Characteristic polynomials of some covers of symmetric digraphs

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ABSTRACT. Let D be a symmetric digraph and A a finite group. We give a formula for the characteristic polynomial of a cyclic A-cover of D. This is a generalization of a formula for the characteristic polynomial of a regular covering of a graph. Furthermore, we discuss cyclic abelian covers of D.

1 Introduction

Graphs and digraphs treated here are finite simple. Two vertices are called *adjacent* if they are joined by an edge (arc). The *adjacency matrix* $\mathbf{A}(G)$ of a graph (digraph) G whose vertex set is $\{v_1, \ldots, v_n\}$ is a square matrix of order n, whose entry a_{ij} at the place (i,j) is equal to 1 if there exists an edge (arc) starting at the vertex v_i and terminating at the vertex v_j, and 0 otherwise. The *characteristic polynomial* $\Phi(G;\lambda)$ of a graph or a digraph G is defined by $\Phi(G;\lambda) = \det(\lambda\mathbf{I} - \mathbf{A}(G))$.

Schwenk [11] studied relations between the characteristic polynomials of some related graphs. Kitamura and Nihei [7] discussed the structure of regular double coverings of graphs by using their eigenvalues. Chae, Kwak and Lee [4] gave the complete computation of the characteristic polynomials of K_2 (or $\overline{K}_2$)-bundles over graphs. Kwak and Lee [8] obtained a formula for the characteristic polynomial of a graph bundle when its voltage assignment takes in an abelian group. Sohn and Lee [12] showed that the characteristic polynomial of a weighted K_2 (or $\overline{K}_2$)-bundles over a weighted graph G can be expressed as a product of characteristic polynomials of two weighted graphs whose underlying graphs are G. Mizuno and Sato [9] established an explicit decomposition formula for the characteristic polynomial of a derived graph covering of a graph G with voltages in any finite group, i.e., any regular covering of G.

Cheng and Wells [2] introduced cyclic triple covers of a complete symmetric digraph KD, and discussed isomorphism classes of cyclic triple covers of KD. Mizuno and Sato [10] defined cyclic p-tuple covers of a symmetric digraph D, where p is prime, and obtained a formula for the number of k-cyclic p-tuple covers of D with respect to a group Γ of automorphisms of D, for any $k \in GF(p)^*$.

Let D be a symmetric digraph, A a finite group and $g \in A$. In Section 2, we introduce cyclic A-covers of D as a generalization of regular graph coverings of a graph and cyclic p-tuple covers of a symmetric digraph, and give an decomposition formula for the characteristic polynomial of a g-cyclic A-cover of D. In Section 3, we consider the case that A is an abelian group, and examine the structure of g-cyclic A-covers of D. Furthermore, we establish another formula for the characteristic polynomial of g-cyclic A-cover of D.

For propositions concerning the representation of groups the reader is referred to [1].

2 Cyclic A-covers of symmetric digraphs

Let D be a symmetric digraph and A a finite group. A function $\alpha: A(D) \to A$ is called *alternating* if $\alpha(y,x) = \alpha(x,y)^{-1}$ for each $(x,y) \in A(D)$. For $g \in A$, a *g-cyclic A-cover* $D_g(\alpha)$ of D is the digraph defined as follows:

$V(D_g(\alpha)) = V(D) \times A$, and $((u,h),(v,k)) \in A(D_g(\alpha))$ if and only if $(u,v) \in A(D)$ and $k^{-1}h\alpha(u,v) = g$,

where $V(D)$ and $A(D)$ is the vertex set and the arc set of D, respectively. The *natural projection* $\pi: D_g(\alpha) \to D$ is a function from $V(D_g(\alpha))$ onto $V(D)$ which erases the second coordinates. A digraph D' is called a *cyclic A-cover* of D if D' is a g-cyclic A-cover of D for some $g \in A$. In the case that A is abelian, $D_g(\alpha)$ is called simply a *cyclic abelian cover*.

