On the support size of 3-designs with repeated blocks

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Abstract. The set of all distinct blocks of a $t-(v,k)$ design is referred to as the support of the design, and its cardinality is denoted by b^*. By generalizing a method on BIB designs called "trade off" to 3-designs, a table for $3-(9,4)$ designs with each $60 \leq b^* \leq 126 = \binom{9}{4}$ is constructed. Also we have produced over 2500 non-isomorphic $3-(9,4)$ designs with $\lambda_3 = 6$. By utilizing this generalized trade off method along with two other methods, a table for $3-(10,4)$ designs with 156 different b^*'s is constructed. By a recursive lower bound on the minimum value of b^* in all $t-(v,k)$ designs, it is shown that $b^*_{\min}[3-(9,4)] \geq 36$, and $b^*_{\min}[3-(10,4)] = 30$.

Introduction

Let $V = \{1,2,\ldots,v\}$ and let $v\sum k$ be the collection of all distinct subsets of V of size k. A t-design (or more specifically a $t-(v,k)$ *design*) with parameters v, k and λ is a collection of b, not necessarily distinct, elements of $v\sum k$, referred to as blocks, with the property that every element of $v\sum t$ occurs in exactly λ blocks. The number of distinct blocks of a t-design is called the *support size* of the design and is denoted by b^*.

A (v,k,t) *trade* or simply a t-trade of volume s consists of two disjoint collections T_1 and T_2, each of s blocks, such that for every element of $v\sum t$, the number of blocks containing this element is the same in both T_1 and T_2.

For a given set of parameters v, k and t, it is an open question to determine possible support sizes of all $t-(v,k)$ designs. See for example [6] page 278. There are several papers in the literature which deal with this question for the cases of $t = 2$ (BIB designs). Here we apply three methods that have been used for BIBD's, to $3-(9,4)$ and $3-(10,4)$ designs. It can easily be seen that there is no nontrivial $3-(9,4)$ design whose blocks are all distinct. (A design is *trivial* if $v\sum k$ constitutes its blocks.) On the other hand it is known that there exists a nontrivial $3-(10,4)$ design whose blocks are all distinct. In fact, there exists a $3-(10,4)$ design with $b = 30$.

The only other nontrivial case for 3-designs with $v \leq 10$ is $3-(8,4)$ designs, which are studied in [10]. The rest of this section will be denoted to the discussion of the methods used to construct $3-(9,4)$ and $3-(10,4)$ designs.

First, the elementary *method of composition*, in which we consider the union of the collections of the blocks of two given t-designs to produce a new t-design, with a possible different b^*. Second, the *method of reducing support size*, which is discussed in [7] and a computer algorithm is given there. Third, the *method of t-trade off* which is discussed in [3] and is applied to BIB designs with $v = 8$ and $k = 4$. This method can easily be generalized for t-designs.

For v, k and t, consider the following polynomial

$$(x_1 - x_2)(x_3 - x_4)\ldots(x_{2t+1} - x_{2t+2})x_{2t+3}\ldots x_{k+t+1}.$$

If we multiply the factors, and identify each x_i with i, the i^{th} element in V, then the resulting expression forms a (v, k, t) trade. For example, in the case of $t = 2$ and $k = 4$ we obtain the following set of blocks:

T_1 :	1357	1467	2367	2457
T_2 :	2467	2357	1457	1367

By applying certain permutations on this (v, k, t) trade, Graham, Li, and Li [1] have obtained a basis for the space of (v, k, t) trades.

In t-trade off method, for a given t-design, we construct a t-trade in such a way that either the blocks in T_1 or the blocks in T_2 are in the given design. Then by replacing the blocks of T_1 (or T_2) by the blocks of T_2 (or T_1, respectively) in the given design, we obtain another design with possibly different b^*. Based on this idea we have utilized a computer algorithm for 3-trade off method.

