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Article

On Domination and 2-packing Numbers in Intersecting Linear Systems

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Abstract: A linear system is a pair $(P, \mathcal{L})$ where $\mathcal{L}$ is a finite family of subsets on a finite ground set P such that any two subsets of $\mathcal{L}$ share at most one element. Furthermore, if for every two subsets of $\mathcal{L}$ share exactly one element, the linear system is called intersecting. A linear system $(P, \mathcal{L})$ has rank r if the maximum size of any element of $\mathcal{L}$ is r. By $\gamma(P, \mathcal{L})$ and $\nu_2(P, \mathcal{L})$ we denote the size of the minimum dominating set and the maximum 2-packing of a linear system $(P, \mathcal{L})$, respectively. It is known that any intersecting linear system $(P, \mathcal{L})$ of rank r is such that $\gamma(P, \mathcal{L}) \leq r - 1$. Li et al. in [S. Li, L. Kang, E. Shan and Y. Dong, *The finite projective plane and the 5-Uniform linear intersecting hypergraphs with domination number four*, Graphs and **34** Combinatorics (2018) , no. 5, 931–945.] proved that every intersecting linear system of rank 5 satisfying $\gamma(P, \mathcal{L}) = 4$ can be constructed from a 4-uniform intersecting linear subsystem $(P', \mathcal{L}')$ of the projective plane of order 3 satisfying $\tau(P', \mathcal{L}') = \nu_2(P', \mathcal{L}') = 4$, where $\tau(P', \mathcal{L}')$ is the transversal number of $(P', \mathcal{L}')$. In this paper, we give an alternative proof of this result given by Li et al., giving a complete characterization of these 4-uniform intersecting linear subsystems. Moreover, we prove a general case, that is, we prove if q is an odd prime power and $(P, \mathcal{L})$ is an intersecting linear system of rank $(q + 2)$ satisfying $\gamma(P, \mathcal{L}) = q+1$, then this linear system can be constructed from a spanning $(q+1)$-uniform intersecting linear subsystem $(P', \mathcal{L}')$ of the projective plane of order q satisfying $\tau(P', \mathcal{L}') = \nu_2(P', \mathcal{L}') = q + 1$.

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1. Introduction

A *set system* is a pair $(X, \mathcal{F})$ where $\mathcal{F}$ is a finite family of subsets on a finite ground set X. A set system can be also thought of as a hypergraph, where the elements of X and $\mathcal{F}$ are called *vertices* and *hyperedges*, respectively. The set system $(X, \mathcal{F})$ is *intersecting* if $E \cap F \neq \emptyset$, for for every pair of distinct subsets $E, F \in \mathcal{F}$. On the other hand, the set system $(X, \mathcal{F})$ is a *linear system* if it satisfies $|E \cap F| \leq 1$, for every pair of distinct subsets $E, F \in \mathcal{F}$; and it is denoted by $(P, \mathcal{L})$. The elements of P and $\mathcal{L}$ are called *points* and *lines* respectively. In this paper, we only consider linear systems, and the most of following definitions can be generalized for set systems.

The *rank* of a linear system $(P, \mathcal{L})$ is the maximum size of a line of $\mathcal{L}$. An *r-uniform* linear system $(P, \mathcal{L})$ is a linear system such that every line contains exactly r points. Hence, a (simple) graph is a 2-uniform linear system. In this paper we only consider linear systems of rank $r \geq 2$.

Let $(P, \mathcal{L})$ be a linear system and $p \in P$ be a point. The *degree* of p is the number of lines containing p and it is denoted by $deg(p)$. The maximum degree overall points of the linear systems is denoted by $\Delta(P, \mathcal{L})$. A point of degree 2 and 3 is called *double point* and *triple point*, respectively. Two points p and q in $(P, \mathcal{L})$ are *adjacent* if there is a line $l \in \mathcal{L}$ such that $p, q \in l$.

