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Article

Total Dominator Colorings of P_4-reducible and P_4-tidy Graphs

Wei Ge[1] **and Jun Yue**[2,*]

[1] Shandong University of Engineering and Vocational Technology, Ji'nan, Shandong, China
[2] School of Mathematics Science, Tiangong University, Tianjin, China

* **Correspondence:** yuejun06@126.com

Abstract: A total dominator coloring of G without isolated vertex is a proper coloring of the vertices of G in which each vertex of G is adjacent to every vertex of some color class. The total dominator chromatic number $\chi_d^t(G)$ of G is the minimum number of colors among all total dominator coloring of G. In this paper, we will give the polynomial time algorithms to computing the total dominator coloring number for P_4-reducible and P_4-tidy graphs.

Keywords: total dominating set, total dominating coloring, P_4-reducible graphs, P_4-tidy graphs
Mathematics Subject Classification: 05C69, 05C75, 05C85

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1. Introduction

Let $G = (V, E)$ be a graph with the vertex set V of order n and the edge set E of size m. The open neighborhood and the closed neighborhood of a vertex $v \in V$ are $N(v) = \{u \in V|uv \in E\}$ and $N[v] = N_G(v) \cup \{v\}$, respectively. The degree of a vertex v is $deg_G(v) = |N_G(v)|$. The minimum and maximum degree of G are denoted by $\delta = \delta(G)$ and $\Delta = \Delta(G)$, respectively. A *dominating set* of a graph G is a vertex subset S in G such that $N_G[S] = V$. The domination number $\gamma(G)$ of G is the cardinality of a minimum dominating set. As a generalization of the dominating set, the *total dominating set* TS of a graph G is a subset of the vertices in G such that $N_G(TS) = V$. The total domination number $\gamma_t(G)$ of G is the cardinality of a minimum total dominating set. The literature on total domination has been surveyed and detailed in the book [1].

A *total dominator coloring* of a graph G without isolated vertex is a proper coloring of G in which each vertex of the graph is adjacent to every vertex of some (other) color class. For convenience, we abbreviated write *TD-coloring* for total dominator coloring. The total dominator chromatic number $\chi_d^t(G)$ of G is the minimum number of color classes in a TD-coloring of G. It was introduced by Kazemi in [2] and studied further, the more details can refer to [3, 4].

A similarly concept has been given by Gera, Horton and Rasmussen [5], which is called *dominator coloring*. A dominator coloring, briefly DC, of a graph G is a proper coloring of G such that every vertex of $V(G)$ dominates all vertices of at least one color class (possibly its own class). The dominator chromatic number $\chi_d(G)$ of G is the minimum number of color classes in a dominator coloring of G. As a consequence result, we have $\chi(G) \leq \chi_d(G) \leq \chi_d^t(G)$ for G without isolate vertex. Since then, many researchers studied on it, the details refer to [6–10].

An induced path on k vertices shall be denoted by P_k. Vertices of degree one (resp. two) in P_k will be called endpoints (resp. midpoints). An induced subgraph of G isomorphic to P_k is simply said to be a P_k in G. A chordless cycle on k vertices is denoted by C_k. A cograph is a graph that does not contain P_4 as an induced subgraph [11]. Several generalizations of cographs have been defined in the literature, such as P_4-sparse [12], P_4-lite [13], P_4-extendible [14] and P_4-reducible graphs [15]. A graph class generalizing all of them is the class of P_4-tidy graphs [16].

The practical applications (to computational semantics, examination scheduling, clustering analysis, group-based cooperation) of these classes of graphs, have certainly motivated the theoretical and algorithmical study. For some NP-hard problem, these classes of graphs have polynomial time algorithms. Bagan et al. [17] proved that determining χ_d^t is NP-hard for general graphs, but polynomial-time solvable for some special graphs, such as cographs and P_4-sparse graphs.

In this paper, we consider the total dominator coloring of graphs. In the next section, we give a polynomial time to compute the total dominator chromatic number of P_4-reducible graphs. In the last section, the algorithm of the total dominator coloring of P_4-tidy graphs is given, which is also a polynomial time algorithm.