Minimum support size

For each set of parameters v, k, and t, it is also an open question to determine $b^*_{\min}$, the minimum possible value of the support sizes of $t-(v, k)$ designs. For some special values of v, k, and t, this question has been answered. The value $\binom{v}{t}$ is settled for the upper bound of $b^*_{\min}$. But as is mentioned in [6, page 278], for almost all v, k, and t, the actual value is far below $\binom{v}{t}$. In [6] the "hard" case of $v = 8$, $k = 3$, and $t = 2$ is mentioned for an evidence of this fact, in which there is no nontrivial design without repeated block. The case $v = 9$, $k = 4$ and $t = 3$ is another "hard" case, and as it is demonstrated in Table 1, $b^*_{\min} \leq 60$, which is again far below $\binom{9}{3} = 84$.

Next we prove a lemma which gives a recursive bound for $b^*_{\min}$.

Lemma. *Let* $b^*_{\min}[t-(k, v)]$ *be the minimum possible value of the support sizes of all* $t-(k, v)$ *designs. Then*

$$b^*_{\min}[(t-1)-(v-1, k-1)] + b^*_{\min}[(t-1)-(v-1, k)] \leq b^*_{\min}[t-(v, k)]$$

Proof: Suppose D is a $t-(v, k)$ design based on the set V with support size $b^*[t-(v, k)]$. We denote the collection of blocks of D by $\mathcal{B}(D)$. With respect to any element $x \in V$, we may obtain from D, the following $(t-1)$-designs D_1 and D_2,

$$\mathcal{B}(D_1) = \{B | B \in \mathcal{B}(D) \text{ and } x \notin B\},$$
$$\mathcal{B}(D_2) = \{B - \{x\} | B \in \mathcal{B}(D) \text{ and } x \in B\}.$$

Then D_1 is a $(t-1)-(v-1,k)$ design, while D_2 is a $(t-1)-(v-1,k-1)$ design. The support size of D is equal to the sum of the support sizes of D_1 and D_2. This implies the statement of the lemma.

Corollary 1. $b^*_{\min}[3-(9,4)] \geq 36$.

Proof: It follows from the previous lemma and the fact that $b^*_{\min}[2-(8,4)] = 14$ (see [3]), and $b^*_{\min}[2-(8,3)] = 22$ (see [4]).

Corollary 2. $b^*_{\min}[3-(10,4)] = 30$.

Proof: We have $b^*_{\min}[2-(9,3)] = 12$ (see for example [9]), and $b^*_{\min}[2-(9,4)] = 18$ (see [8]). Therefore the statement follows by the above lemma and the fact that a $3-(10,4)$ design with $b^* = 30$ (a Steiner system $S(3,4,10)$) exists (see Table 2).

$3-(9,4)$ designs

Let D be a $t-(v,k)$ design based on the set of elements V. It is well known that for each s $(1 \leq s \leq t)$, every element of $v \sum s$ occurs in the same number of blocks. We denote this number by λ_s, and let $\lambda_0 = b$ and $\lambda_t = \lambda$. By an elementary counting argument we have,

$$\lambda_s = \lambda \frac{\binom{v-s}{t-s}}{\binom{k-s}{t-s}} \quad 0 \leq s \leq t. \tag{1}$$

Therefore, for the existence of a $t-(v,k)$ design with parameter λ, the basic necessary conditions are that, for each s, the value λ_s given in (1) be an integer. Thus, for $3-(9,4)$ designs, the minimum possible value for λ_s is 6, and the minimum number of blocks is $\binom{9}{4} = 126$. By applying the method of 3-trade off on trivial design we have obtained all $3-(9,4)$ designs with $60 \leq b^* \leq 126$ and $b = 126$, except for $B^* \in A \cup B$, whre $A = \{117, 119, 120, \ldots, 125\}$ and $B = \{113, 115, 116\}$. The designs with $b = 126$ and $b^* \in A$ do not exist because it is proven that there exists no (v,k,t) trade with volume equal to $1, 2, \ldots, 2^t - 1$, and $2^t + 1$ (see [5]). In Table 1 we have constructed designs with $b = 2(126) = 252$ for each $b^* \in A \cup B$.

