Further Notes on: Largest Triangle-free Subgraphs in Powers of Cycles

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ABSTRACT. We derive upper bounds for the number of edges in a triangle-free subgraph of a power of a cycle, extending results of an earlier paper by Bondy and Locke. In particular, the solution found for the case $m = 20$ is a decomposition of $3C_n^{20}$ into odd complete graphs.

Introduction

We recall some of the terminology which was introduced in [1]. For positive integers m and n such that $n \geq 2m+1$, we denote by $C_{m,n}$ the graph with vertex set $\{0, 1, \ldots, n-1\}$ and edge set $\{ij : i-j \equiv \pm k (\mod n), 1 \leq k \leq m\}$; the graph $C_{m,n}$ is a circulant, the m-th power of the n-cycle $C_{1,n}$, and is sometimes denoted C_n^m. For a graph G and a positive integer x, we write xG for the graph obtained from G by replacing every edge by x parallel edges. Let $T_{m,n}$ be a triangle-free subgraph of $C_{m,n}$, with the maximum number of edges, and put

$$t_{m,n} := \frac{| E(T_{m,n}) |}{| E(C_{m,n}) |} = \frac{| E(T_{m,n}) |}{mn}.$$

If $n = 2m+1$, $C_{m,n}$ is isomorphic to the complete graph K_n. In this case, by Turán's theorem [3],

$$| E(T_{m,n}) | = \left\lfloor \frac{n^2}{4} \right\rfloor = \frac{n^2-1}{4}$$

and hence

$$t_{m,n} = \frac{n+1}{2n} = \frac{m+1}{2m+1}.$$

ARS COMBINATORIA 49(1998), pp. 65-77

For $n >> m$, Chung and Trotter [2] prove that

$$.586 \approx 2-\sqrt{2} \leq t_{m,n} \leq \frac{5+\sqrt{3}}{11} \approx .612.$$

Chung and Trotter's lower bound results from alternately colouring d vertices along $C_{m,n}$ white, followed by d vertices black for a suitable choice of d. This colouring induces a bipartite subgraph, $B_{m,n,d}$, of $C_{m,n}$. The maximum value of $\mid E(B_{m,n,d}) \mid$ is attained when

$$d = d_m = \left\lfloor 0.5+\sqrt{\binom{m+1}{2}} \right\rfloor \quad \textit{and} \quad 2d \mid n .$$

Aaron Meyerowitz (personal communication) noted that there are two values of d which achieve the maximum value of $\mid E(B_{m,n,d}) \mid$ for some specific m. We give a short argument to demonstrate this.

Let $f_{m,d} = \frac{1}{n} \mid E(B_{m,n,d}) \mid$. Then

$$\begin{aligned} f_{m,d} &= \frac{1}{d}\left(\sum_{i=1}^{d} i + \sum_{i=1}^{m-d}(d-i)\right) \\ &= \frac{1}{d}\left(\frac{d(d+1)}{2} + (m-d)d - \frac{(m-d)(m-d+1)}{2}\right) \\ &= 2m-d+1-\frac{m^2+m}{2d}. \end{aligned}$$

Thus

$$f_{m,d+1} = 2m-d-\frac{m^2+m}{2d+2}.$$

Now,

$$\begin{aligned} & f_{m,d} = f_{m,d+1} \\ \iff & -d+1-\frac{m^2+m}{2} = -d-\frac{m^2+m}{2} \\ \iff & 2d(d+1) = m^2+m \\ \iff & 2(2d+1)^2 = (2m+1)^2+1. \end{aligned}$$

This last equation is a Pell equation in the variables $\alpha = 2d+1$ and $\beta = 2m+1$. Solutions are given by

$$\alpha_0 = 1,\ \alpha_1 = 5,\ \beta_0 = 1,\ \beta_1 = 7,\ \alpha_{i+2} = 6\alpha_{i+1} - \alpha_i,\ \beta_{i+2} = 6\beta_{i+1} - \beta_i.$$

The first few values of m for which there are two choices of d are thus $m = 3$, $m = 20$ and $m = 119$.

In [1], Bondy and Locke introduced a method to compute a good upper bound on $t_{m,n}$. In the case $2d \mid n$, the upper bound coincides with the lower bound and thus exact values of $t_{m,n}$ were calculated for $2 \le m \le 16$ in these cases.

