A few more resolvable spouse-avoiding mixed-doubles round robin tournaments

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Abstract. It has been shown that there exists a resolvable spouse-avoiding mixed-doubles round robin tournament for any positive integer $v \neq 2,3,6$ with 27 possible exceptions. We show that such designs exist for 19 of these values and the only values for which the existence is undecided are: 10,14,46,54,58,62,66 and 70.

1. Introduction

The terminology and notation in this paper follow from that of [2,4,5]. A *spouse-avoiding mixed-doubles round robin tournament* is an arrangment for couples to play mixed-doubles tennis so that no player is partnered by, or opposes, his or her spouse; otherwise, every player has each other player as an opponent exactly once and has each other player of the opposite sex as a partner exactly once. The tournament is *resolvable* if its matches can be partitioned into rounds so that every player can play at the same time within a round. A resolvable spouse-avoiding mixed-doubles round robin tournament for n couples is denoted $R(n)$.

It shown in [4] that the existence of an $R(n)$ is equivalent to the existence of a self-orthogonal Latin square of order n with a symmetric orthogonal mate when n is odd, and a self-orthogonal Latin square of order n with a unipotent (that is, some fixed element, say 0, must occur on the main diagonal) symmetric orthogonal mate when n is even.

For convenience we denote by SOLSSOM(v) (USOLSSOM(v)) a self-orthogonal Latin square of order v with a (unipotent) symmetric orthogonal mate. We further denote by ISOLSSOM(v,n) (IUSOLSSOM(v,n)) a SOLSSOM(v) (USOLSSOM(v)) with a sub-SOLSSOM(n) (sub-USOLSSOM(n)) missing in the lower right corner. The first letter I stands for incomplete. An ISOLSSOM($v,0$) (IUSOLSSOM($v,0$)) and ISOLSSOM($v,1$) always exist if a SOLSSOM(v) (USOLSSOM(v)) exists.

From Wang [4], Lindner, Mullin and Stinson [2] and Zhu [5] we have

Lemma 1.1. *A SOLSSOM(v) does not exist for $v \in \{2,3,6\}$, but a SOLSSOM(v) does exist for positive integer v with the possible exception of $v \in E$*

$$E = \{10,14,46,54,58,62,66,70\}.$$

For the existence of USOLSSOM we have (see [4])

Lemma 1.2. *USOLSSOM(v) does not exist for $v \in \{2,6\}$, but a USOLSSOM(v) does exist for even integer v with the possible exception of $v \in E \cup F$*

$$F = \{74,82,98,102,118,142,174,194,202,214,\\ 230,258,278,282,394,398,402,422,1322\}.$$

We then have

Lemma 1.3. *An $R(n)$ does not exist for $n \in \{2,3,6\}$, but an $R(n)$ does exist for positive integer n with the possible exception of $n \in E \cup F$.*

It is our purpose here to show that such designs exist for $v \in F$, and reduce this number of possible exceptions to 8.

2. Construction

The following lemma follow mutatis mutandis from Lemma 1 in [5] which are the variants of Theorem 1 in [1]. We give its proof in detail.

Lemma 2.1. *Suppose q is an even prime power, $q \geq 8$, and there exist USOLSSOM(m), USOLSSOM(m + k) and ISOLSSOM(m + k_t, k_t) where m is even,*

$$k_t = 0 \text{ or } k_t \text{ odd} > 0, t = 1,2,\ldots\frac{q-4}{2}, k = \sum_{t=1}^{\frac{q-4}{2}}(2k_t).$$

Then there exists a USOLSSOM(qm + k).

Proof: Let $L_\lambda = (a_{ij}), a_{ij} = a_i + \lambda a_j, a_i, a_j, \lambda \in GF(q) = \{a_0, a_1, \ldots, a_{q-1}\}$ such that $a_0 = 0$ and $a_i = \alpha^{i-1}$ $1 \leq i \leq q-1$ where α is a primitive element of $GF(q)$. It is easy to see that the Latin squares $L_1, L_\alpha 1, L_\alpha 2, \ldots L_\alpha q - 2$ are pairwise orthogonal and that the squares $L_\alpha, L_\alpha 2, \ldots, L_\alpha d, d = \frac{q-2}{2}$ are all self-orthogonal, the square L_1 is unipotent symmetric. The cells with entry 0 in $L_\alpha t$ determine a common transversal of $L_\alpha d$ and L_1, a USOLSSOM(q), say tth transveral, $t = 1,2,\ldots,\frac{q-4}{2}$. The transpose of the tth transversal is also a common transversal of the USOLSSOM(q), say (t')th transversal. We know that all these transversals intersect in cell $(0,0)$ and are disjoint elsewhere.