2. TD-coloring in P_4-reducible graphs

In this section, we first give some basic lemmas, which will be used in the following section. And then we give a polynomial time algorithm to compute the value of the total dominator chromatic number of the P_4-reducible graph. Lerchs el at. [18] proved that the cographs are precisely the graphs obtained from single-vertex graphs by a finite sequence of $\cup$ and $\vee$ operations defined as follows.

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be arbitrary graphs with $V_1 \cap V_2 = \emptyset$. Now,

– $G_1 \cup G_2$ is the union of G_1 and G_2.

– $G_1 \vee G_2$ is the join of G_1 and G_2.

For the purpose of constructing the P_4-reducible graphs, Jamison el at. [15] defined yet another graph operation, denoted by $\oplus$, as follows. Let the graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)(V_1 \cap V_2 = \emptyset)$ be such that $V_1 = \{a, d\}, E_1 = \emptyset$, and some adjacent vertices b, c in V_2 are adjacent to all the remaining vertices in V_2.

– $G_1 \oplus G_2 = (V_1 \cup V_2, \{ab, cd\} \cup E_2)$.

A graph G is $B - P_4$ if there exists a unique $P_4 = abcd$ in G such that every vertex of G outside $\{a, b, c, d\}$ is adjacent to both b and c and nonadjacent to both a and d.

Proposition 1. *[15] A graph G is P_4-reducible if and only if, G is obtained from single-vertex graphs by a finite sequence of operations $\cup$, $\vee$ and $\oplus$.*

In the same paper, Jamison el at. also gave the following characterization of the P_4-reducible graphs.

Theorem 1. *[15] A graph G is P_4-reducible if, and only if, for every induced subgraph H of G exactly one of the following conditions are satisfied*

(i) H is disconnected;

(ii) $\overline{H}$ is disconnected;

(iii) H is a $B - P_4$ graph.

Before giving the main result, we need a new notation as follows.Given a graph G with a TD-coloring f. Let q_f to be the number of colors used by f. Since f is a TD-coloring, there exists a set of color classes of f such that every vertex of G total dominates at least one color class in that set; and we denote by p_f the size of a smallest such set. Obviously, $p_f < q_f$.

In a graph G, a complete bipartite graph $G[X, Y]$ is called *universal* if every vertex in $V(G) \setminus \{X, Y\}$ is adjacent to every vertex of at least one set of X and Y.

Lemma 1. *Let G be a connected graph,Then*

(i) The following are equivalent.

— G has a TDC f with $p_f = 2$;

— G has a universal complete bipartite graph;

— Every proper coloring of G is a TD-coloring with $p_f = 2$.

(ii) If $\overline{G}$ is not connected, then every proper coloring of G is a TDC and $\chi_d^t = \chi(G)$. Moreover, G must has a universal complete bipartite graph with $p_f = 2$.

Proof. (i) Suppose that G has a TDC f with $p_f = 2$. So there are two color classes X and Y such that every vertex of G total dominates at least one of X and Y. In particular, every vertex of X must dominate every vertex of Y, every vertex of Y must dominate every vertex of X, Thus $G[X, Y]$ is a universal complete bipartite graph. Conversely, if G has a universal complete bipartite graph, then every proper coloring f is obviously a TDC with $p_f = 2$.

(ii) Since $\overline{G}$ is not connected, there is a partition of $V(G)$ into two non-empty sets V_1 and V_2 such that every vertex of V_1 is adjacent in G to every vertex of V_2. Let f be any proper coloring of G. Then some color α appears only in V_1 and some color β appears only in V_2. So every vertex of V_1 totally dominates color β,and every vertex of V_2 totally dominates color α, Thus f is a TDC. and $\chi_d^t = \chi(G)$. Moreover, the same argument shows that $p_f \leq 2$, and every TDC f of G satisfies $p_f \geq 2$, then G must has a universal complete bipartite graph with $p_f = 2$. □

Lemma 2. *Let G be a non-connected graph with components $G_1, \ldots, G_k$, $k \geq 2$. Then*

$$\chi_d^t = min\left\{\sum_{i=1}^{k} p_{f_i} + \max_{1\leq i\leq k}\{q_{f_i} - p_{f_i}\}\right\},$$

where the minimum is over all choices of a TD-coloring f_i for each G_i.