In this note, we establish similar results for $17 \le m \le 24$. We use the same general method as in [1]. The upper bound will be derived from a linear program. The method used in that paper generated $O(2^{m-2})$ constraints, tested each constraint to see if passed through the proposed primal solution, $B_{m,n}$, and then ran a linear program on these constraints. For example, in the case $m = 16$, approximately 2^{14} constraints are generated, but only 353 constraints are needed for the linear program. Of these, only 10 actually show up in the dual solution that was found.

To extend the values of m for which we can compute the upper bound, it becomes necessary to reduce the number of constraints to which we apply the complementary slackness test. To avoid storage and time problems, it would also be desirable to reduce the number of constraints that are used in the linear program. The results in [1] were obtained by using a mainframe. An additional goal of this project was to trim the work down sufficiently that it became possible to use a personal computer. The programming was done in *maple* using exact arithmetic and the revised simplex algorithm (rather than maple's simplex package).

Upper Bounds

Again, we recall the method of [1]. Our goal is to find a suitable integer x and a decomposition of $xC_{m,n}$ into edge-disjoint complete graphs and from this decomposition determine the upper bound for the size of a largest triangle-free subgraph of $C_{m,n}$. We say that an edge ij of $C_{m,n}$ is of *type* k, $1 \le k \le m$, if $i - j \equiv \pm k (\mod n)$. There are exactly n edges of type k in $C_{m,n}$ for each k, $1 \le k \le m$. Let x_k denote the proportion of these edges belonging to the triangle-free subgraph $T_{m,n}$, $1 \le k \le m$. Then

$$t_{m,n} = \frac{|E(T_{m,n})|}{mn} = \frac{1}{m}\sum_{k=1}^{m} x_k.$$

We derive an upper bound c_m for $t_{m,n}$, valid for all $n > 2m+1$, by deriving an upper bound for $\sum_{k=1}^{m} x_k$.

Let $K[a_1, a_2, \dots, a_r]$ be the complete subgraph of $C_{m,n}$ with vertex set $\{a_1, a_2, \dots, a_r\}$, where each a_i is a nonnegative integer and $a_1 < a_2 < \dots < a_r \le a_1 + m$. Consider the rotations $K[a_1 + \mu, a_2 + \mu, \dots, a_r + \mu]$ of this graph, where $\mu = 0, 1, \dots, n-1$, and additions are performed modulo n. These n complete graphs together cover each edge of type k precisely once for each each pair i, j such that $1 \le i < j \le r$ and $a_j - a_i = k$. On the other hand, by Turán's theorem, each of these complete graphs contains at most $\left\lfloor \frac{r^2}{4} \right\rfloor$ edges of $T_{m,n}$. Thus, $K[a_1, a_2, \dots, a_r]$ determines an linear inequality $I[a_1, a_2, \ldots, a_r]$ in the x_k's, namely

$$\sum_{1 \le i < j \le r} x_{a_j - a_i} \le \left\lfloor \frac{r^2}{4} \right\rfloor .$$

For instance, $I[0, 1, 2, 3, 5]$ is the inequality

$$3x_1 + 3x_2 + 2x_3 + x_4 + x_5 \le 6.$$

In each $K[a_1, a_2, \ldots, a_r]$, we may assume that $a_1 = 0$ and thus that $\{a_1, a_2, \ldots, a_r\} \subseteq \{0, 1, \ldots, m\}$. Even subsets (of cardinality greater than two) do not help, since $K_{2\alpha}$ can be $(2\alpha - 2)$-covered by copies of $K_{2\alpha-1}$ and the resulting constraints are just as strong. Thus we need only consider 2^{m-1} subsets.

We are only interested in the those inequalities that are tight at the proposed primal solution $B_{m,n,d}$, namely

$$x* = \frac{1}{d}(1, 2, 3, \ldots, d-1, d, d-1, \ldots, 2d-m).$$

In [1], each of the 2^{m-1} subsets of odd cardinality was tested. To save some calculation time, it was noted that

$$I[a_1, a_2, \ldots, a_{2r+1}] \text{ and } I[0, a_{2r+1} - a_{2r}, a_{2r+1} - a_{2r-1}, \ldots, a_{2r+1} - a_1]$$
$$\text{are the same inequality} \qquad (O_1)$$

and thus we may assume that $2a_{r+1} \le a_1 + a_{2r+1}$. Thus, we check approximately 2^{m-2} constraints.

For $m = 16$, this is a large number of constraints to generate and test. We now show that this number can be severely reduced. For the first few values of m, the inequalities $I[a_1, a_2, \ldots, a_{2r+1}]$ which are tight at the primal solution led to the following observation.

