Hamiltonian Connected Line Graphs

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Abstract. Let G be a simple graph with n vertices. Let $L(G)$ denote the line graph of G. We show that if $\kappa'(G) \geq 2$ and if for every pair of nonadjacent vertices $v, u \in V(G)$, $d(v) + d(u) > 2n/3 - 2$, then for any pair of vertices $e, e' \in V(L(G))$, either $L(G)$ has a hamilton (e, e')-path, or $\{e, e'\}$ is a vertex-cut of $L(G)$. When G is a triangle-free graph, this bound can be reduced to $n/3$. These bounds are all best possible and they partially improve prior results in [J. Graph Theory, 10 (1986), 411–425] and [Discrete Math. 76 (1989) 95–116].

1. Introduction.

We shall follow the notation of Bondy and Murty [2], unless otherwise stated. Let G be a graph and e, e' be two edges of G. A trail in G whose first edge is e and whose last edge is e' is called an *(e, e')-trail*. An (e, e')-trail T is called a *spanning* (e, e')-trail of G if $V(T) = V(G)$ and if every edge of G is incident with an internal vertex of T. A trial T of G is *dominating* if $G - V(T)$ is edgeless. For convenience, the graph K_1 is regarded as having a closed trail.

The *line graph* of G, denoted by $L(G)$, has $E(G)$ as its vertex set in which two vertices are adjacent in $L(G)$ if and only if the corresponding edges are adjacent in G.

Theorem A. *(Harary and Nash-Williams [12]) Let G be a graph with at least three edges. Then G has a dominating closed trial if and only if $L(G)$ is hamiltonian.*

Theorem B. *(Lesniak-Foster and Williamson [13], Zhan [14]) Let G be a graph and let e, e' be in $E(G)$. If G has a spanning (e, e')-trail, then $L(G)$ has a spanning (e, e')-path.*

The definition of spanning (e, e')-trails was used in [9]. We shall say a few words about this definition. Let G be the 4-cycle and e, e' be two nonadjacent edges in G. Then G has an (e, e')-trail that is spanning in G but $L(G)$ does not have a hamilton (e, e')-path. This is why we define a spanning (e, e')-trail in the way above.

If for every pair of vertices u, v of G, G has a spanning (u, v)-path, then G is said to be *hamiltonian connected*. With the help of Theorem B, Zhan showed the following:

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Theorem C. *(Zhan [14]) If $\kappa'(G) \geq 4$, then for every pair of edges $e, e' \in E(G)$, G has a spanning (e, e')-trail and so $L(G)$ is hamiltonian connected.*

If $X \subseteq E(G)$ is an edge-cut such that at least two components of $G - X$ have edges, then X is called an *essential edge-cut*. It is easy to see that if $\{e, e'\}$ is an essential edge-cut of G, then G cannot have any spanning (e, e')-trails. It has been noted by Catlin [7], (and by Zhan [14], for the case when $k = 2$), that G is $2k$-edge-connected if and only if $|E(G)| \geq k$ and for any k-subset $X \subseteq E(G)$, $G - X$ has k edge-disjoint spanning trees. In particular, 4-edge-connected graphs always have 2 edge-disjoint spanning trees. Thus, the following improves Theorem C:

Theorem D. *(Catlin and Lai [9]) Let G be a graph with 2 edge-disjoint spanning trees. For two edges $e, e' \in E(G)$, one of the following holds:*

(i) *G has a spanning (e, e')-trail;*
(ii) *$\{e, e'\}$ is an essential edge-cut of G.*

2. Main results.

The proofs of the following theorems appear in the last section.

Theorem 1. *Let G be a simple graph with $|V(G)| = n \geq 27$ and with $\kappa'(G) \geq 2$. If for every pair of nonadjacent vertices $u, v \in V(G)$,*

$$d(u) + d(v) > \frac{2n}{3} - 2, \tag{1}$$

then for every pair of edges $e, e' \in E(G)$, exactly one of the following holds:

(i) *G has a spanning (e, e')-trail;*
(ii) *$\{e, e'\}$ is an essential edge-cut of G.*

Corollary 1A. *Let G satisfy the hypothesis of Theorem 1. Then either $L(G)$ has a 2-vertex-cut or $L(G)$ is hamiltonian connected.*

Corollary 1B. *(Catlin [4] and Benhocine, Clark, Köler, and Veldman [1]) Let G be a 2-edge-connected simple graph with $n = |V(G)| \geq 27$. If for every pair of nonadjacent vertices $u, v \in V(G)$,*

$$d(u) + d(v) > \frac{2n}{3} - 2, \tag{2}$$

then $L(G)$ is hamiltonian.