Proof. Let f be any TD-coloring of G. For $i = 1, \ldots, k$, any color class whose vertices are all in G_i will be called a *private color* of G_i. Let p_i be the number of private colors in G_i. Let f_i be the coloring induced by f on G_i. Consider any vertex v of G_i. Since f is a TD-coloring, v totally dominates a color class, which must therefore be a private color of G_i. By the same argument , f_i is a TD-coloring of G_i. Then $p_i \geq p_{f_i}$. Moreover, if R_i denotes the set of vertices of all private colors of G_i, then f uses $q_{f_i} - p_i$ colors on the vertices of $G_i \setminus R_i$. Such colors can be used on several components. So f uses at least

$$\sum_{i=1}^{k} p_i + \max_{1\leq i\leq k}\{q_{f_i} - p_i\}$$

colors. This value is larger than or equal to

$$\sum_{i=1}^{k} p_{f_i} + \max_{1\leq i\leq k}\{q_{f_i} - p_{f_i}\}.$$

Because replacing p_i by p_{f_i} can only decrease the sum, and if it increases the max then it decreases the sum correspondingly. On the other hand, we can turn f into a TD-coloring f' with a number of colors equal to the second displayed value. Indeed, let $q = \max_{1\leq i\leq k}\{q_{f_i} - p_{f_i}\}$. It suffices to use in each G_i precisely p_{c_i} private colors and to rename $1, \ldots, q_{f_i} - p_{f_i}$ the other colors of f_i. Since f is any TD-coloring, it follows that χ_d^t is equal to the minimum of this value. □

If the graph G is constructed by operation $\oplus$, then we give the following lemma.

Lemma 3. *Let G be a connected $B - P_4$ graph, and let G^* be the vertex-induced subgraph by the outside of $\{a, b, c, d\}$. Then*

$$\chi_d^t(G) = 2 + \chi(G^*).$$

Moreover, every TD-coloring of G has a universal bipartite graph with $p_f = 2$.

Proof. Color G^* using $\chi(G^*)$ such that G^* is proper coloring and color a, b with the two colors in $\chi(G^*)$, and then color b and c with two different new colors, respectively. For any vertex $v \in V(G)$, v must be adjacent to one of $\{b, c\}$, since G is a $B - P_4$ graph. This is to say, every vertex in G total dominates a color set. Then $\chi_d^t(G) \leq 2 + \chi(G^*)$. Suppose that $\chi_d^t(G) \leq 1 + \chi(G^*)$. Then there must be two adjacent vertices b, c in G have the same color by pigeonhole principle, a contradiction. So the lemma holds. □

Combining the above lemmas, we give the following theorem.

Theorem 2. *Let G be a P_4-reducible graph.*
(i) If G is $B - P_4$ graph and G^* defined as Lemma 3, then $\chi_d^t(G) = 2 + \chi(G^*)$;
(ii) If G is connected and not a $B - P_4$ graph, then $\chi_d^t = \chi(G)$;
(iii) If G is not connected, and let k (≥ 2) be the number of components in G, then

$$\chi_d^t(G) = \chi(G) + 2k - 2.$$

Proof. (i) From the Lemma 3, we have $\chi_d^t(G) = 2 + \chi(G^*)$.

(ii) Let G be a connected graph and not a $B - P_4$ graph. Then $\overline{G}$ is not connected by Theorem 1. From Lemma 1 (ii), we have $\chi_d^t(G) = \chi(G)$.