A graph H is called a *covering* of a graph G with projection $\pi: H \to G$ if there is a surjection $\pi: H \to G$ such that $\pi|_{N(v')}: N(v') \to N(v)$ is a bijection for all vertices $v \in V(G)$ and $v' \in \pi^{-1}(v)$. When a finite group Π acts on a graph (digraph) G, the *quotient graph (digraph)* G/Π is a simple graph (digraph) whose vertices are the Π-orbits on $V(G)$, with two vertices adjacent in G/Π if and only if some two of their representatives are adjacent in G. A covering $\pi: H \to G$ is said to be *regular* if there is a subgroup B

of the automorphism group $Aut\, H$ of H acting freely on H such that the quotient graph H/B is isomorphic to G.

Let G be a graph and A a finite group. Let $D(G)$ be the arc set of the symmetric digraph corresponding to G. Then a mapping $\alpha\colon D(G) \rightarrow A$ is called an *ordinary voltage assignment* if $\alpha(v,u) = \alpha(u,v)^{-1}$ for each $(u,v) \in D(G)$. The pair (G,α) is called an *ordinary voltage graph*. The *derived graph* G^{α} of the ordinary voltage graph (G,α) is defined as follows:

$V(G^{\alpha}) = V(G) \times A$ and $((u,h),(v,k)) \in D(G^{\alpha})$ if and only if $(u,v) \in D(G)$ and $k = h\alpha(u,v)$,

where $V(G)$ is the vertex set of G. Similarly to the case of a cyclic A-cover of a symmetric digraph, the *natural projection* $\pi\colon G^{\alpha} \rightarrow G$ is defined. The graph G^{α} is called a *derived graph covering* of G with voltages in A or an *A-covering* of G. The A-covering G^{α} is an $|A|$-fold regular covering of G. Furthermore, every regular covering of a graph G is an A-covering of G for some group A (see [5]).

The pair (D,α) of D and α can be considered as the ordinary voltage graph $(\tilde{D},\alpha)$ of the underlying graph $\tilde{D}$ of D. Thus the 1-cyclic A-cover $D_1(\alpha)$ corresponds to the A-covering $\tilde{D}^{\alpha}$. Furthermore, if p is prime, $A = GF(p)$ and $k \in A$, then the k-cyclic A-cover $D_k(\alpha)$ is the k-cyclic p-tuple cover of D (see [10]).

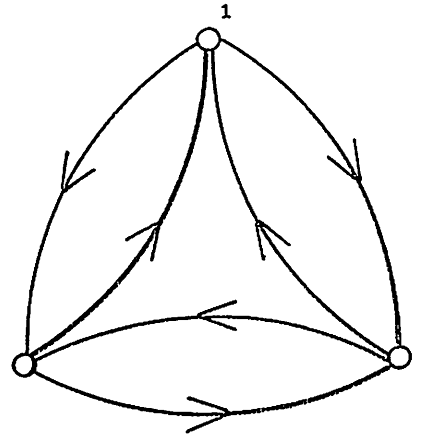

Figure 1. A symmetric digraph

Now, we give an example. Let D be the symmetric digraph of Figure 1 and $A = Z_3 = \{0,1,-1\}$ (the additive group). Furthermore, let $\alpha\colon A(D) \rightarrow Z_3$ be the alternating function such that $\alpha(1,2) = 0$, $\alpha(2,3) = 1$ and $\alpha(3,1) = -1$. Then the 1-cyclic Z_3-cover (or 1-cyclic 3-tuple cover) $D_1(\alpha)$ is shown in Figure 2. Arrange the vertices of $D_1(\alpha)$ in three blocks:

$$(1,0),(2,0),(3,0);(1,1),(2,1),(3,1);(1,-1),(2,-1),(3,-1).$$

We consider the adjacency matrix $\mathbf{A}(D_1(\alpha))$ under this order. Then $\mathbf{A}(D_1(\alpha))$ is given as follows:

$$\mathbf{A}(D_1(\alpha)) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

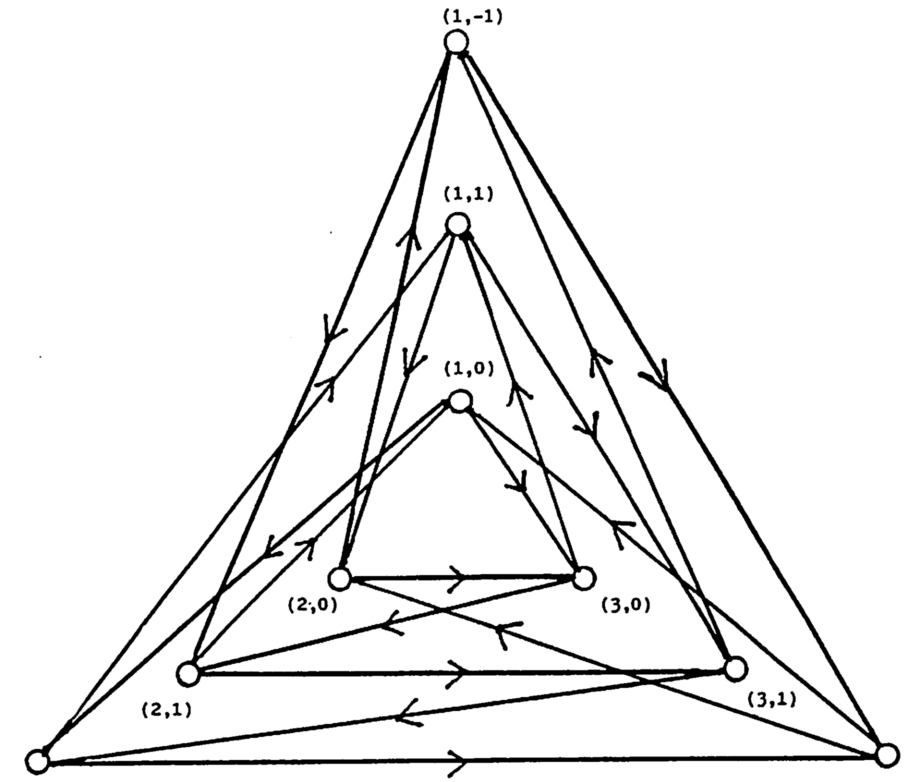

Figure 2. The 1-cyclic Z_3-cover $D_1(\alpha)$

The block diagonal sum $\mathbf{M}_1 \dot{+} \dots \dot{+} \mathbf{M}_s$ of square matrices $\mathbf{M}_1, \dots, \mathbf{M}_s$ is defined as the square matrix

$$\begin{bmatrix} \mathbf{M}_1 & & 0 \\ & \ddots & \\ 0 & & \mathbf{M}_s \end{bmatrix}$$

If $\mathbf{M}_1 = \mathbf{M}_2 = \dots = \mathbf{M}_s = \mathbf{N}$, then we write $s \circ \mathbf{N} = \mathbf{M}_1 \dot{+} \dots \dot{+} \mathbf{M}_s$. Furthermore, if $\mathbf{M}_1 = \dots = \mathbf{M}_{a_1} = \mathbf{N}_1$, $\mathbf{M}_{a_1+1} = \dots = \mathbf{M}_{a_1+a_2} = \mathbf{N}_2, \dots, \mathbf{M}_{s-a_t+1} = \dots = \mathbf{M}_s = \mathbf{N}_t$ $(a_1 + a_2 + \dots + a_t = s,\ a_i \geq 0,\ 1 \leq i \leq t)$, then we write $a_1 \circ \mathbf{N}_1 \dot{+} a_2 \circ \mathbf{N}_2 \dot{+} \dots \dot{+} a_t \circ \mathbf{N}_t = \mathbf{M}_1 \dot{+} \dots \dot{+} \mathbf{M}_s$. For a square matrix $\mathbf{B}$, we define $\Phi(\mathbf{B}; \lambda) := \det(\lambda \mathbf{I} - \mathbf{B})$. The *Kronecker product* $\mathbf{A} \otimes \mathbf{B}$ of matrices $\mathbf{A}$ and $\mathbf{B}$ is considered as the matrix $\mathbf{A}$ having the element a_{ij} replaced by the matrix $a_{ij}\mathbf{B}$.