Observation

$I[a_1, a_2, \ldots, a_{2r+1}]$ is tight at the proposed primal solution if and only if for any set S of d consecutive integers, chosen from $\{0, 1, 2, \ldots, m\}$, $| \{a_1, a_2, \ldots, a_{2r+1}\} \cap S | \in \{r, r+1\}$.

We restate this in a format that is useful for implementing on a computer, allowing $I[a_1, a_2, \ldots, a_{2r+1}]$ to be tested in $O(r)$ steps without generating the inequality. Simply generating the inequality takes $O(r^2)$ steps. A further saving is achieved by grouping inequalities and testing several at the same time.

Theorem 1. *$I[a_1, a_2, \ldots, a_{2r+1}]$ is tight at the proposed primal solution only if*

(i) $\{a_1, a_2, \ldots, a_r\} \subseteq \{0, 1, \ldots, m-d\}$ *and* $\{a_{r+2}, a_{r+3}, \ldots, a_{2r+1}\} \subseteq \{d, d+1, \ldots, m\}$;

(ii) *furthermore,* $a_{r+j+1} \geq a_j + d$, *for* $j = 1, 2, \ldots, r$ *and* $a_{r+j} \leq a_{j+1} + d - 1$, *for* $j = 1, 2, \ldots, r\}$.

For a given r, we generate two subsets

$$S_1 = \{a_2, \ldots, a_r\} \subseteq \{0, 1, \ldots, m-d\}$$

and

$$S_2 = \{a_{r+2}, a_{r+3}, \ldots, a_{2r+1}\} \subseteq \{d, d+1, \ldots, m\}.$$

We have already noted that we may assume that $a_1 = 0$. We now test whether or not the $2r$ values chosen satisfy condition (ii) of Theorem 1. If not, we generate other sets (usually in a nested fashion). If the values chosen so far do satisfy Theorem 1(ii), then we generate the inequality $I[a_1, a_2, \ldots, a_{r+1}, \ldots, a_{2r+1}]$ for each a_{r+1} with $a_{r+1} \leq \frac{1}{2}a_{2r+1}$, $a_{r+1} > a_r$ and $a_{r+1} \geq a_{2r} - d + 1$.

During the running of the program, it became obvious that if (O_1) is avoided and if $a_1 = 0$, then very few of the generated inequalities were the same. For example, $I[0, 3, 5, 6, 15, 19, 20]$ and $I[0, 1, 3, 6, 15, 16, 20]$ are identical, but, for $m = 20$, this is the only inequality generated twice. This permits a time saving during the setup phase, since we do not need to check that each generated inequality is distinct from the previously generated inequalities. However, it may add a few columns to the linear programming problem.

The number of inequalities generated by this method is at most

$$\begin{aligned}(d+1-\frac{m}{2}) \sum_{r=0}^{m-d+1} \binom{m-d}{r-1} \binom{m-d+1}{r} \\ \leq \left(\frac{2d+2-m}{2}\right) \sum_{r=0}^{m-d+1} \binom{m-d+1}{r}^2 \\ \leq \left(\frac{2d+2-m}{2}\right) \binom{2m-2d+2}{m-d+1}.\end{aligned}$$

We compare this with the number of distinct inequalities through the primal solution in Table 1. It is obvious that we have grossly overcounted the number of inequalities and that $\binom{2m-2d+2}{m-d+1}$ is closer to the true order of magnitude. The factor $\binom{2m-2d+2}{m-d+1}$ helps explain why the number of inequalities makes a large jump when $m-d+1$ increases. One should observe the difference between the actual of number inequalities and the column for 2^{m-2}.

m	d	actual	$m-d+1$	$\binom{2m-2d+1}{m-d+1}$	$\left(\frac{2d+2-m}{2}\right)$	$\left(\frac{2d+2-m}{2}\right)\binom{2m-2d+1}{m-d+1}$	2^{m-2}
2	2	2	1	2	2	4	1
3	2	3	2	6	1.5	9	2
4	3	5	2	6	2	12	4
5	4	6	2	6	2.5	15	8
6	5	8	2	6	3	18	16
7	5	17	3	20	2.5	50	32
8	6	23	3	20	3	60	64
9	7	26	3	20	3.5	70	128
10	7	65	4	70	3	210	256
11	8	77	4	70	3.5	245	512
12	9	92	4	70	4	280	1024
13	10	104	4	70	4.5	315	2048
14	10	272	5	252	4	1008	4096
15	11	308	5	252	4.5	1134	8192
16	12	353	5	252	5	1260	16384
17	12	919	6	924	4.5	4158	32768
18	13	1045	6	924	5	4620	65536
19	14	1162	6	924	5.5	5082	131072
20	15	1288	6	924	6	5544	262144
20	14	3622	7	3432	5	17160	524288
20	14 & 15	701					

