odot K_2$.

Case 2. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r \in \mathbb{N}$. Assign the label to u_i, v_i, w_i $(1 \le i \le n)$ as in Theorem 6. Finally the vertex w_{4r+2} is relabelled by 4. Obviously this vertex labels is a 4-total prime cordial of $C_n \odot K_2$.

Case 3. $n = 2, 3, 4, 5, 6, 7$.

A 4-total prime cordial labeling follows from Table 9.

n	2	3	4	5	6	7
u_1	4	2	4	4	4	4
u_2	3	3	2	4	4	4
u_3	-	3	3	2	2	2
u_4	-	-	1	3	3	2
u_5	-	-	-	3	3	3
u_6	-	-	-	-	1	3
u_7	-	-	-	-	-	4
v_1	4	4	4	4	4	4
v_2	3	4	2	2	4	4
v_3	-	4	3	2	2	2
v_4	-	-	3	3	3	3
v_5	-	-	-	1	3	3
v_6	-	-	-	-	1	4
v_7	-	-	-	-	-	3
w_1	2	4	4	4	4	4
w_2	4	3	2	2	2	2
w_3	-	2	3	3	2	2
w_4	-	-	4	3	3	3
w_5	-	-	-	1	1	3
w_6	-	-	-	-	4	1
w_7	-	-	-	-	-	1

Table 9. Your caption here

□

Example 2. *A* 4*-total prime cordial labeling of* $C_5 \odot K_2$ *is given in Figure 2.*

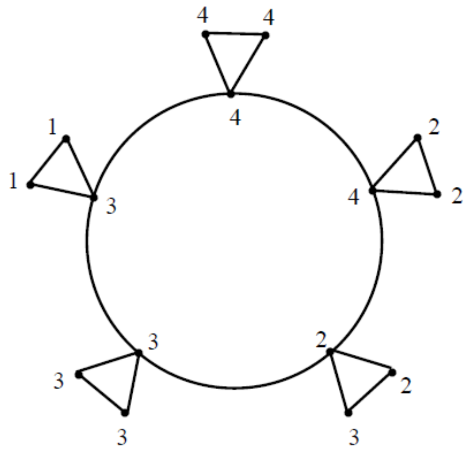

Figure 2

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