Theorem 1. *Let D be a symmetric digraph, A a finite group, $g \in A$ and $\alpha\colon A(D) \to A$ an alternating function. Furthermore, let $\rho_1 = 1, \rho_2, \dots, \rho_t$ be the irreducible representations of A, and f_i the degree of ρ_i for each i, where $f_1 = 1$. For $h \in A$, the matrix $\mathbf{A}_h = (a_{uv}^{(h)})$ is defined as follows:*

$$a_{uv}^{(h)} := \begin{cases} 1 & \text{if } \alpha(u,v) = h \text{ and } (u,v) \in A(D), \\ 0 & \text{otherwise.} \end{cases}$$

Then the characteristic polynomial of the g-cyclic A-cover $D_g(\alpha)$ of D is

$$\Phi(D_g(\alpha); \lambda) = \Phi(D; \lambda) \cdot \prod_{j=2}^{t} \left\{ \Phi\left(\sum_{h \in A} \rho_j(h) \otimes \mathbf{A}_{hg}; \lambda \right) \right\}^{f_i},$$

where $\otimes$ is the Kronecker product of matrices.

Proof: Set $V(D) = \{v_1, \dots, v_m\}$ and $A = \{1 = g_1, g_2, \dots, g_n\}$. Arrange vertices of $D_g(\alpha)$ in n blocks:

$$(v_1, 1), \dots, (v_m, 1); (v_1, g_2), \dots, (v_m, g_2); \dots; (v_1, g_n), \dots, (v_m, g_n).$$

We consider the adjacency matrix $\mathbf{A}(D_g(\alpha))$ under this order. For $h \in A$, the matrix $\mathbf{P}_h = (p_{ij}^{(h)})$ is defined as follows:

$$p_{ij}^{(h)} = \begin{cases} 1 & \text{if } g_i h = g_j, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that $p_{ij}^{(h)} = 1$, i.e., $g_j = g_i h$. Then $((u, g_i), (v, g_j)) \in A(D_g(\alpha))$ if and only if $(u,v) \in A(D)$ and $g_j^{-1} g_i \alpha(u,v) = g$, i.e., $\alpha(u,v) = g_i^{-1} g_j g = g_i^{-1} g_i h g = hg$. Thus we have

$$\mathbf{A}(D_g(\alpha)) = \sum_{h \in A} \mathbf{P}_h \otimes \mathbf{A}_{hg}.$$

Let ρ be the right regular representation of A. Then we have $\rho(h) = \mathbf{P}_h$ for $h \in A$. Furthermore, there exists a regular matrix $\mathbf{P}$ such that

$$\mathbf{P}^{-1}\rho(h)\mathbf{P} = (1) \dotplus f_2 \circ \rho_2(h) \dotplus \ldots \dotplus f_t \circ \rho_t(h) \text{ for each } h \in A,$$

where t is the number of distinct irreducible representations of A. Putting

$$\mathbf{B} = (\mathbf{P}^{-1} \otimes \mathbf{I}_m)\mathbf{A}(D_g(\alpha))(\mathbf{P} \otimes \mathbf{I}_m),$$

we have

$$\mathbf{B} = \sum_{h \in A} \{(1) \dotplus f_2 \circ \rho_2(h) \dotplus \ldots \dotplus f_t \circ \rho_t(h)\} \otimes \mathbf{A}_{hg}.$$

Note that $\mathbf{A}(D) = \sum_{h \in A} \mathbf{A}_{hg}$. Therefore it follows that

$$\Phi(D_g(\alpha); \lambda) = \Phi(\mathbf{B}; \lambda) = \Phi(D; \lambda) \cdot \prod_{j=2}^{t} \left\{ \Phi\left(\sum_h \rho_j(h) \otimes \mathbf{A}_{hg}; \lambda \right) \right\}^{f_j}.$$

□

Corollary 1. $\Phi(D; \lambda) | \Phi(D_g(\alpha); \lambda)$.