Proof: The truth of Corollary 1A follows immediately from Theorem B and Theorem 1. Call an *end-block* of G a maximal 2-connected subgraph H such that either $H = G$ or H contains exactly one cut-vertex of G. If G satisfies the conclusion of Theorem 1 but $L(G)$ is not hamiltonian, then it would follow that every pair of adjacent edges are incident with a cut-vertex of G, which leads to an obvious contradiction, since in an end block of G, one can always find two adjacent edges that are not both incident with a cut-vertex of G. Thus, Corollary 1B follows from Theorem 1 and Theorem A. ∎

In fact, it was proved in [1] and in [4] that G has a spanning closed trail with $|V(G)| \geq 4$ and with a weaker lower bound $(2n+1)/3$, and in [8], Catlin showed that when $n = |V(G)| \geq 20$, then bound in (2) can be lowered $(2n-9)/5$.

Theorem 2. *Let G be a 2-edge-connected triangle-free simple graph with $n \geq 33$ vertices. If for every pair of distinct nonadjacent vertices $u, v \in V(G)$,*

$$d(u) + d(v) > \frac{n}{3}, \tag{3}$$

then for every pair of edges $e, e' \in E(G)$, exactly one of the following holds:

(i) *G has a spanning (e, e')-trail;*
(ii) *$\{e, e'\}$ is an essential edge-cut of G.*

Corollary 2. *Let G be a graph satisfying the hypothesis of Theorem 2, then $L(G)$ is either hamiltonian connected or has a vertex-cut of size 2.*

Theorem 1, Theorem 2, and Corollary 2, are best possible in some sense. Let $s \geq 10$ be an integer, and let $G(s)$ and $G(s, s)$ be defined as follows.

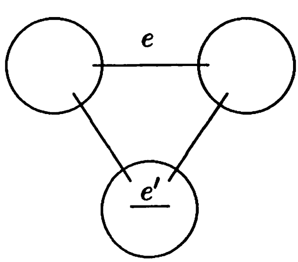

Figure 1: $G(s)$ or $G(s, s)$ with edges e and e'.

For $G(s)$, each circle in Figure 1 denotes a complete subgraph K_s, and a line joining two circles denotes a single edge joining two vertices in two distinct K_s's. Let $n = |V(G(s))| = 3s$. Apparently for every pair of nonadjacent $u, v \in V(G(s))$, $d(v)+d(u) \geq (2n)/3-2$. But for the given edges e, e', neither (i) nor (ii) of Theorem 1 holds. We then obtain $G(s, s)$ by replacing each circle in Figure 1 by a complete bipartite subgraph $K_{s,s}$ and by arranging the 3 edges between the 3 $K_{s,s}$'s so that the resulting graph is a bipartite one. Let $n = |V(G(s, s))| = 6s$ this time. Then for every pair of nonadjacent $u, v \in V(G(s, s))$, $d(v) + d(u) \geq n/3$. But for the given edges e, e', neither (i) nor (ii) of Theorem 2, nor the conclusion of Corollary 2, holds.

We shall also consider the computational complexity of the following decision problem: given a graph G and a pair of edges e, e', does G have a spanning (e, e')-trial?

Theorem 3. *The problem to determine if G has a spanning (e, e')-trail is NP-complete.*

3. Collapsible and reduced graphs.

Let G be a graph and let $X \subseteq E(G)$. The *contraction* G/X is the graph obtained from G by identifying the ends of each edge of X and deleting the resulting loops. If H is a subgraph of G, then we use G/H for $G/E(H)$.