As the designs in Table 1 indicate, $3-(9,4)$ designs exist for each $60 \leq b^* \leq 126$, whose number of blocks b, is given in the table. Besides this, all $3-(9,4)$ designs with $60 \leq b^* \leq 126$ exist for any possible $b > 126$. For, if $60 \leq b^* \leq 126$ and $b^* \notin A \cup B$, then one may take l copies of the design listed in Table 1 with the given b^*, and obtain a design with $b = 126\,l$. If a design with $b^* \in A \cup B$ and with a larger b is desired, then one note that the design with $b^* = 61$ is embedded in the designs with $b^* \in A \cup B$ in Table 1. Thus by adding copies of this subdesign to the design with desired b^*, one can increase the number of blocks without altering the support size. We should mention that how the designs can be reconstructed from the entries in Table 1. The first four lines list the blocks vertically, in lexicographical order. And that x in the column beneath the block indicates that that block occurs x times in the design, i.e. the frequency of that block. Each design with a given b and b^* is listed in a row, with its block frequencies.

Note: With a long search by the 3-trade off method and the method of reducing support sizes of designs, we were unable to find designs with $b^* \in B$ and $b = 126$. A similar result was observed in the case of $3-(10,4)$ designs. This fact indicates that there might not be any 3-trade of volume 10, 11, or 13. This conjecture is under investigation.

The method of t-trade off is very useful in constructing non-isomorphic designs. Utilizing this method, by a computer program, we have obtained over 2500 non-isomorphic $3-(9,4)$ designs with $\lambda_3 = 6$ (see [12]). The previous known lower bound on this was 50 (see [2]).

$3-(10,4)$ designs

In [11], it is shown that there exist five mutually disjoint $S(3,4,10)$, i.e. $3-(10,4)$ designs with $\lambda_3 = 1$ ($b = 30$).

In Table 2, we have demonstrated the result. In this Table first we introduce an $S(3,4,10)$. Then we present all the designs which may be found by composition. There, we list the value of b and b^* for the desired design, then b^* of a design D, followed by the image of a permutation on ten elements. If we apply this premutation on the design whose b^* is given in the last column and consider the collection of blocks of the resulting design together with the blocks of D, we obtain the desired design.

In the last part of Table 2, we list all the designs which are found by t-trade off method. In this part we list the value of b and b^* of the desired design, then the support size of a design D, that can be traded to obtain the desired design. Needed trade is introduced in the polynomial notation. The list of all $3-(10,4)$ designs is available from the author.

Note: With a long search by computer, we were not able to find $3-(10,4)$ designs with $b^* \in C \cup E$, where $C = \{31, \ldots, 45, 47, 49, 50, 51, 53, 55, 57, 59\}$ and $E = \{61, 63\}$. This raises an interesting question: Following [13] we denote,

$J(v) = \{k|$ there exists a pair of $3-(v,4)$ designs with $\lambda_3 = 1$, having exactly k quadruples in common$\}$ and

$R(v) = \{k |$ there exists a $3-(v,4)$ design with $\lambda_3 = 2$ and having exactly k repeated blocks$\}$.

Rosa and Hoffman [13] have shown that, in the case of Steiner triple systems $J(v) = R(v)$. Now the problem is to prove similar result in the case of $3-(v,4)$ designs. This is the fact in the case of $3-(8,4)$ designs [10].