A *linear subsystem* $(P', \mathcal{L}')$ of a linear system $(P, \mathcal{L})$ satisfies that for any line $l' \in \mathcal{L}'$ there exists a line $l \in \mathcal{L}$ such that $l' = l \cap P'$. The *linear subsystem induced* by a set of lines $\mathcal{L}' \subseteq \mathcal{L}$ is the linear subsystem $(P', \mathcal{L}')$ where $P' = \bigcup_{l \in \mathcal{L}'} l$. The linear subsystem $(P', \mathcal{L}')$ of $(P, \mathcal{L})$ is called *spanning linear subsystem* if $P' = P$. Given a linear system $(P, \mathcal{L})$, and a point $p \in P$, the linear system obtained from $(P, \mathcal{L})$ by *deleting the point* p is the linear system $(P', \mathcal{L}')$ induced by $\mathcal{L}' = \{l \backslash \{p\} : l \in \mathcal{L}\}$. Given a linear system $(P, \mathcal{L})$ and a line $l \in \mathcal{L}$, the linear system obtained from $(P, \mathcal{L})$ by *deleting the line* l is the linear system $(P', \mathcal{L}')$ induced by $\mathcal{L}' = \mathcal{L} \setminus \{l\}$. Let $(P', \mathcal{L}')$ and $(P, \mathcal{L})$ be two linear systems. The linear systems $(P', \mathcal{L}')$ and $(P, \mathcal{L})$ are isomorphic, denoted by $(P', \mathcal{L}') \simeq (P, \mathcal{L})$, if after deleting points of degree 1 or 0 from both, the systems $(P', \mathcal{L}')$ and $(P, \mathcal{L})$ are isomorphic as hypergraphs, see [1].

A subset D of points of a linear system $(P, \mathcal{L})$ is a *dominating set* of $(P, \mathcal{L})$ if for every $p \in P \setminus D$ there exists $q \in D$ such that p and q are adjacent. The minimum cardinality of a dominating set of a linear system $(P, \mathcal{L})$ is called *domination number*, denoted by $\gamma(P, \mathcal{L})$. Domination in set systems (hypergraphs) was introduced by Acharya [2] and studied further in [3–7].

A subset T of points of a linear system $(P, \mathcal{L})$ is a *transversal* of $(P, \mathcal{L})$ (also called *vertex cover* or *hitting set*) if $T \cap l \neq \emptyset$, for every line $l \in \mathcal{L}$. The minimum cardinality of a transversal of a linear system $(P, \mathcal{L})$ is called *transversal number*, denoted by $\tau(P, \mathcal{L})$. Since any transversal of a linear system $(P, \mathcal{L})$ is a dominating set, then $\gamma(P, \mathcal{L}) \leq \tau(P, \mathcal{L})$.

A subset R of lines of a linear system $(P, \mathcal{L})$ is a 2*-packing* of $(P, \mathcal{L})$ if the elements of R are triplewise disjoint, that is, if any three elements are chosen in R then they are not incidents in a common point. The *2-packing number* of $(P, \mathcal{L})$ is the maximum cardinality of a 2-packing of $(P, \mathcal{L})$, denoted by $\nu_2(P, \mathcal{L})$. There are some works related on transversal and 2-packing numbers in linear systems, see for example [1, 8–11].

Kang et al. [12] proved if $(P, \mathcal{L})$ is an intersecting linear system of rank $r \geq 2$, then $\gamma(P, \mathcal{L}) \leq r - 1$. Shan et al. [13] gave a characterization of set systems $(X, \mathcal{F})$ holding the equality when $r = 3$. On the other hand, Dong et al. [6] shown all intersecting linear systems $(P, \mathcal{L})$ of rank 4, satisfying $\gamma(P, \mathcal{L}) = 3$, can be constructed by the Fano plane. In this paper, we prove that if $(P, \mathcal{L})$ is an intersecting linear system of rank 5 satisfying $\gamma(P, \mathcal{L}) = 4$, then this linear system can be constructed from an 4-uniform intersecting linear subsystem $(P', \mathcal{L}')$ of the projective plane of order 3 satisfying $\tau(P', \mathcal{L}') = \nu_2(P', \mathcal{L}') = 4$, and we give a complete characterization of this 4-uniform intersecting linear subsystems. The result was also obtained by Li et al. [14]. Furthermore, we prove that if q is an odd prime power and if $(P, \mathcal{L})$ is an intersecting linear system of rank $(q + 2)$ satisfying $\gamma(P, \mathcal{L}) = q + 1$ then, this linear system can be constructed from a spanning $(q + 1)$-uniform intersecting linear subsystem $(P', \mathcal{L}')$ of the projective plane of order q satisfying $\tau(P', \mathcal{L}') = \nu_2(P', \mathcal{L}') = q + 1$.

