Counterexample to a Conjecture on Hamilton Cycles

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Abstract

A counterexample is presented to the following conjecture of Jackson: If G is a 2-connected graph on at most $3k+2$ vertices with degree sequence $(k, k, \ldots k, k+1, k+1)$ then G is hamiltonian.

Counterexample

All graphs considered here are finite and simple. A cycle of a graph G is said to be *hamilton* if it contains all the vertices of G. A graph containing a hamilton cycle is called *hamiltonian*. Definitions which are not given may be found in [1]. In [3], Jackson has proved that every 2-connected k-regular graph on at most $3k$ vertices is hamiltonian. In [2], he conjectured that "if G is a 2-connected graph on $3k+2$ vertices with the degree sequence $(k, k, \ldots k, k+1, k+1)$, then G is hamiltonian". In this note we disprove it by means of a counterexample.

Let $A = K_{k+1} \setminus \{e\}$ where $V(A) = \{v_1, v_2, \ldots v_{k+1}\}$ and $e = v_1v_{k+1}$. Let $B = K_{k,k}$ with bipartition $X = \{x_1, x_2, \ldots x_k\}$, $Y = \{y_1, y_2, \ldots y_k\}$ and disjoint from A. We define a graph $G = A \cup B \cup \{v_1x_1, v_{k+1}x_k\}$ (see Fig. 1). Clearly all the vertices of G are of degree k except x_1 and x_k which are of degree $k+1$. It is easy to see that $G \setminus X$ has $|X|+1$ components, and hence G is nonhamiltonian [1, Theorem 4.2].

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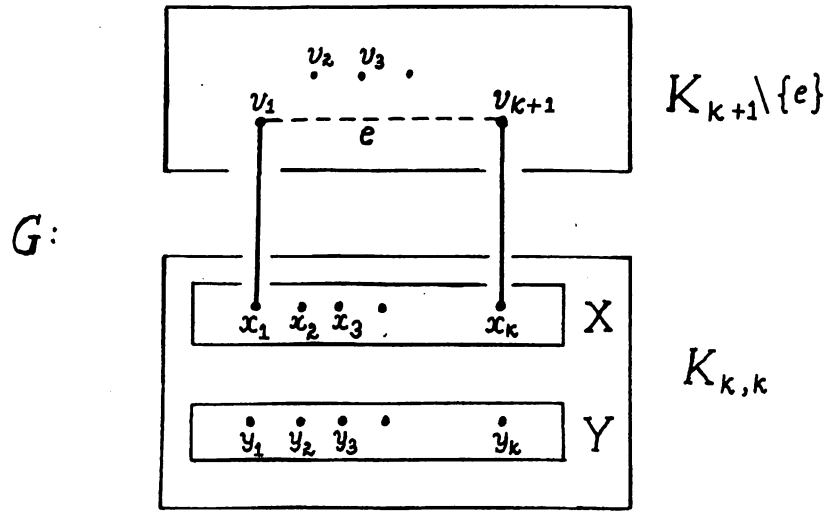

Figure 1

REFERENCES

[1] J.A. Bondy and U.S.R. Murty, *Graph Theory with Applications*, Macmillan Press, (1976).

[2] B. Jackson, Hamilton cycles in regular 2-connected graphs, *Graph Theory and Related Topics* (Eds. J.A. Bondy and U.S.R. Murty), Academic Press, (1979), 261 – 265.

[3] B. Jackson, Hamilton cycles in regular 2-connected graphs, *J. Combinatorial Theory (ser. B) 29 (1980), 27 – 46.*