Let $O(G)$ denote the set of vertices of odd degree in G. A graph G is *eulerian* if G is connected and $O(G) = \emptyset$. A graph is *supereulerian* if it has a spanning eulerian subgraph. Let $R \subseteq V(G)$ be a subset with even cardinality. An *R-subgraph* of G is a subgraph Γ of G such that $G - E(\Gamma)$ is connected and such that $O(\Gamma) = R$. A graph G is *collapsible* if for every $R \subseteq V(G)$ with $|R|$ even, G has an R-subgraph. Note that by definition, K_1 is both collapsible and supereulerian. In [5], Catlin proved that every graph G has a unique collection of maximal collapsible subgraphs, say $H_1, H_2, \ldots, H_c$. Thus, the graph $G' = G/(\cup_{i=1}^{c} E(H_i))$ is unique, and is called the *reduction* of G. A vertex v in the reduction of G is *trivial* if its preimage in G under the contraction is a K_1 in G. A graph is *reduced* if it is the reduction of some graph.

Theorem E. *(Catlin [5]) Let G be a graph, and let $F(G)$ denote the minimum number of extra edges that must be added to G so that the resulting graph has 2 edge-disjoint spanning trees.*

(i) *Let H be a collapsible subgraph of G. Then G is supereulerian if and only if G/H is supereulerian; and G is collapsible if and only if G/H is collapsible.*
(ii) *G is reduced if and only if G has no nontrivial collapsible subgraphs; if and only if the reduction of G is G itself. In particular, a reduced graph does not contain 2-cycles and 3-cycles.*
(iii) *G is collapsible if and only if the reduction of G is K_1.*
(iv) *If G has 2 edge-disjoint spanning trees, that is $F(G) = 0$, then G is collapsible.*
(v) *If G is reduced, then $\delta(G) \leq 3$.*
(vi) *If G is reduced and if $F(G) = 1$, then $G = K_2$.*

In [9], it is noted that if G is reduced, then

$$F(G) = 2|V(G)| - |E(G)| - 2. \tag{4}$$

The following result, conjectured by Catlin in [3] and recently proved by Catlin, Han, and Lai, will be applied in this paper.

Theorem F. *(Catlin, Han, and Lai [10]) If G is a connected reduced graph with* $F(G) \leq 2$*, then either* $G = K_1$*, or* $G = K_2$*, or there is an integer* $t \geq 1$ *such that* $G = K_{2,t}$.

4. The proofs.

The following notation and terminology will be used in this section. For a graph G and an integer $i \geq 1$, $D_i(G)$ denotes the number of vertices of degree i in G.

We say that an edge $e \in E(G)$ is *subdivided* when it is replaced by a path of length 2 whose internal vertex, denoted by $v(e)$, has degree 2 in the resulting graph. This process is called *subdividing* e. For a graph G and distinct edges e, $e' \in E(G)$, let $G(e,e')$ denote the graph obtained from G by subdividing both e and e'. Thus,

$$V(G(e,e')) - V(G) = \{v(e), v(e')\}.$$

The reason for introducing $G(e,e')$ can be found in Lemma 1 below.

Lemma 1. *For a graph* G *and* $e, e' \in E(G)$, G *has a spanning* (e,e')*-trail if and only if either* $G(e,e')$ *has a spanning* $(v(e), v(e'))$*-trail, or both* e *and* e' *are incident with the same vertex* v *in* G *such that* $G(e,e') - v$ *has a spanning* $(v(e), v(e'))$*-trail.*

Proof: The proof is straightforward and so is omitted. ∎

Lemma 2. *Let* G *be a reduced graph with* n *vertices. Then*

$$2F(G) + 4 \leq \sum_{i=1}^{3}(4-i)|D_i(G)|. \tag{5}$$

Proof: This follows by counting the incidences of G and by (4). ∎

Lemma 3. *Let* G *be a graph and let* G' *be the reduction of* G*. For vertices* u, $v \in V(G)$*, define* u', v' *to be vertices in* G' *whose preimages contain* u *and* v*, respectively. (Note even* $v \neq v$*, it may still happen that* $u' = v'$*). Then* G *has a spanning* (u,v)*-trail if and only if* G' *has a spanning* (u', v')*-trail.*

Proof: Let u, v, u', v', and G satisfy the hypothesis of Lemma 3. Let x be a vertex not in $V(G)$. Define a new graph H from G with $V(H) = V(G) \cup \{x\}$ and $E(H) = E(G) \cup \{ux, xv\}$. Then G has a spanning (u,v)-trail if and only if H is supereulerian, if and only if the reduction of H is supereulerian (by (i) of Theorem E), if and only if G' has a spanning (u', v')-trail. ∎