Begin with the USOLSSOM(q) and replace each of its cells with an $m \times m$ array labelled by the element in that cell, the array will be a USOLSSOM(m) if the cell is not on any transversals mentioned above, otherwise the array will be the upper left part of an ISOLSSOM($m + k_t, k_t$) if the cell is on the tth or (t')th transversal but not $(0,0)$ and the array in $(0,0)$ will be the upper left part of a USOLSSOM($m + k$). We suppose that each of the above ISOLSSOM($m + k_t, k_t$) and USOLSSOM($m + k$) is based on the same elements as the USOLSSOM(m)

and some other new elements, and the new elements remain unchanged when labelling. Then we obtain the upper left part of the required USOLSSOM($qm+k$). Suppose its four corners are occupied by the USOLSSOM($m+k$) as shown in Fig.1, so what we need now is to describe the right part and the lower part.

The right part consists of the columns $C_1, C_2, \ldots, C_t, \ldots, C_{d-1}$ where column C_t comes from the right parts of the ISOLSSOM($m+k_t, k_t$) on tth and (t')th transversals in such an order: tth transversal left and (t')th transversal right. The lower part consists of the rows $R_1, R_2, \ldots, R_t, \ldots, R_{d-1}$, where R_t comes from the lower parts of the ISOLSSOM($m+k_t, k_t$) on tth and (t')th transversals in another order, i.e. tth transversal below and (t')th transversal above.

Now we get a self-orthogonal Latin square with a unipotent orthogonal mate which is almost symmetric. The only problem is that in the orthogonal mate some positions occupied by a new element x in a cell of the tth transversal have their symmetric positions occupied by another new element y in the symmetric cell of the (t')th transversal. Since m is even and k_t odd > 0, we can replace the element x by y in the positions above the diagonal of the cell. For the cells of (t')th transversal, replace the correspondent element y by x. It is a routine matter to see that the final squares are the required USOLSSOM($qm+k$).

We have the following corollary

Corollary 2.2. *There exists a USOLSSOM(v) for* $v \in \{142, 194, 202, 258, 278, 282, 394, 398, 402, 422\}$.

Proof: In Table 1 we use Lemma 2.1 to get the required USOLSSOMs. All the input ISOLSSOM($m+k_t, k_t$), USOLSSOM(m) and USOLSSOM($m+k$) are obtained from Lemmas 1.1 and 1.2.

Table 1

Equation	$k = \sum(2k_t)$
$142 = 32 \times 4 + 14$	$7 \times (2 \times 1)$
$194 = 8 \times 24 + 2$	2×1
$202 = 16 \times 12 + 10$	$5 \times (2 \times 1)$
$258 = 16 \times 16 + 2$	2×1
$278 = 32 \times 8 + 22$	$11 \times (2 \times 1)$
$282 = 32 \times 8 + 26$	$13 \times (2 \times 1)$
$394 = 32 \times 12 + 10$	$5 \times (2 \times 1)$
$398 = 32 \times 12 + 14$	$7 \times (2 \times 1)$
$402 = 32 \times 12 + 18$	$9 \times (2 \times 1)$
$422 = 16 \times 26 + 6$	$3 \times (2 \times 1)$

To apply the second construction we need some input designs, which we state below. From [4] we have

Lemma 2.3. *There exists an IUSOLSSOM(v,n) for* $(v,n) \in \{(18,4),(22,4),(26,4),(30,4),(34,8),(38,8)\}$.

We give another construction below, using "frame-type" SOLSSOMs (FSOLSSOMs). A t-FSOLSSOM(u) is defined as follows.

Let V be an tu-set and let π be a partition of V into u sets $T_1, T_2, \ldots, T_u$, each of size t. Then a t-SOLSSOM(u) is a pair of tu by tu arrays, A and B both indexed by V, which satisfy the following.

(1) Each cell of A and B which is indexed by a pair (t_1, t_2) where t_1 and t_2 belong to the same partition block is empty, and all other cells of each array contain a member of V,
(2) Each line (row or column) of each array indexed by a member of block T of the partition contains each member of $V \backslash T$,
(3) The array B is symmetric, and
(4) If A' denotes the transpose of A, then $\{A, A', B\}$ is a set of pairwise orthogonal partial Latin squares.

Loosely speaking, a t-FS0LSSOM(u) can be considered to be a SOLSSOM(tu) "missing" a set of u disjoint sub-SOLSSOM(t)s.

For u odd, a 1-FSOLSSOM(u) is equivalent to a SOLSSOM(u), whereas for u even, a 1-FSOLSSOM(u) cannot exist.

From [2,3] we have

Lemma 2.4. *There exists a 2-FSOLSSOM(u) for* $u = 5,7$.