(iii) Suppose that G is not connected. Let $G_1, \ldots, G_k$ be the components of G. By Lemma 2, we have

$$\chi_d^t = min\left\{\sum_{i=1}^{k} p_{f_i} + \max_{1\leq i\leq k}\{q_{f_i} - p_{f_i}\}\right\},$$

for some appropriate choice of a TD-coloring f_i for each G_i. Consider any component G_i of G. If G_i has a universal bipartite graph, then we have $p_{f_i} = 2$ (whatever the choice of f_i) since Lemma 1. If G_i has no universal bipartite graph, then $\overline{G_i}$ is not connected from the Lemma 1. And further we have $p_{f_i} = 2$ (whatever the choice of f_i again) by Lemma 1. It follows that in the formula above, the sum term is equal to $2k$. Moreover, we may assume that $q_{f_i} = \chi(G_i)$ for each i since this is the best way to minimize the value in the formula. More precisely, if some component G_i of G has a universal bipartite graph and satisfies $\chi(G_i) = \chi(G)$, then the max term in the formula is equal to $\chi(G) - 2$, so we obtain the conclusion. □

3. TD-coloring in P_4-tidy graphs

In this section, we mainly study on the total dominator coloring of the P_4-tidy graphs. Now, we begin with some preliminaries. Let $G = (V, E)$ be a graph. Set $F = \{e \in E | e$ belongs to an induced P_4 of $G\}$ and $G_p = (V, F)$. A connected component of G_p having exactly one vertex is called a weak vertex. Any connected component of G_p distinct from a weak vertex is called a p-component of G. A graph G is p-connected if it has only one p-component and no weak vertices.

An p-connected graph $G = (V, E)$ is p-separable if V can be partitioned into two sets (C, S) such that each P_4 that contains vertices from C and from S has its midpoints in C and its endpoints in S. we will call it a p-partition. An urchin (resp. starfish) of size k (≥ 2), is a p-separable graph with p-partition (C, S), where $C = \{c_1, \ldots, c_k\}$ is a clique; $S = \{s_1, \ldots, s_k\}$ is a stable set; s_i is adjacent to c_i if and only if $i = j$(resp. $i \neq j$). A quasi-urchin (resp. starfish) of size k is a graph obtained from an urchin (resp.starfish) of size k by replacing at most one vertex by K_2 or S_2. Note that the new vertices result in true or false twins, respectively, and they are in the same set of the new p-partiton (C^*, S^*). The elements of S^* are called legs and C^* is called the body of the quasi-starfish or quasi-urchin.

Note that there are five possible quasi-starfishes of size two, and they are also the five possible quasi-urchins of size two: P_4, P, $\bar{P}$, fork and kite (see Figure 1). To avoid ambiguity, we will consider these five graphs as quasi-starfishes, while quasi-urchins will be always of size at least three.

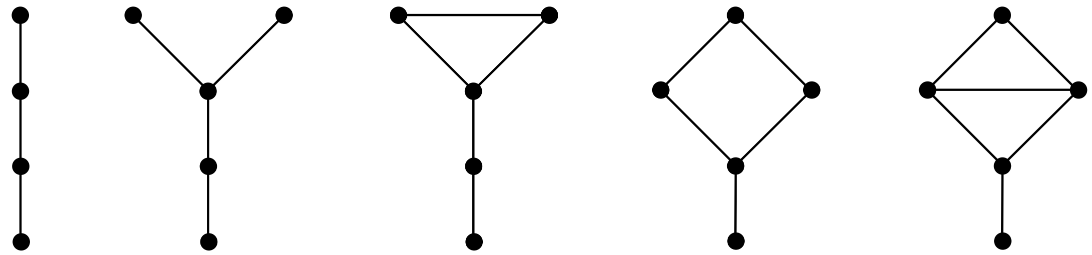

Figure 1. Possible quasi-starfishes of size two. From left to right: P_4, fork, P,$\bar{P}$ and kite.

When considering quasi-urchins and quasi-starfishes, we have ten kinds of them. We will call Type 1 (resp. Type 2) the urchins (resp. starfish); Type 3 (resp. Type 4) the urchins (resp. starfish), where a vertex in the body was replaced by K_2; Type 5 (resp. Type 6) the urchins (resp. starfish), where a vertex in the body was replaced by S_2; Type 7 (resp. Type 8) the urchins (resp. starfish), where a leg was replaced by K_2; and Type 9 (resp. Type 10) the urchins (resp. starfish), where a leg was replaced by S_2. To avoid ambiguity, we will let the graph of odd type have size at least three.