Table 1

For $m = 20$ we stored a 20×1308 matrix. It is obvious that space considerations become important. We can still run larger problems without storing the system of inequalities. We use the revised simplex algorithm, keep the basis matrix B and either B^{-1} or the eta-matrix factorization of B. For each pivot, regenerate the inequalities until a suitable entering column is found. For many of the pivots, we hope that the entering column will be found near the beginning of the search through the possible inequalities.

This approach was used for the cases $21 \leq m \leq 24$. We store B, B^{-1}, a list $T_1, \dots, T_m$ such that $I[T_i]$ corresponds to the i^{th} column of B and, for each T_i we store a number p_i, where p_i is the number of inequalities generated up to and including T_i. The numbers p_i are used to ensure that the simplex algorithm does not cycle. Since we generate the T_i in the same order for each pivot, p_i is simply the index for the column which would correspond to $I[T_i]$ if we were to store the entire matrix. The sets $T_i, \dots, T_m$ are stored for the purposes of verification of the solution. At this stage of the process, one of the tests for redundant columns was removed since it takes as long to perform the test as to decide whether the generated column corresponds to an entering variable. One version of the linear program was run which generated the T_i from largest to smallest, but this version generated more columns than were generated by running from smallest to largest.

There are two seemingly reasonable ways to get an initial feasible solution. An obvious initial basic feasible solution is obtained by beginning with

$B = I$ and c an m-vector of ones. An alternate initialization is provided by setting

$$B_m = \begin{pmatrix} B_{m-1} & 0 \\ 0 & 1 \end{pmatrix}$$

and setting c to be the appropriate vector. We do not assume that any of the inequalities in the solution for the case $m - 1$ are actually used in the solution for the case m. However, this may save several of the initial iterations. Of course, it is also possible that this might lead to a particularly bad basic solution that we would not normally have hit.

It might be possible that some random choice of inequalities might provide a better initial solution for the simplex method. However, rather than choosing a random set of inequalities, perhaps a better yet initialization for case m might be to *stretch* the covering from case $m - 1$. Thus, for each $I[T_i]$ obtained in case $m - 1$, use $I[T_i']$ for case m, where

$$T_i' = \{j \in T_i : j < d\} \cup \{j + 1 : j \in T_i \wedge j \geq d\}.$$

A curious feature is that the 20 complete graphs from $m = 20$ when stretched only gave rise to a rank 19 matrix. There would be no way to choose one more constraint to obtain a rank 21 matrix. Even if there is a column that can be appended to the stretched basis from the previous case, it is not immediately obvious that the initial basis matrix thus formed will give a non-negative solution.

However, we can combine the previous two ideas.

For $m = 23$, we started with

$$B_{23} = \begin{pmatrix} B_{22} & 0 \\ 0 & 1 \end{pmatrix},$$

and set up a special matrix A' representing the stretched inequalities (called $T_{23}^{stretched}$ in appendix) obtained from the solution to the case $m = 22$. Each of these inequalities satisfies the conditions of Theorem 1 and, therefore, is an equality at the presumed primal solution. This will be true whenever $d_{m+1} > d_m$.

For each pivot, we look first at the columns of A' before generating the remaining inequalities. Since the number of pivots taken using this method for the case $m = 23$ was 255 and the number of pivots taken for the case $m = 22$ without this modification was 1278, it appears to have been a useful refinement. This seems especially likely since a slight error in the first input of $T_{23}^{stretched}$ led to a significantly larger number of columns generated.

We note that $d_{24} = 17 = d_{23}$ and thus the stretched inequalities from the case $m = 23$ might not pass through the primal solution for the case $m = 24$. However, the stretched inequalities approach for the case $m = 24$

does indeed give fewer columns generated than we had for the case $m = 21$ with either of the two starting bases. It would seem that manipulating a solution to the case $m - 1$ is a worthwhile enhancement for any case m.

The case $m = 20$ admits two values of d. In most cases there is only one value of d, but $\sqrt{\binom{m+1}{2}}$ falls between two values d_1 and d_2. One of these will be chosen as d. Let x^1 and x^2 denote the primal solutions corresponding to d_1 and d_2. It may be that taking the inequalities that are tight at both x^1 and x^2, together with a few extra inequalities (perhaps from triangles) that are tight at x^*, might speed up the computations even further. This approach has not been tried by the author.