The following lemma follow mutatis mutandis from Construction 2.3 in [2]. So we state it without proof. For the detail the reader is referred to [2].

Lemma 2.5. *Suppose that for positive integers* t, u, v, w, a *with* $0 \leq a \leq w$ *there exist:*

(1) *a t-FSOLSSOM(u),*
(2) *an IUSOLSSOM(v, w),*
(3) *a USOLSSOM($u(w-a)+a$),*
(4) *three pairwise orthogonal Latin squares of order* $\frac{v-a}{t}$ *containing common subsquares of order* $\frac{w-a}{t}$.

Then there exists a USOLSSOM($u(v-a)+a$).

We have the following corollary

Corollary 2.6. *There exists a USOLSSOM(v) for* $v \in \{74,82,98,102,118,174,214,230\}$.

Proof: In Table 2, we use Lemma 2.5 to get the required USOLSSOMs. All the input t-FSOLSSOM(u), IUSOLSSOM(v,w), USOLSSOM($u(w-a)+a$) and $\frac{v-a}{t}$ sub $\frac{w-a}{t}$ are obtained from Lemmas 1.1, 1.2, 2.3 and 2.4.

Table 2

Equation	w	t	$\frac{v-a}{t}$ sub $\frac{w-a}{t}$
$74 = 5(18-4)+4$	4	2	7
$82 = 5(18-2)+2$	4	2	8 sub 1
$98 = 5(22-3)+3$	4	1	19 sub 1
$102 = 7(18-4)+4$	4	2	7
$118 = 5(26-3)+3$	4	1	23 sub 1
$174 = 9(22-3)+3$	4	1	19 sub 1
$214 = 7(34-4)+4$	8	1	30 sub 4
$230 = 7(38-6)+6$	8	2	16 sub 1

From USOLSSOM(82) we have

Lemma 2.7. *There exists a USOLSSOM(1322).*

Proof: Write $1322 = 16 \times 82 + 5 \times (2 \times 1)$. Since there exist ISOLSSOM(83,1), USOLSSOM(92) from Lemmas 1.1, 1.2, respectively, a USOLSSOM(1322) exists from Lemma 2.1.

3. Conclusion

Up to now, it has been shown that a USOLSSOM(v) exists for $v \in F$. Updating Lemma 12 we obtain the following

Theorem 3.1. *A USOLSSOM(v) does not exist for* $v \in \{2,6\}$, *but a USOLSSOM(v) does exist for even integer* v *with the possible exception of* $v \in E$.

We then have

Theorem 3.2. *An* $R(n)$ *does not exist for* $n \in \{2,3,6\}$, *but an* $R(n)$ *does exist for positive integer* n *with the possible exception of* $n \in E$.

$$E = \{10, 14, 46, 54, 58, 62, 66, 70\}$$

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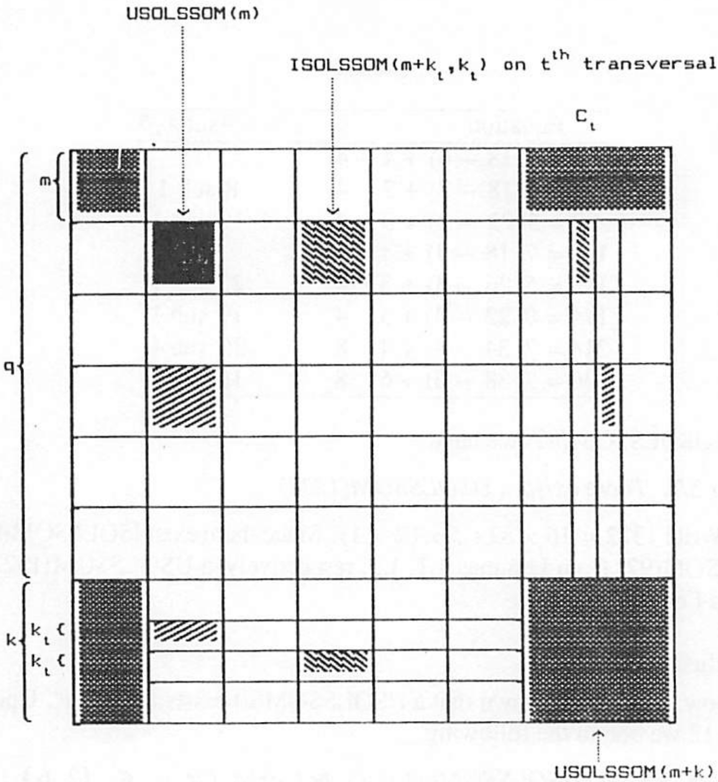

Figure 1

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