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs with $V_1 \cap V_2 = \emptyset$, such that G_1 is p-separable with partition (V_1^1, V_1^2). Consider the graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2 \cup \{xy | x \in V_1^1, y \in V_2\}$. we shall denote this graph by $G_1 \underline{\vee} G_2$.

Proposition 2. *[16] A graph G is P_4-tidy if and only if every p-component is isomorphic to either P_5 or $\overline{P_5}$ or C_5 or a quasi-starfish or a quasi-urchin. Quasi-starfishes or quasi-urchins are the p-separable p-components of G.*

Lemma 4. *Let G be a quasi-starfish or quasi-urchin of size k. Then*
(1) If G is type $1, 2, 5, 6, 7, 9$ *or* 10*, then* $\chi(G) = k$.
(2) If G is type $3, 4$ *or* 8*, then* $\chi(G) = k + 1$.

Proof. Since a proper coloring of the maximum clique in G can be extended to G without adding other colors, then the results are hold. □

In [19], Corneil el at. gave the chromatic number of the union and join graph operations.

Theorem 3. *[19] If G is the trivial graph, then* $\chi(G) = 1$. *Let* $G_1 = (V_1, E_1)$ *and* $G_2 = (V_2, E_2)$ *be two graphs such that* $V_1 \cap V_2 = \emptyset$. *Then,*
i. $\chi(G_1 \cup G_2) = max\{\chi(G_1), \chi(G_2)\}$;
ii. $\chi(G_1 \vee G_2) = \chi(G_1) + \chi(G_2)$.

Lemma 5. *Let G be a quasi-starfish or quasi-urchin of size k. Then*
(1) If G is type $1, 2, 5, 6, 9$ *or* 10*, then* $\chi_d^t(G) = k + 1$;
(2) If G is type $3, 4, 7$ *or* 8*, then* $\chi_d^t(G) = k + 2$.

Proof. (1) By the configuration of G, let (C^*, S^*) be the p-partition of G. Now we give a coloring f of G as follows. If G is in type 1 (resp. type 2, type 9 or type 10), then color every vertex of C^* a new color and color all vertices of S^* another new color. Then f is a TD-coloring in G since every vertex is total dominator a color set, and $\chi_d^t(G) \leq k + 1$. If G is in type 5 (resp. type 6), then color each S_2 with a different colors and color all the vertices in S^* another new colors. Then f is a TD-coloring of G and $\chi_d^t(G) \leq k + 1$.

Suppose $\chi_d^t(G) = k$. We obtain $\chi_d^t(C^*) = k$, since C^* is a clique. Then there must be two vertices $c_i (\in C^*)$, $s_j (\in S^*)$ such that they have been colored by the same color and they are not adjacent. Let $s_i (\in S^*)$ is adjacent to c_i, we can get c_i is not adjacent to all the vertices in any color, a contradiction. Then $\chi_d^t(G) = k + 1$.

(2) Since the proof of type 3 and type 4 are similar to type 1, we limit them. Now assume that G is in type 7 (resp. type 8). Let s_1, s_1' be two adjacent vertices in S^*. We give the following coloring

f of G. Color each vertex in C^* a new color; color the vertices s_1, s_1' two new colors and color the remained vertex in S^* by the same color of s_1 or s_1'. Then f is a TD-coloring of G with $|f| = k + 2$. Then $\chi_d^t(G) \le k + 2$.

Suppose $\chi_d^t(G) = k + 1$. Since C^* is a clique of size k, then color C^* must use k colors. All vertices of S^* do not color the same color, since s_1 or s_1' is adjacent. Then either s_1 or s_1' must have been colored by the same color of a vertex c_i in C^*. Then there are must be a vertex in $S^* \setminus \{s_1, s_1'\}$ that not dominate the color class which contains s_1, a contradiction. Then $\chi_d^t(G) = k + 2$.

□

A *redundant color* of G is a color in a total dominating coloring of G that no vertex in G total dominating the vertices in this color. Let $r(G)$ be the number of redundant colors in a total dominating coloring of G. Now, we give the lemmas as follows.