2. Previous Results

Let $\mathcal{I}_r$ be the family of intersecting linear systems $(P, \mathcal{L})$ of rank r with $\gamma(P, \mathcal{L}) = r - 1$. To better understand this paper, we need the following results:

Lemma 1 (Lemma 2.1 in [6])**.** *For every linear system $(P, \mathcal{L}) \in \mathcal{I}_r$, there exists a spanning intersecting r-uniform linear subsystem $(P^*, \mathcal{L}^*)$ of $(P, \mathcal{L})$ such that every line in $\mathcal{L}^*$ contains one point of degree one.*

Let $(P^*, \mathcal{L}^*)$ be the spanning intersecting r-uniformlinear subsystem obtained from Lemma 1. Furthermore, let $(P', \mathcal{L}')$ be the intersecting $(r - 1)$-uniform linear subsystem obtained from $(P^*, \mathcal{L}^*)$ by deleting the point of degree one of each line of $\mathcal{L}^*$, see [6].

Lemma 2 (Lemma 2.2 in [6]). *For every linear system* $(P, \mathcal{L}) \in \mathcal{I}_r$ *it satisfies*

$$\gamma(P, \mathcal{L}) = \gamma(P^*, \mathcal{L}^*) = \tau(P^*, \mathcal{L}^*) = \tau(P', \mathcal{L}') = r - 1.$$

Lemma 3 (Lemma 2.4 in [6]). *Let* $(P, \mathcal{L}) \in \mathcal{I}_r$ $(r \geq 3)$*, then every line of* $(P', \mathcal{L}')$ *has at most one point of degree 2 and* $\Delta(P', \mathcal{L}') = r - 1$.

Lemma 4 (Lemma 2.5 in [6]). *Let* $(P, \mathcal{L}) \in \mathcal{I}_r$ $(r \geq 3)$*, then*

$$3(r-2) \leq |\mathcal{L}'| \leq (r-1)^2 - (r-1) + 1 \text{ and } |P'| = (r-1)^2 - (r-1) + 1,$$

and so $\gamma(P', \mathcal{L}') = 1$.

Proposition 1 (Proposition 2.1 and Proposition 2.2 in [1]). *Let* $(P, \mathcal{L})$ *be a linear system with* $|\mathcal{L}| > \nu_2(P, \mathcal{L})$. *If* $\nu_2(P, \mathcal{L}) \in \{2, 3\}$*, then* $\tau(P, \mathcal{L}) \leq \nu_2(P, \mathcal{L}) - 1$.

Theorem 1 (Theorem 2.1 in [8]). *Let* $(P, \mathcal{L})$ *be a linear system and* $p, q \in P$ *be two points such that* $deg(p) = \Delta(P, \mathcal{L})$ *and* $deg(q) = \max\{deg(x) : x \in P \setminus \{p\}\}$. *If* $|\mathcal{L}| \leq deg(p) + deg(q) + \nu_2(P, \mathcal{L}) - 3$*, then* $\tau(P, \mathcal{L}) \leq \nu_2(P, \mathcal{L}) - 1$.