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Appendix

Table 1

$3-$ designs with $v = 9, k = 4$

			111111111111111111111111111111111111111111111111111111112222222222222222222222222222222222233333333333333333333444444444455556
			222222222222222222222333333333333333444444444455555566673333333333333334444444444555555666744444444445555556667555555666766677
			333333444445555666778444445555666778555566677866677877884444455556667785555666778666778778855556667786667787788666778778877888
			456789567896789789899567896789789899678978989978989989995678967897898996789789899789899899967897898997898998999789899899989999
NO.	BI	B	
1	60	126	0014010500100330002010103230030102000600000000303000015115001000101220120010001331411020102000103110302013110021024002300000031
2	61	126	120003000230400240000004100040213000500100120100102211031400020010005110140200030103002012102030200032200200211001210222030110
3	62	126	120003000230400240000004100040213000400201120010101201041400020010005110140200030103002012112020100041200301210001210222030110
4	63	126	010014301200020402100112111300031100100103103020210300031410020020005000200101014040040012000311000222100120320104300300010012
5	64	126	010014303000020222100110311300211100100103103020210300031410020020005000020101014220040012002111000220300120320104300300010012
6	65	126	001014303000030212100110311400111100100103103020200310032310020020005000010111014220040012001112000220300120320104400200010012
7	66	126	002013303000030112110100321400111100100113102020200310032310010030005000010111014320030012011102000220300120320004410200010012
8	67	126	031101050010210000213010320003310200042000002103300012111131001100220120014000301231010130030200010302012111021201001313000040
9	68	126	230001002200003240020003101110212003400201101200150002011200120000025110320013011310003010100322110111200401010101110300201302
10	69	126	202011301000030112130000132400110102110113121010200311010310010050005000210111014320010012011402000210200100330004110200120031
11	70	126	1111111111111111111111111111111111111111111111111111111100500000505000000050005000500000000055000000000500050005000000050500000500
12	71	126	101121203000004140010003112112111001400001100400050004012200112100022110020013211410002010200312110111110202012101140300000401
13	72	126	100221311000021113110000144200110101111012121010002311110320010050202000210011014320100020211402000110002100331004020300120210
14	73	126	050010101130000321111003211000104201310112000102023201010420000100200420122011200303201001111220010022111213000110110131200202
15	74	126	011121203010003140010003212102111001400001100300050104011400102110012110030002211411001011201311010111210102112101040401000310
16	75	126	020022321000100112211103023000030120001102220130011402000312002023000021011000132032210111001311020010122101112311020201100112
17	76	126	010203311100011131110001324100110101111010311010012311110420000230202000210001014312100020211221000111002100331022020400110210
18	77	126	010122321000101112201103023010030020001102220130011302011302002023000021010000142032210111001211021010122101112311021201000112
19	78	126	020103212100011131110001323100210101211000311010012311110420000220202100211001004312100020211221000111012100231021020410110210
20	79	126	020022321000100112211103022010130020001102220130111301010312002023000021011000132032210111011210021010122101112310021211000112
21	80	126	110220300021001221111003112120004001200122010201023111010320010130200210120012211301201011111220110022001103102111120120100401
22	81	126	010032321000200112111103023010120020001102220120111311010312002033000011011000132032200111111210021010112101122310021211001111
23	82	126	100221400011001221111012112220003001100122011201122111010230020130200110020013211300211011111220110021001103202211120110100401
24	83	126	010221400021000221111012212210003001100122011101122211010330010130200110020012211301211011111220010021101103202211120111100301
25	84	126	001032321000201111111103023020110020001102220120111211111212002033000011010001132032200111111110121010112101122310022210001111
26	85	126	011121203010012131010012212102101101111102200121020102031400102110012112010000213211111121001311000211210102121102120311000220
27	86	126	0212012111100201210210112211111111012110101111102121201104011100300004010211010121311101022003211000111300100321013001311110030
28	87	126	011211400110001221111012122120002101110022001201222111010231010130200111010003211310211011111220110012001102212111121210100301
29	88	126	0002223300001111111110302302011100100110212112011211111121201103210001011000113212220011111110121010102201212311012210000211
30	89	126	110031212001002211111101122200121020110102220111020302011210112012011021020011122021220111000312111100121111012301021101111102
31	90	126	102021300110201211111003023120001111101123001100221111111220001131101111010003311230011011111210210021011102211201121110110211
32	91	126	01022123001011111111103022020112001101102021120112111111121201003210011021000112212220011111110121011102201112211012220000211
33	92	126	020202211110011131110011222110111101211010111110112211110321010120201110211002102311101020211221000111012101221021020311110210
34	93	126	202011200110211211111002023120001211111123001100211111111120001131201112010003311220011011111211210021011101211101121110120211