In the proof below we often need to go back and forth from subgraphs L' of $G(e,e')$ and subgraphs L of G. For any subgraph L' of $G(e,e')$ such that $d_{L'}(v(e)) = 2$ whenever $v(e) \in V(L')$ ($d_{L'}(v(e')) = 2$ whenever $v(e') \in V(L')$), let L denote the corresponding subgraph of G such that $L = L'$ if $V(L') \cap \{v(e), v(e')\} = \emptyset$, and L is the graph obtained from L' by contracting exactly one edge incident with $v(e)$ if $v(e) \in V(L')$ (with $v(e')$ if $v(e') \in V(L')$). We say that L is obtained from L' by *undoing the subdivision*. For any $v \in V(G)$, the *neighborhood* of v in G, denoted by $N(v)$, consists of the vertices in G that are adjacent to v.

Proof of Theorem 1: Suppose that G, e, e' satisfy the hypothesis of Theorem 1. Let G'' denote the reduction of $G(e,e')$. If $G(e,e')$ is collapsible, that is $G'' = K_1$, then by Lemma 3, $G(e,e')$ has a spanning $(v(e), v(e'))$-trail and so (i) of Theorem 1 follows from Lemma 1. Hence, we assume that $G'' \neq K_1$. Let w, w' denote the vertices in G'' whose preimages contain $v(e)$, $v(e')$, respectively. Thus, when w and w' are trivial vertices, $w = v(e)$ and $w' = v(e')$.

By $\kappa'(G) \geq 2$, $D_1(G'') = \emptyset$. By $\kappa'(G) \geq 2$ and by (vi) of Theorem E, $F(G'') \geq 2$, and so by Lemma 2, $|D_2(G'') \cup D_3(G'')| \geq 4$, where equality holds only if $D_3(G'') = \emptyset$.

Claim 1: Let $v_{H'} \in D_2(G'') \cup D_3(G'')$ be a nontrivial vertex with preimage H' in $G(e,e')$, and let H be the subgraph of G obtained from H' by undoing the subdivision. If $D_2(G'') \cup D_3(G'')$ has a trivial vertex $v' \notin \{w, w'\}$, then $|V(H)| > 2n/3 - 5$. Moreover, if $v_{H'} \notin \{w, w'\}$, then $|V(H)| \geq 2n/3 - 3$.

Note that H is a simple collapsible subgraph of G and so $|V(H)| \geq 3$. Choose a vertex $v \in V(H)$ such that $vv' \notin E(G)$ and v is incident with at most one edge in $E(G'')$. By $vv' \notin E(G)$, $|V(H)| \geq d(v)$ and so Claim 1 follows from (1).

Claim 2: If $D_2(G'') \cup D_3(G'')$ has a trivial vertex not in $\{v(e), v(e')\}$, then $D_2(G'') \cup D_3(G'')$ has at most one nontrivial vertex.

Claim 2 follows from Claim 1 and the hypothesis of $n \geq 27$.

Claim 3: $D_2(G'') \cup D_3(G'') - \{w, w'\}$ cannot have 3 trivial vertices.

By (ii) of Theorem E, G'' is reduced and so has no cycles of length less than 4. Thus, if $D_2(G'') \cup D_3(G'')$ has 3 trivial vertices other than $v(e)$, $v(e')$, then two of them must be nonadjacent and so by (1), $n < 12$, contrary to $n \geq 27$. This proves Claim 3.

Claim 4: $D_2(G'')$ has at most 2 nontrivial vertices.

Suppose that $D_2(G'')$ has three nontrivial vertices v_1', v_2', v_3' whose preimages in $G(e,e')$ are H_1', H_2', and H_3', respectively. Let H_i denote the subgraph of G obtained from H_i' by undoing the subdivision and let $n_i = |V(H_i)|$, $(1 \leq i \leq 3)$. Since $v_i' \in D_2(G'')$, each H_i has a vertex v_i that not incident with edges in G'' and so by (1),

$$n_i + n_j \geq 2 + d(v_i) + d(v_j) > \frac{2n}{3}. \tag{6}$$

Thus, $2n \geq 2\sum_{i=1}^{3} n_i > 2n$, a contradiction. This proves Claim 4.