Lemma 5. *Let* $(P, \mathcal{L})$ *be an* r*-uniform intersecting linear system with* $r \geq 2$ *be an even integer. If* $\nu_2(P, \mathcal{L}) = r + 1$ *then* $\tau(P, \mathcal{L}) = \frac{r+2}{2}$.

Proof. Let $(P, \mathcal{L})$ be a r-uniform intersecting linear system with $\nu_2(P, \mathcal{L}) = r + 1$, where $r \geq 2$ is an even integer. Let $R = \{l_1, \ldots, l_{r+1}\}$ be a maximum 2-packing of $(P, \mathcal{L})$. Since $(P, \mathcal{L})$ is an intersecting linear system then $l_i \cap l_j \neq \emptyset$, for $1 \leq i < j \leq r + 1$, hence $|l_i| = r$, for $i = 1, \ldots, r + 1$. Let $l \in \mathcal{L} \setminus R$. Since $(P, \mathcal{L})$ is an intersecting linear system it satisfies $l \cap l_i = l \cap l_i \cap l_{j_i} \neq \emptyset$, for $i = 1, \ldots, r + 1$ and for some $j_i \in \{1, \ldots, r + 1\} \setminus \{i\}$, however, by the pigeonhole principle there are a line $l_s \in R$ such that $l \cap l_s = \emptyset$, since there are an odd number of lines in R and by linearity of $(P, \mathcal{L})$. Therefore $|\mathcal{L}| = |R|$ and $\tau(P, \mathcal{L}) = \lceil \nu_2/2 \rceil = \frac{r+2}{2}$ (see [9]). □

3. Intersecting Linear Systems $(P, \mathcal{L})$ of Rank 5 with $\gamma(P, \mathcal{L}) = 4$

In this section, we give a complete characterization of the linear systems $(P', \mathcal{L}')$ of $(P, \mathcal{L}) \in \mathcal{I}_5$.

Notice that any line $l \in \mathcal{L}'$ satisfies $|l| \geq \nu_2(P', \mathcal{L}') - 1$ (since $(P', \mathcal{L}')$ is intersecting), which implies that. $\nu_2(P', \mathcal{L}') \leq r$. It is clear, by Lemma 2 and Proposition 1, for every $(P, \mathcal{L}) \in \mathcal{I}_5$ satisfies $4 \leq \nu_2(P', \mathcal{L}') \leq 5$. In fact, by the following lemma, Lemma 6, it implies that $\nu_2(P', \mathcal{L}') = 4$.

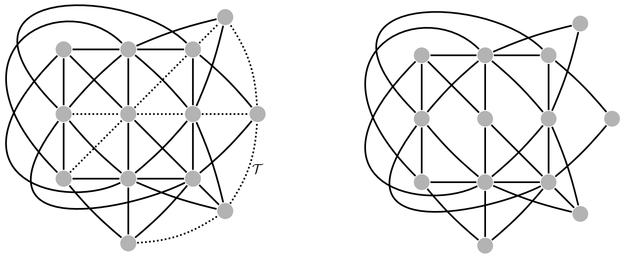
Lemma 6. *Any* $(P, \mathcal{L}) \in \mathcal{I}_5$ *satisfies* $\nu_2(P', \mathcal{L}') = 4$.

Proof. Let $(P, \mathcal{L}) \in \mathcal{I}_5$ and suppose that $\nu_2(P, \mathcal{L}) = 5$. Then, by Lemma 5 it satisfies $\tau(P', \mathcal{L}') = 3$, a contradiction since $\tau(P', \mathcal{L}') = 4$. Therefore $\nu_2(P', \mathcal{L}') = 4$. □

Hence, by the following Theorem 2, if $(P, \mathcal{L}) \in \mathcal{I}_5$ then $(P', \mathcal{L}')$ is a linear subsystem of the projective plane of order 3.