Table 1 continued

35	94	126	0410010131100111211110113101012111113111001011111211111113101000101141111210100123111110210022110101123101101110021104111100121
36	95	126	0201211310101111111111031120101120111012020111201211111111112110023100111210001112022210111111110131001102211012210012220101201
37	96	126	0011313110111011111111111212111110101100021211111103120112201210210010200201111120212201210003111111011111101122302021101110111
38	97	126	0101312210111011111111111212011210101110011211111103120112201210110110200211101120212201210003111111011111111112302020102110111
39	98	126	100221310011101121111102112111112001101021111201122111010210221211100110120111211102111111111120212001001111212211021011120201
40	99	126	0111212120110111111110122121021011011111021011110211121111310102110111111011010212211111121001311010111210102112202110211100220
41	100	126	0031111301110111111111112021130001111100113011120211111111112201103010111101010311122011111111111111100211111022111111221110100211
42	101	126	00013222101210111011111211111211111010011101121111110212112120121011111010020111112011221121000211111111111111112312020101110111
43	102	126	0212012111100111211110112211111111012110101111111121111103111110120102110211011111311101021112211000111111101221012011311110120
44	103	126	200031111012201111111110111211211020111111121101110211111120021021111111021111112010211121001211121111111011012211020101112111
45	104	126	0121112111101111111111120211200111111011120111202111111111220100311011111100031112201111111111111110021112101211111121210100211
46	105	126	0021213011111011111111121111121100111010112111111211111111112121111001110102111110122111111111110122001111011212210032101111101
47	106	126	021111121110111111111112021020111111111102011120211111111220100211111111111002111220111111111111110021112201111111111220100211
48	107	126	1100311210112011111111111111111211020111101121111110211111120121011111111102111011201121111210012111121011111111101221102011111111
49	108	126	1011212011121011111111112011121111011101112111110121111111111111112210011111011211101121111111111110122011111101112210022110111201
50	109	126	2100210210112111111111111110111121102111120112111111011111111110121011112111112111011201111121001211121111111111011211010111111121
51	110	126	021111112111011111111111211101111111111111101111121111113101011101121111110101122111111210012110111112210111111112110211100121
52	111	126	101121211011111111111111111121111101110111112111111111111111111122100111110111111201221011111111101221011110111112210022111011111
53	112	126	0111212111111011111111111211111111110111011111111111112111111111121111121001111101111110122111111111110122001111111111221002211111101
54	113	252	1243023321222210124141213431321203310512340013124212251132231103322123212301142324220424120221331231141132102611441122220420142
55	114	126	11111111111111111111111111120110211111111111110211112011111111111112011021111111111111021111201111111111111110121112101111111111210101211
56	115	252	412302231111233231223121221333124401421202144242001314212024041311231422321304124113231501112155310223212111124032411222113332
57	116	252	123222152122322011323120442214311410153131002304311124324241002221331231024112413441121241131301221514122132132214221413111151
58	117	252	033222162121322011323121442114311410153121002314311124324241102211331231024111413442121241131301221414122232132214221423110151
59	118	126	11111111111111111111111111111111111111111111111111111111210111110112111111110112111111121001211111112110111111112110111110121
60	119	252	321231412114011411135123212106232311132113412313152113113211213301341410113234511322233201131313111343123122122332132210202315
61	120	252	2322120612123221013241204321144114111631111042043111333131411112211431231125112412341121241131312221413123221132313102413212151
62	121	252	0423212422202221232222233220132222222222211222243122222222222332112222222222222211222332222222222232122202222242222221232224222 02222
63	122	252	32221222122222222223220222332322212222422222212122122232232220023222422221222222242422122202232132212232222212222122312232122232
64	123	252	13231224122023222222222232220322222222231202232242212222222222322212222221322222211232233221222222222222122222331222222222213332111232
65	124	252	12223203422323211212232123122232222132221212222122223232322222221222222222321221222122224222121232231224222202232222222232222221
66	125	252	03223233222121222222222242221222222122212212222232232222222222233221232112222221212222133322222222222212321122232222223321133322212212
67	126	126	111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111

Table 2

A 3-design with $v = 10, k = 4$

B^*	B	
		111111111111222222223333344456
		222233344556333445584455756767
		346746958787467566795668878998
		5A9878A699AA8A9978AAA979A8AAA9
30	30	111111111111111111111111111111

