

# A Construction of balanced ternary designs

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**Abstract.** A new method of construction of balanced ternary designs from PBIB designs, which yields two new designs, is given.

## 1. Introduction

Balanced ternary designs, introduced by Tocher (1952), were studied by Murty and Das (1967), Das and Rao (1968), Dey (1970), Saha and Dey (1973), Nigam (1974), Saha (1975), Sharma and Agarwal (1976), Morgan (1977), Sinha and Saha (1979), Surendran and Sunny (1979), Tyagi and Rizwi (1979), Kageyama (1980), Billington and Robinson (1983), Billington (1984), Das (1986), Donovan (1986), Agarwal and Das (1987), Malika and Surendran (1987), Shah and Gujarathi (1989). Billington and Robinson also tabulated the designs with  $R \leq 15$ .

Here we give a new method of construction of balanced ternary designs with frequencies 0, 1, 2, which yields two new designs listed as unknown in the table of Billington and Robinson (1983).

## 2. Balanced Ternary Designs

**Theorem 2.1.** *The existence of two PBIB designs with the same association scheme and having parameters:*

$$v, b_i, r_i, k_i, \lambda_{ij}, i, j = 1, 2 \quad (2.1.i)$$

*such that  $k_1 = 2k_2$  and  $\lambda_{11} + 4\lambda_{21} = \lambda_{12} + 4\lambda_{22}$  ( $= \wedge$  say) implies the existence of a balanced ternary design with parameters:*

$$\begin{aligned} V &= v, B = b_1 + b_2, R = r_1 + 2r_2, \rho_1 = r_1, \rho_2 = r_2, \\ K &= k_1, \wedge = \lambda_{12} + 4\lambda_{22}, \Delta = r_1 + 4r_2 \end{aligned} \quad (2.2)$$

**Proof:** Let  $N_i$  ( $i = 1, 2$ ) be the incidence matrices of the PBIB designs with parameters (2.1.i) respectively. Then,  $N^* = [N_1, 2N_2]$  is the incidence matrix of the balanced ternary design with parameters (2.2). Table 1 is a table of new balanced ternary designs with  $R \leq 15$  and listed as unknown in the table of Billington and Robinson (1983). Efficiency factor ( $E$ ) =  $\frac{\Delta V}{RK}$ .

**Table 1.** New balanced ternary designs with frequencies 0, 1, 2.

Number	V	B	R	K	$\rho_1$	$\rho_2$	$\Lambda$	E	Source
BR209	18	42	14	6	12	1	4	0.857	Duplicate of SR72, 2(6.3) SR44, S62
BR245	16	28	14	8	6	4	6	0.857	

*BR* numbers are from Billington and Robinson (1983). Under the source column: (*m*, *n*) stands for the disconnected design of *mn* treatments *hm* blocks of *n* treatments each; *SR*, *S* numbers refer to PBIB designs from Clatworthy (1973).

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