A *finite projective plane* (or merely *projective plane*) is an uniform linear system satisfying that any pair of points have a common line, any pair of lines have a common point and there exist four points in general position (there are not three collinear points). It is well known that if $(P, \mathcal{L})$ is a projective plane then there exists a number $q \in \mathbb{N}$, called *order of projective plane*, such that every point (line, respectively) of $(P, \mathcal{L})$ is incident to exactly $q + 1$ lines (points, respectively), and $(P, \mathcal{L})$ contains exactly $q^2 + q + 1$ points (lines, respectively). In addition to this, it is well known that projective planes of order q, denoted by Π_q, exist when q is a power prime. For more information about the existence and the unicity of projective planes see, for instance, [15, 16].

Theorem 2 (Theorem 2.1 in [1]). *Let* $(P, \mathcal{L})$ *be a linear system with* $|\mathcal{L}| > 4$. *If* $\nu_2(P, \mathcal{L}) = 4$*, then* $\tau(P, \mathcal{L}) \leq 4$. *Moreover, if* $\tau(P, \mathcal{L}) = \nu_2(P, \mathcal{L}) = 4$*, then* $(P, \mathcal{L})$ *is a linear subsystem of* Π_3.



(a) (b)

Figure 1. (a) Projective Plane of Order 3, Π_3 and (b) Linear System Obtained from Π_3 by Deleting the Lines of the Triangle $\mathcal{T}$. Figure obtained from [8]

Araujo-Pardo et al. [1] proved that the linear system $(P, \mathcal{L})$ with $|\mathcal{L}| > 4$ satisfying $\tau(P, \mathcal{L}) = \nu_2(P, \mathcal{L}) = 4$ are a special family of linear subsystems of Π_3.

Given a linear system $(P, \mathcal{L})$, a *triangle* $\mathcal{T}$ of $(P, \mathcal{L})$, is the linear subsystem of $(P, \mathcal{L})$ induced by three points in general position (non collinear) and the three lines induced by them. Consider the projective plane Π_3 and a triangle $\mathcal{T}$ of Π_3 (see (a) of Figure 1). Araujo-Pardo et al. [1] defined the intersecting 4-uniform linear system $\hat{C} = (P_{\hat{C}}, \mathcal{L}_{\hat{C}})$ induced by $\mathcal{L}_{\hat{C}} = \mathcal{L} \setminus \mathcal{T}$ (see (b) of Figure 1). The linear system $\hat{C}$ just defined has thirteen points and ten lines. In the same paper, [1], it was defined $\hat{C}_{4,4}$ to be the family of spanning intersecting 4-uniform linear systems $(P, \mathcal{L})$ such that:

i) $\hat{C}$ is a linear subsystem of $(P, \mathcal{L})$; and
ii) $(P, \mathcal{L})$ is a linear subsystem of Π_3,

Proposition 2 (Proposition 4.1 in [1])**.** *If* $(P, \mathcal{L}) \in \hat{C}_{4,4}$ *then* $\tau(P, \mathcal{L}) = \nu_2(P, \mathcal{L}) = 4$.

It is easy to check that if $(P, \mathcal{L}) \in \hat{C}_{4,4}$ then any point p of degree 4 of $(P, \mathcal{L})$ is a minimum dominating set, which implies $\gamma(P, \mathcal{L}) = 1$. Therefore

Corollary 1. *If* $(P, \mathcal{L}) \in \mathcal{I}_5$ *then* $(P', \mathcal{L}') \in \hat{C}_{4,4}$.

4. Intersecting Linear Systems $(P, \mathcal{L})$ of Rank $q + 2$ with $\gamma(P, \mathcal{L}) = q + 1$

In this section, we prove if q is an odd prime power and $(P, \mathcal{L})$ is an intersecting linear system of rank $(q + 2)$ satisfying $\gamma(P, \mathcal{L}) = q + 1$, then this linear system can be constructed from a spanning $(q+1)$-uniform intersecting linear subsystem $(P', \mathcal{L}')$ of the projective plane of order q, Π_q, satisfying $\tau(P', \mathcal{L}') = \nu_2(P', \mathcal{L}') = q + 1$.