3-designs with $v = 10, k = 4$

B	B^*	= B^*	+ Permutation 1	2	3	4	5	6	7	8	9	A	on B^*
60	46	30	3	9	A	4	1	6	7	2	8	5	30
60	48	30	6	1	2	7	5	4	8	9	3	A	30
60	52	30	2	6	5	4	9	7	1	8	3	A	30
60	54	30	2	5	6	A	9	7	3	4	1	8	30
60	56	30	5	4	2	8	7	9	6	3	1	A	30
60	58	30	4	A	9	2	6	5	7	1	8	3	30
60	60	30	2	6	5	4	1	7	3	8	9	A	30
90	62	30	3	1	9	6	5	7	A	2	4	8	46
90	64	30	4	7	3	8	5	2	A	1	6	9	48
90	65	30	4	6	3	1	8	7	2	9	A	5	46
90	66	30	8	4	1	3	9	A	7	2	5	6	46
90	67	30	7	1	9	4	3	2	6	8	5	A	46
90	68	30	8	4	A	2	5	7	6	3	1	9	46
90	71	30	9	3	8	2	1	7	5	4	6	A	46
90	73	30	1	2	9	6	5	A	4	7	3	8	46
90	77	30	A	2	5	8	1	7	9	3	4	6	58
120	89	30	2	1	7	3	9	8	4	A	5	6	64
90	90	60	3	7	5	6	4	1	2	8	9	A	30
120	120	90	7	1	6	2	5	4	3	8	9	A	30
150	150	120	1	4	5	3	7	2	6	8	9	A	30
150	123	30	3	6	1	5	9	8	4	A	7	2	100
240	203	30	7	1	9	4	3	2	6	8	5	A	202
240	204	30	4	2	5	3	7	1	8	A	9	6	202
270	208	60	4	2	5	3	7	1	8	A	9	6	202