Lemma 6. *(1) Let $G = G_1 \cup G_2$. Then*

$$\chi_d^t(G) = \chi_d^t(G_1) + \chi_d^t(G_2) - min\{r(G_1), r(G_2)\}.$$

(2) Let $G = G_1 \vee G_2$. Then

$$\chi_d^t(G) = \chi(G) = \chi(G_1) + \chi(G_2).$$

Proof. (1)Let f_i be a total dominator coloring of G_i with $r(G_i)$ redundant colors, where $i \in \{1, 2\}$. Without loss of generality, we assume that $r(G_1) \le r(G_2)$. Now, we define the coloring f in G as follows. Color G_i as f_i and then using $r(G_1)$ redundant colors to repeat some redundant colors in G_2. Because f_1 and f_2 are total dominator coloring of G_1 and G_2, respective. Then f is total dominator coloring of G and then $\chi_d^t(G) = \chi_d^t(G_1) + \chi_d^t(G_2) - r(G_1)$. On the other hand, suppose that $\chi_d^t(G) \le \chi_d^t(G_1) + \chi_d^t(G_2) - \min\{r(G_1), r(G_2)\} - 1$. By Pigeonhole Principle, there are some vertex which is not total dominating. Then the result holds.

(2) By Theorem 3 (ii), $\chi(G) = \chi(G_1) + \chi(G_2)$. Let f be a proper coloring of G using $\chi(G)$ colors. Since $G = G_1 \vee G_2$ contains a complete bipartite graph as a induced subgraph, then f is a total dominator coloring and $\chi_d^t(G) \le \chi(G)$. So $\chi_d^t(G) = \chi(G) = \chi(G_1) + \chi(G_2)$, since $\chi_d^t(G) \ge \chi(G)$. □

Note that we can compute the number of redundant color in a total dominator coloring in polynomial time.

Lemma 7. *Let $G_1 = (V_1, E_1)$ be a p-separable p_4-tidy graph, and $G_2 = (V_2, E_2)$ a graph such that $V_1 \cap V_2 = \emptyset$. Then,*

(1) If G is not type 7, then $\chi_d^t(G_1 \underline{\vee} G_2) = \chi(G_1) + \chi(G_2) + 1$;

(2) If G is type 7, then $\chi_d^t(G_1 \underline{\vee} G_2) = \chi(G_1) + \chi(G_2) + 2$.

Proof. Let $G = G_1 \underline{\vee} G_2$. By Proposition 2, G_1 is a quasi-urchin or a quasi-starfish and set (C^*, S^*) be its p-partition. Then G contains $G_1[C^*] \vee G_2$ as an induced subgraph. Thus $\chi_d^t(G) \ge \chi_d^t(G_1[C^*] \vee G_2)$. On the other hand, if G is type 7 or type 8, a TDC of $G_1[C^*] \vee G_2$ can be extended to G by adding two new colors. Hence $\chi_d^t(G) = \chi_d^t(G_1[C^*] \vee G_2) + 2$; if G is not type 7 or type 8, a TDC of $G_1[C^*] \vee G_2$ can be extended to G by adding one new colors. Hence $\chi_d^t(G) = \chi_d^t(G_1[C^*] \vee G_2) + 1$. By Lemma 6, we get $\chi_d^t(G_1 \vee G_2) = \chi(G_1) + \chi(G_2)$, so by Lemma 4, we can know if G_1 is type 8, then $\chi(G_1[C^*]) = \chi(G_1) - 1$; otherwise, $\chi(G_1[C^*]) = \chi(G_1)$.

Combining the above, we can get that if G is type 7, then $\chi_d^t(G) = \chi(G_1) + \chi(G_2) + 2$; otherwise, $\chi_d^t(G) = \chi(G_1) + \chi(G_2) + 1$. □

Theorem 4. *[20] Every graph G either is p-connected or can be obtained uniquely from its p-components and weak vertices by a finite sequence of $\cup$, $\vee$, $\underline{\vee}$ operations.*

Combining the above lemmas and theorems, we get the following theorem.

Theorem 5. *Total dominator chromatic number of P_4-tidy can be computed in polynomial time on P_4-tidy graphs.*

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Conflict of Interest

The authors declares no conflict of interests.

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