Araujo-Pardo et al. [1] proved if q is an even prime power then $\tau(\Pi_q) \leq \nu_2(\Pi_q) - 1 = q + 1$, however, if q is an odd prime power then $\tau(\Pi_q) = \nu_2(\Pi_q) = q + 1$.

Lemma 7. *Let q be an odd integer. For every* $(P, \mathcal{L}) \in \mathcal{I}_{q+2}$ *it satisfies* $\tau(P', \mathcal{L}') = \nu_2(P', \mathcal{L}') = q + 1$.

Proof. Since $(P', \mathcal{L}')$ is an intersecting $(q+1)$-uniform linear system then $|l'| \geq \nu_2(P', \mathcal{L}') - 1$, for any line $l' \in \mathcal{L}'$. Hence $\nu_2(P', \mathcal{L}') \leq q + 2$. On the other hand, if $\nu_2(P', \mathcal{L}') = q + 2$ then by Lemma 5 it follows that $\tau(P', \mathcal{L}') = \frac{q+3}{2}$, which is a contradiction, since $\tau(P', \mathcal{L}') = q + 1$. Therefore $\nu_2(P', \mathcal{L}') \leq q + 1$. On the other hand, let $p \in P'$ be a point such that $deg(p) = \Delta(P', \mathcal{L}')$ and $\Delta'(P', \mathcal{L}') = \max\{deg(x) : x \in P' \setminus \{p\}\}$. By Theorem 1 if $|\mathcal{L}'| \leq \Delta(P', \mathcal{L}') + \Delta'(P', \mathcal{L}') + \nu_2(P, \mathcal{L}) - 3 \leq 3q$ then $\tau(P', \mathcal{L}') \leq \nu_2(P', \mathcal{L}') - 1$, which implies $q + 2 \leq \nu_2(P', \mathcal{L}')$, a contradiction, since $\nu_2(P', \mathcal{L}') \leq q + 1$. Hence, if $\tau(P', \mathcal{L}') \geq \nu_2(P', \mathcal{L}')$ then $|\mathcal{L}'| \geq \Delta(P', \mathcal{L}') + \Delta'(P', \mathcal{L}') + \nu_2(P, \mathcal{L}) - 2 \geq 3q + 1$, which implies $\nu_2(P', \mathcal{L}') \geq q + 1$. Hence $\nu_2(P', \mathcal{L}') = q + 1$. □

Theorem 3. *Let q be an odd prime power. For every $(P, \mathcal{L}) \in \mathcal{I}_{q+2}$, the linear system $(P', \mathcal{L}')$ is a spanning $(q+1)$-uniform linear subsystem of Π_q such that $\tau(P', \mathcal{L}') = \nu_2(P', \mathcal{L}') = q+1$ with $|\mathcal{L}'| \geq 3q+1$.*

Proof. Let $(P, \mathcal{L}) \in \mathcal{I}_{q+2}$. Then $(P', \mathcal{L}')$ is an $(q+1)$-uniform intersecting linear system with $|P'| = q^2+q+1$ (by Lemma 4). Furthermore, If $|\mathcal{L}'| = q^2+q+1$ then all points of $(P', \mathcal{L}')$ have degree $q+1$ (it is a consequence of Lemma 4, see [6]). Since projective planes are dual systems, the 2-packing number coincides with the cardinality of an oval, which is the maximum number of points in general position (no three of them collinear), and it is equal to $q+1 = \nu_2(P', \mathcal{L}')$ (Lemma 7), when q is odd, see for example [16]. Hence, the linear system $(P', \mathcal{L}')$ is a projective plane of order q, Π_q. Therefore, if $(P, \mathcal{L}) \in \mathcal{I}_{q+2}$ then $(P', \mathcal{L}')$ is a spanning linear subsystem of Π_q satisfying $\tau(P', \mathcal{L}') = \nu_2(P', \mathcal{L}') = q+1$ with $|\mathcal{L}'| \geq 3q+1$ (see proof of Lemma 7). □

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Conflict of Interest

The author declares no conflict of interest.

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