3-designs with $v = 10$, $k = 4$

B	B^* =	D +	T	B	B^* =	D +	T
90	69	70	$(1-6)(2-3)(5-A)(7-9)$	90	70	72	$(1-3)(2-8)(5-7)(9-A)$
90	72	76	$(3-8)(4-6)(5-9)(A-7)$	90	74	76	$(1-3)(2-8)(5-7)(9-A)$
90	75	76	$(1-4)(2-8)(3-A)(5-6)$	90	76	81	$(1-6)(2-3)(4-9)(8-7)$
90	78	81	$(1-6)(2-3)(4-5)(A-7)$	90	79	81	$(1-4)(2-8)(3-A)(5-6)$
90	80	81	$(1-9)(2-8)(3-7)(4-6)$	90	81	85	$(1-9)(2-7)(4-A)(8-5)$
90	82	85	$(1-4)(2-8)(3-A)(5-6)$	90	83	85	$(1-A)(2-8)(4-9)(7-5)$
90	84	88	$(1-A)(2-5)(3-9)(7-6)$	90	85	88	$(1-8)(2-4)(3-5)(6-A)$
90	86	88	$(1-4)(2-8)(3-6)(5-A)$	90	87	85	$(1-6)(2-3)(5-9)(8-A)$
90	88	90	$(1-5)(2-A)(3-7)(6-9)$	120	91	92	$(1-3)(2-A)(4-8)(5-7)$
120	92	93	$(1-4)(2-9)(3-7)(6-A)$	120	93	96	$(1-6)(2-5)(3-9)(A-8)$
120	94	93	$(1-7)(2-4)(3-5)(8-A)$	120	95	93	$(1-9)(2-8)(3-6)(5-A)$
120	96	97	$(1-9)(2-A)(3-6)(8-5)$	120	97	100	$(1-6)(2-7)(3-9)(5-4)$
120	98	97	$(1-7)(2-4)(3-6)(8-5)$	120	99	97	$(1-A)(2-9)(3-8)(6-5)$
120	100	101	$(1-6)(2-8)(3-9)(4-A)$	120	101	102	$(1-8)(2-6)(3-4)(A-9)$
120	102	103	$(1-4)(2-7)(3-8)(A-5)$	120	103	104	$(1-9)(2-5)(3-6)(A-7)$
120	104	109	$(1-A)(2-8)(3-7)(5-4)$	120	105	104	$(1-8)(2-A)(3-4)(7-5)$
120	106	103	$(1-8)(2-A)(3-4)(7-5)$	120	107	104	$(1-7)(2-4)(3-6)(8-5)$
120	108	104	$(1-8)(2-6)(3-4)(A-5)$	120	109	110	$(1-9)(2-A)(3-5)(4-7)$
120	110	111	$(1-A)(2-9)(3-6)(7-5)$	120	111	114	$(1-8)(2-6)(3-4)(5-A)$
120	112	113	$(1-8)(2-4)(3-7)(6-9)$	120	113	118	$(1-5)(2-9)(3-8)(6-A)$
120	114	118	$(1-4)(2-5)(3-A)(7-6)$	120	115	118	$(1-4)(2-8)(3-6)(7-A)$
120	116	118	$(1-5)(2-A)(3-7)(4-9)$	120	117	118	$(1-8)(2-4)(3-7)(6-9)$
120	118	120	$(1-7)(2-A)(3-5)(6-4)$	120	119	118	$(1-A)(2-9)(4-8)(6-5)$
150	121	122	$(1-8)(2-4)(3-7)(9-5)$	150	122	123	$(1-8)(2-4)(3-7)(5-A)$
150	124	123	$(1-7)(2-5)(3-8)(6-A)$	150	125	123	$(1-9)(2-A)(3-4)(8-6)$
150	126	123	$(1-8)(2-4)(3-7)(5-6)$	150	127	128	$(1-8)(2-4)(3-7)(5-A)$
150	128	123	$(1-9)(2-7)(3-4)(8-6)$	150	129	128	$(1-7)(2-5)(3-8)(6-A)$
150	130	128	$(1-8)(2-4)(3-7)(5-6)$	150	131	132	$(1-8)(2-4)(3-7)(5-A)$
150	132	128	$(1-5)(2-4)(3-A)(9-6)$	150	133	132	$(1-5)(2-7)(3-6)(8-A)$
150	134	132	$(1-8)(2-6)(3-7)(5-A)$	150	135	132	$(1-5)(2-6)(4-9)(7-8)$
150	136	132	$(1-8)(2-6)(5-A)(9-7)$	150	137	142	$(1-8)(2-7)(3-9)(5-6)$
150	138	142	$(1-8)(2-6)(3-4)(5-9)$	150	139	142	$(1-8)(2-6)(3-4)(5-A)$
150	140	145	$(1-8)(2-6)(3-4)(5-A)$	150	141	146	$(1-8)(2-6)(3-4)(5-9)$
150	142	146	$(1-7)(2-A)(3-4)(6-5)$	150	143	146	$(1-8)(2-4)(3-7)(A-5)$
150	144	150	$(1-8)(2-A)(3-6)(5-4)$	150	145	146	$(1-8)(2-4)(3-7)(6-5)$
150	146	150	$(1-9)(2-8)(3-6)(5-4)$	150	147	149	$(1-5)(2-9)(3-7)(4-6)$
150	148	150	$(1-A)(2-9)(3-6)(7-4)$	150	149	148	$(1-4)(2-A)(3-5)(6-7)$
180	151	157	$(1-8)(2-4)(3-9)(A-5)$	180	152	157	$(1-9)(2-6)(3-4)(5-8)$
180	153	157	$(1-4)(2-6)(3-7)(9-5)$	180	154	157	$(1-9)(2-A)(3-6)(4-5)$
180	155	157	$(1-7)(2-A)(3-6)(4-5)$	180	156	157	$(1-7)(2-9)(3-6)(4-5)$
180	157	165	$(1-6)(2-A)(3-8)(5-7)$	180	158	165	$(1-8)(2-6)(3-9)(5-7)$
180	159	165	$(1-9)(2-A)(3-4)(5-7)$	180	160	165	$(1-9)(2-6)(3-4)(5-8)$
180	161	165	$(1-9)(2-A)(3-6)(4-5)$	180	162	165	$(1-7)(2-A)(3-6)(4-5)$
180	163	165	$(1-6)(2-8)(3-7)(4-5)$	180	164	165	$(1-7)(2-9)(3-6)(4-5)$
180	165	173	$(1-7)(2-9)(3-A)(5-6)$	180	166	173	$(1-7)(2-6)(3-4)(5-8)$
180	167	173	$(1-9)(2-6)(3-4)(5-7)$	180	168	174	$(1-9)(2-A)(3-4)(5-6)$