Let D be the symmetric digraph corresponding to a graph G. Then, note that $\mathbf{A}(D) = \mathbf{A}(G)$.

Corollary 2 (9, Theorem 1). *Let G be a graph, A a finite group and $\alpha\colon A(D) \to A$ an ordinary voltage assignment. Let ρ_i, f_i be as in Theorem 1. Then the characteristic polynomial of the A-covering G^α of G is*

$$\Phi(G^\alpha; \lambda) = \Phi(G; \lambda) \cdot \prod_{j=2}^{t} \left\{ \Phi\left(\sum_h \rho_j(h) \otimes \mathbf{A}_h; \lambda \right) \right\}^{f_j}.$$

3 Cyclic abelian covers of symmetric digraphs

Let D be a symmetric digraph, A a finite abelian group and A^* the character group of A. For the mapping $f\colon A(D) \to A$, a pair $D_f = (D, f)$ is called a *weighted symmetric digraph*. Given any weighted symmetric digraph D_f, the adjacency matrix $\mathbf{A}(D_f) = (a_{f,uv})$ of D_f is the square matrix of order $|V(D)|$ defined by

$$a_{f,uv} = a_{uv} \cdot f(u, v).$$

The characteristic polynomial of D_f is that of its adjacency matrix, and is denoted $\Phi(D_f; \lambda)$ (see [12]).

Corollary 3. *Let D be a symmetric digraph, α an alternating function from $A(D)$ to a finite abelian group A, and $g \in A$. Then we have*

$$\Phi(D_g(\alpha);\lambda) = \prod_{\chi \in A^*} \Phi(D_{\chi(g)^{-1}(\chi\circ\alpha)};\lambda).$$

Proof: Each irreducible representation of A is linear, and these constitute the character group A^*. By Theorem 1, we have

$$\Phi(D_g(\alpha);\lambda) = \Phi(D;\lambda) \cdot \prod_{\chi \in A^* \setminus \{1\}} \Phi\left(\sum_{h \in A} \chi(h)\mathbf{A}_{hg};\lambda\right).$$

Since

$$\sum_h \chi(h)\mathbf{A}_{hg} = \sum_h \chi(g^{-1})\chi(hg)\mathbf{A}_{hg} = \mathbf{A}(D_{\chi(g)^{-1}(\chi\circ\alpha)}),$$

it follows that

$$\Phi(D_g(\alpha);\lambda) = \prod_{\chi \in A^*} \Phi(D_{\chi(g)^{-1}(\chi\circ\alpha)};\lambda).$$

□

For example, we consider the 1-cyclic Z_3-cover $D_1(\alpha)$ of Figure 2. By Corollary 3, we have

$$\begin{aligned}\Phi(D_1(\alpha);\lambda) &= (\lambda^3 - 3\lambda - 2)(\lambda^3 - 3\zeta\lambda - 2)(\lambda^3 - 3\zeta^2\lambda - 2)\\ &= (\lambda^3 - 3\lambda - 2)(\lambda^6 + 3\lambda^4 - 4\lambda^3 + 9\lambda^2 - 6\lambda + 4)\\ &= \lambda^9 - 6\lambda^6 - 15\lambda^3 - 8,\end{aligned}$$

where $\zeta = (-1 + \sqrt{3}i)/2$.