If $F(G'') \leq 2$, then by Theorem F and by $\kappa'(G') \geq 2$, there is some integer $t \geq 2$ such that $G'' = K_{2,t}$. Assume first that $t = 2$. By Lemma 1 and Lemma 3, G has a spanning (e,e')-trail unless $v(e)$ and $v(e')$ are contained in the preimages of two distinct nonadjacent vertices of w, w' G''. In the latter case, at least one of the two vertices x and y in $V(G'') - \{w, w'\}$ is nontrivial by (1). If both x and y are nontrivial, then by Claim 4, w and w' must be trivial, whence (ii) of Theorem 1 holds. If x or y is trivial, then by Claim 2, both w and w' are trivial, whence G has a spanning (e, e')-trail.

Thus, we assume that $t \geq 3$. By Claim 4 and by $t \geq 3$, $D_2(G'')$ has at least $t - 2$ trivial vertices. If $D_2(G'') - \{w, w'\}$ has a trivial vertex, then by Claim 2, $D_2(G'') \cup D_3(G'')$ has at most one nontrivial vertex. Thus, if $t \geq 5$, then $D_2(G'')$ must have at least two trivial vertices u and v (say), and so by (1), $4 = d(u) + d(v) > (2n + 1)/3$, contrary to the assumption that $n \geq 27$. Similarly, if $t = 4$, then w, w' must be two trivial vertices in $D_2(G'')$, and so G'' has a spanning (w, w')-trail, which implies (i) of Theorem 1 by Lemma 1 and Lemma 3.

Therefore, we assume that $t = 3$ and that G'' does not have a spanning (w, w')-trail, whence w and w' cannot be both in $D_2(G'')$. Hence, we assume that $w \in D_3(G'')$, and so w is nontrivial, and that either $w' = w$ or $w' \in D_2(G'')$. If $D_2(G'') - \{w'\}$ has a trivial vertex, then by Claim 2 and $w \in D_3(G'')$ being nontrivial, $D_2(G'') - \{w'\}$ must have 2 trivial vertices, contrary to the assumption that $n \geq 27$, by (1). Note that by Claim 4, if $w = w'$, then $D_2(G'')$ must have a trivial vertex, which would lead to the same contradiction. It follows that $w' \in D_2$ and there are two nontrivial vertices $v_1, v_2 \in D_2(G'')$. Let v_3 denote the vertex in $D_3(G'') - \{w\}$. Let H_i $(1 \leq i \leq 3)$ denote the preimages of v_i in G, and let H_0 denote the subgraph of G obtained from the preimage of w in $G(e, e')$ by undoing the subdivision. If v_3 is trivial, then by Claim 1, we have

$$n - 1 \geq |V(H_0)| + |V(H_1)| + |V(H_2)| \geq 2n/3 - 5 + 4n/3 - 6 = 2n - 11,$$

and so $n \leq 10$, a contradiction. Thus, v_3 is also nontrivial. By choosing $u_i \in V(H_i)$ such that u_i is incident with as few edges in $E(G'')$ as possible, we have by (1)

$$|V(H_1)| + |V(H_2)| \geq d(u_1) + d(u_2) > \frac{2n}{3} - 2 \text{ and}$$
$$|V(H_0)| + |V(H_3)| \geq d(u_0) + d(u_3) > \frac{2n}{3} - 2 - 3,$$

and so $n \geq \sum_{i=0}^{3} |V(H_i)| \geq 4n/3 - 7$. It follows that $n \leq 21$, a contradiction.

Hence, we may assume that $F(G'') \geq 3$ and so by Lemma 2, $|D_2(G'') \cup D_3(G'')| \geq 5$ where equality holds only if $D_3(G'') = \emptyset$.

Case 1: $D_2(G'') \cup D_3(G'')$ has at least 4 nontrivial vertices.

Let H_i', $(1 \leq i \leq 4)$ denote the preimages in $G(e, e')$ of the 4 nontrivial vertives in $D_2(G'') \cup D_3(G'')$. Let H_i denote the subgraph of G obtained from H_i' by undoing the subdivision. Simce G is simple, $|V(H_i)| \geq 3$, and so for H_i,

H_j, there are vertices $v_i \in V(H_i)$ and $v_j \in V(H_j)$ such that $v_iv_j \notin E(G)$ and each of v_i and v_j is incident with at most one edge in $E(G'')$. It follows by (1) that

$$2n \geq 2\sum_{i=1}^{4}|V(H_i)| > 4\left(\frac{2n}{3}-2\right) = \frac{8n}{3} - 8,$$

and so $n < 12$, a contradiction.