3-designs with $v = 10$, $k = 4$

B	B^* =	D +	T	B	B^* =	D +	T
180	169	174	(1 − 7)(2 − 9)(3 − 4)(6 − 5)	180	170	174	(1 − 7)(2 − 9)(3 − 6)(4 − 5)
180	171	176	(1 − 7)(2 − 9)(3 − 4)(6 − 5)	180	172	180	(1 − 9)(2 − 6)(3 − 5)(4 − 7)
180	173	176	(1 − 8)(2 − 7)(3 − 4)(9 − 5)	180	174	180	(1 − 8)(2 − 6)(3 − 7)(5 − 4)
210	175	179	(1 − 9)(2 − 6)(3 − 8)(5 − 4)	180	176	180	(1 − *A*)(2 − 8)(3 − 7)(5 − 4)
210	177	182	(1 − 7)(2 − 9)(3 − 8)(5 − 6)	210	178	182	(1 − *A*)(2 − 8)(3 − 4)(7 − 5)
210	179	182	(1 − 9)(2 − 6)(3 − 7)(4 − 5)	210	181	186	(1 − 7)(2 − 6)(3 − 4)(5 − 8)
210	182	186	(1 − *A*)(2 − 6)(3 − 4)(5 − 9)	210	183	186	(1 − 7)(2 − *A*)(3 − 4)(5 − 9)
210	184	190	(1 − 9)(2 − 5)(3 − 7)(6 − 4)	210	185	190	(1 − 9)(2 − 8)(3 − 6)(5 − 4)
210	186	194	(1 − 8)(2 − *A*)(3 − 5)(6 − 4)	210	187	194	(1 − 9)(2 − 8)(3 − 6)(5 − 4)
210	188	194	(1 − 9)(2 − 7)(3 − 6)(5 − 4)	210	189	194	(1 − 8)(2 − *A*)(3 − 5)(4 − 6)
210	190	194	(1 − 7)(2 − 9)(3 − 5)(4 − 6)	210	191	194	(1 − *A*)(2 − 8)(3 − 6)(5 − 4)
210	192	198	(1 − 9)(2 − 7)(3 − 5)(4 − 6)	210	193	198	(1 − 8)(2 − 6)(3 − 5)(4 − 7)
210	194	195	(1 − *A*)(2 − 7)(3 − 6)(4 − 5)	210	195	196	(1 − 6)(2 − 9)(3 − 4)(5 − 7)
210	196	198	(1 − 7)(2 − 9)(3 − 5)(6 − 4)	240	197	203	(1 − *A*)(2 − 7)(3 − 6)(4 − 5)
210	198	202	(1 − 7)(2 − 9)(3 − 6)(4 − 5)	240	199	203	(1 − 8)(2 − 7)(3 − 6)(4 − 5)
240	200	203	(1 − 8)(2 − 7)(3 − 5)(4 − 6)	240	201	203	(1 − 8)(2 − 9)(3 − 7)(4 − 5)
210	202	210	(1 − 8)(2 − 7)(3 − 6)(5 − 4)	270	205	208	(1 − *A*)(2 − 9)(3 − 5)(6 − 4)
240	206	203	(1 − 6)(2 − 9)(3 − 8)(7 − 4)	270	207	208	(1 − *A*)(2 − 9)(3 − 6)(5 − 4)
270	209	208	(1 − 6)(2 − 9)(3 − 8)(*A* − 5)				

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