Corollary 4. *Let D be a symmetric digraph with m vertices, $A = \langle g \rangle$ a cyclic group and $ord(g) = n$ the order of g. Let $\mathbf{1}\colon A(D) \to A$ be the function such that $\mathbf{1}(u,v) = 1$ for each $(u,v) \in A(D)$. Furthermore, set $\zeta = \exp(2\pi i/n)$. Then we have*

$$\Phi(D_g(\mathbf{1});\lambda) = \zeta^{-mn(n-1)/2} \prod_{k=0}^{n-1} \Phi(D;\zeta^k\lambda).$$

Proof: For $g^k \in A$, the character χ_k corresponding to g^k is defined by $\chi_k(g^j) = (\zeta^k)^j$. By Corollary 3, we have

$$\Phi(D_g(\mathbf{1});\lambda) = \prod_{k=0}^{n-1} \Phi(D_{\zeta^{-k}(\chi_k\circ\mathbf{1})};\lambda).$$

Since $\mathbf{1}(u,v) = 1$ for each $(\mu,v) \in A(D)$, we have $\zeta^{-k}(\chi_k \circ \mathbf{1}))(u,v) = \zeta^{-k}\chi_k(1) = \zeta^{-k} \cdot 1 = \zeta^{-k}$ for each $(u,v) \in A(D)$ and each k. Then the adjacency matrix $\mathbf{A}(D_{\zeta^{-k}}) = (a_{\zeta^{-k},uv})$ is given as follows:

$$a_{\zeta^{-k},uv} = \begin{cases} \zeta^{-k} & \text{if } (u,v) \in A(D), \\ 0 & \text{otherwise.} \end{cases}$$

Therefore it follows that

$$\Phi(D_{\zeta^{-k}};\lambda) = \det(\lambda\mathbf{I} - \zeta^{-k}\mathbf{A}(D)) = \zeta^{-mk}\Phi(D;\zeta^k\lambda).$$

Thus the result follows. □

We shall examine the structure of a cyclic abelian cover of a symmetric digraph. Let D be a symmetric digraph, A a finite abelian group, $g \in A$ and $\alpha\colon A(D) \to A$ an alternating function. The right action of A on the g-cyclic A-cover $D_g(\alpha)$ of D is defined as follows:

$$(u,k)^h = (u,hk)$$

for each $h \in A$ and $(u,k) \in V(D_g(\alpha))$. Since a 1-cyclic A-cover of D is an A-covering of the underlying graph $\tilde{D}$, we state the structure of nonunit-cyclic A-cover of D. For convenience' sake, we identify the 1-cyclic A-cover $D_1(\alpha)$ with the A-covering $\tilde{D}^\alpha$ of $\tilde{D}$.

Theorem 2 (The Structure Theorem). *Let D be a symmetric digraph, A a finite abelian group, $g \neq 1 \in A$ and $\alpha\colon A(D) \to A$ an alternating function. Let $ord(g) = n$ and $H = \langle g \rangle$ the subgroup of A generated by g. Furthermore, let $\beta\colon A(D) \to A/H$ be the alternating function such that $\beta(x,y) = \alpha(x,y)H$ for each $(x,y) \in A(D)$, and $\tilde{D}$ the underlying graph of D. Then the g-cyclic A-cover $D_g(\alpha)$ of D is the g-cyclic H-cover $(\tilde{D}^\beta)_g(\mathbf{1})$ of the A/H-covering $\tilde{D}^\beta$ of $\tilde{D}$.*

Proof: Let (x,y) be any arc of D. We shall examine the structure of the induced subdigraph $\langle\pi^{-1}(\{x,y\})\rangle_{D_g(\alpha)}$. Let $|A/H| = m$ and $\{h_1 = 1, h_2, \ldots, h_m\}$ the representatives of all (right) cosets of H in A. Set $\alpha(x,y) = c$. Then $((x,h),(y,k)) \in A(D_g(\alpha))$ if and only if $k^{-1}hc = g$, i.e., $k = hcg^{-1}$. Furthermore, $((y,h),(x,k)) \in A(D_g(\alpha))$ if and only if $k^{-1}hc^{-1} = g$, i.e., $k = hc^{-1}g^{-1}$. For each $i = 1,\ldots,m$ and $g^j \in H$, $((x,h_ig^j),(y,k)) \in A(D_g(\alpha))$ if and only if $k = h_ig^jcg^{-1} = h_icg^{j-1}$, etc. Thus we have

$$((x,h_ig^j),(y,h_icg^{j-1})) \text{ and } ((y,h_icg^j),(x,h_ig^{j-1})) \in A(D_g(\alpha)).$$

for each $i = 1,\ldots,m$ and $g^j \in H$. Therefore, for $i = 1,\ldots,m$, we obtain a dicycle or a union of two dicycles as follows:

$((x,h_i),(y,h_icg^{-1}),(x,h_ig^{-2}),(y,h_icg^{-3}),\ldots,(y,h_icg),(x,h_i))(n\colon \text{ odd});$
$((x,h_i),(y,h_icg^{-1}),(x,h_ig^{-2}),\ldots,(y,h_icg),(x,h_i))$
$\quad\cup((x,h_ig^{-1}),(y,h_icg^{-2}),(x,h_ig^{-3}),\ldots,(y,h_ic),(x,h_ig^{-1}))(n\colon \text{ even}).$

Now, we consider the quotient digraph $D_H = D_g(\alpha)/H$. Set $H_i = Hh_i$ $(1 \le i \le m)$. Then we have

$$A(\langle\pi^{-1}(\{x,y\})\rangle_{D_H}) = \{((x,H_i),(y,cH_i)),((y,cH_i),(x,H_i))|1 \le i \le m\}.$$

However, in the A/H-covering $\tilde{D}^\beta$ of $\tilde{D}$, $((x,H_i),(y,K)) \in D(\tilde{D}^\beta)$ if and only if $K = H_i\beta(x,y) = H_icH = Hh_icH = c(Hh_i) = cH_i$, for each $i = 1,\ldots,m$. Thus we have $((x,H_i),(y,cH_i))$ and $((y,cH_i),(x,H_i)) \in D(\tilde{D}^\beta)$ $(1 \le i \le m)$. Therefore it follows that $D_g((\alpha)/H = \tilde{D}^\beta$.

Next, we consider the g-cyclic A/H-cover $(\tilde{D}^\beta)_g(1)$ of $\tilde{D}^\beta$. Two vertices $((x,H_i),g^j),((y,cH_i),h)$ are adjacent in $(\tilde{D}^\beta)_g(1)$ if and only if $h^{-1}g^j1 = g$, i.e., $h = g^{j-1}$. Thus we have

$$(((x,H_i),g^j),((y,cH_i),g^{j-1})) \in A((\tilde{D}^\beta)_g(1))$$

for each $i = 1,\ldots,m$ and $j = 0,1,\ldots,n-1$. Identifying $((y,cH_i),g^j)$ with (y,h_icg^j), etc, we have

$$((x,h_ig^j),(y,h_icg^{j-1})) \in A((\tilde{D}^\beta)_g(1))$$

for each i and j. Hence it follows that $D_g(\alpha) = (\tilde{D}^\beta)_g(1)$. □

Corollary 5. $\Phi(\tilde{D}^\beta;\lambda)|\Phi(D_g(\alpha);\lambda)$.

Finally, we shall apply Theorem 2 for the characteristic polynomial of a cyclic abelian cover of a symmetric digraph.

Corollary 6. *Let D be a symmetric digraph, A a finite abelian group, $g \ne 1 \in A$ and $\alpha\colon A(D) \to A$ an alternating function. Set $|V(D)| = t$, $ord(g) = n$, $|A| = nq$ and $H = \langle g\rangle$. Furthermore, let $\beta\colon A(D) \to A/H$ be the alternating function such that $\beta(x,y) = \alpha(x,y)H$ for each $(x,y) \in A(D)$. Then the characteristic polynomial of the g-cyclic A-cover $D_g(\alpha)$ of D is*

$$\Phi(D_g(\alpha);\lambda) = \zeta^{-qtn(n-1)/2}\prod_{k=0}^{n-1}\prod_{\chi_k\in(A/H)^*}\Phi(D_{\chi_k\circ\beta};\zeta^k\lambda),$$

where $(A/H)^*$ is the character group of A/H and $\zeta = \exp(2\pi i/n)$.

Proof: By Theorem 2, Corollaries 3,4 and the fact that $\tilde{D}^\beta = D_1(\beta)$. □

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