Case 2: $D_2(G'') \cup D_3(G'')$ has exactly 3 nontrivial vertices.

By Claim 4, we must have $D_3(G'') \neq \emptyset$. Thus, $|D_2(G'') \cup D_3(G'')| \geq 6$ and so there is a trivial vertex $v \in D_2(G'') \cup D_3(G'') - \{w, w'\}$. Now the conclusion of Claim 2 contradicts the hypothesis of Case 2.

Case 3: $D_2(G'') \cup D_3(G'')$ has at most two nontrivial vertices.

By Lemma 2 and by $F(G'') \geq 3$, $|D_2(G'') \cup D_3(G'')| \geq 5$. By Claim 2 and Claim 3, we must have $D_3(G'') = \emptyset$ and $v(e), v(e') \in D_2(G'')$, and $D_2(G'')$ must have exactly one nontrivial vertex and two trivial vertices other than $v(e)$, $v(e')$. Let v, u be the two trivial vertices in $D_2(G'') - \{v(e), v(e')\}$. It follows by (1) and $n \geq 27$ that

$$uv \in E(G). \tag{7}$$

If $V(G'') = D_2(G'')$, then G'' is a 5-cycle and so (ii) of Theorem 1 must hold. Otherwise, let H be the preimage in G of the unique nontrivial vertex in $D_2(G'')$. By Claim 1, $|V(H)| \geq 2n/3 - 3$. Pick a vertex y that is in the preimage of some vertex in $V(G'') - D_2(G'')$. We may assume that $yu \notin E(G)$ (or $yv \notin E(G)$), since that yu and yv are both in $E(G'')$ implies that G'' has a 3-cycle by (7), contrary to (ii) of Theorem E. By (1) and by $yu \notin E(G)$, we have $d(y) \geq 2n/3 - 4$. On the other hand, since $|N(y) \cap V(H)| \leq 1$, one has $d(y) \leq n - (|V(H)| - 1) - 2 < n/3 + 2$. This, together with $d(y) \geq 2n/3 - 4$, implies that $n < 18$, a contradiction.

This completes the proof of Theorem 1. ∎

Proof of Theorem 2: The proof of Theorem 2 is analogous to that of Theorem 1 and so it is omitted. ∎

Proof of Theorem 3: Consider a special case of the problem when G is a cubic graph. Let e, e' be given. Note that when e, e' are not adjacent in G, G has a spanning (e, e')-trail if and only if $G(e, e')$ has a spanning $(v(e), v(e'))$-trail by Lemma 1. If e, e' are not adjacent, then define $G^\star$ to be the graph obtained from $G(e, e')$ as indicated in Figure 2. If e, e' are adjacent in G, then define $G^\star = G$. Thus, for any given e, $e' \in E(G)$, G has a spanning (e, e')-trail if and only if $G^\star$ is hamiltonian. In Theorem 2.2 of [11], Garey *et al*. show that the problem of determining if an undirected 3-regular graph is hamiltonian is NP-complete. Thus, this NP-complete problem reduces to a special case of the problem of determining if a graph has a spanning (e, e')-trail. ∎

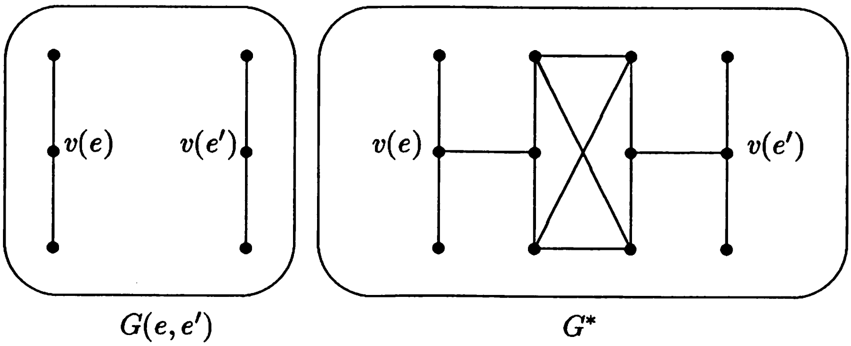

Figure 2: The graphs $G(e, e')$ and G^* with e, e' nonadjacent.

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