The coverings of $C_{21,n}$ starting from different bases cover each edge 20 times, and use many of the same complete graphs. This may suggest some robustness to the solution process. The coverings of $C_{23,n}$ starting with the same basis, but favouring slightly different complete graphs, again use many of the same complete graphs. But here we see some evidence that conditions we impose can affect the complexity of the solution.

We note that the dual solutions are more complex than the primal solutions. It would be nice to cover $C_{m,n}$ with complete graphs so as to minimize the total number of times each edge is covered, subject to the covering being a solution to the dual problem. In the linear programming approach, any solution is expressed in terms of rational numbers, and a suitable multiple yields an integer covering. One could conceivably minimize the covering with an integer programming approach, although the running time would probably be prohibitive.

We should perhaps note that the covering of $C_{20,n}$ obtained by the linear program covers each edge three times. One might hazard a guess that $C_{119,n}$ will also have a small covering.

References

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[2] F.R.K. Chung and W.T. Trotter, Jr. Triangle-free graphs with restricted bandwidth. *Progress in Graph Theory* (J.A. Bondy and U.S.R. Murty, eds.), 175–190. Academic Press, Toronto, 1984.

[3] P. Turán. On an extremal problem in graph theory (in Hungarian). *Mat. Fig. Lapok.* 48 (1941), 436–452.

Appendix

$n = 17$ (919 inequalities) $d = 12$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	$\cdot \mathbf{x} \leq$
6I[0,3,13]	0	0	6	0	0	0	0	0	0	6	0	0	6	0	0	0	0	12
2I[0,3,14]	0	0	2	0	0	0	0	0	0	0	2	0	0	2	0	0	0	4
2I[0,5,15]	0	0	0	0	2	0	0	0	0	2	0	0	0	0	2	0	0	4
8I[0,5,16]	0	0	0	0	8	0	0	0	0	0	8	0	0	0	0	8	0	16
12I[0,6,15]	0	0	0	0	0	12	0	0	12	0	0	0	0	0	12	0	0	24
5I[0,6,14]	0	0	0	0	0	5	0	5	0	0	0	0	0	5	0	0	0	10
13I[0,6,16]	0	0	0	0	0	13	0	0	0	13	0	0	0	0	0	13	0	26
7I[0,6,17]	0	0	0	0	0	7	0	0	0	0	7	0	0	0	0	0	7	14
5I[0,1,10,12,17]	5	5	0	0	5	0	5	0	5	5	5	5	0	0	0	5	5	30
3I[0,2, 7,14,17]	0	3	3	0	3	0	6	0	0	3	0	3	0	3	3	0	3	18
2I[0,2, 7,14,16]	0	4	0	0	2	0	4	0	2	0	0	2	0	4	0	2	0	12
5I[0,2,10,14,17]	0	5	5	5	0	0	5	5	0	5	0	5	0	5	5	0	5	30
3I[0,3, 8,14,17]	0	0	6	0	3	3	0	3	3	0	3	0	0	6	0	0	3	18
2I[0,4, 8,13,17]	0	0	0	6	2	0	0	2	4	0	0	0	4	0	0	0	2	12
10I[0,1, 2, 9,13,14,17]	30	10	10	20	10	0	10	20	10	0	10	20	20	10	10	10	10	120
2I[0,1, 4, 8,13,15,17]	2	4	2	6	2	0	4	2	4	0	2	2	4	2	2	2	2	24
3I[0,2, 3,10,13,15,17]	3	9	6	3	3	0	6	3	0	6	3	3	6	3	6	0	3	36
	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	410

$$40 \sum_{i=1}^{17} x_i \leq 410$$

$$c_{17} \leq \frac{41}{68}$$

$m = 18$ (1045 inequalities) $d = 13$

																		$.\mathbf{x} \leq$
7I[0,4,15]	0	0	0	7	0	0	0	0	0	0	7	0	0	0	7	0	0	0	14
I[0,5,15]	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	2
6I[0,6,16]	0	0	0	0	0	6	0	0	0	6	0	0	0	0	0	6	0	0	12
I[0,7,17]	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	2
4I[0,7,18]	0	0	0	0	0	0	4	0	0	0	4	0	0	0	0	0	0	4	8
29I[0,8,17]	0	0	0	0	0	0	0	29	29	0	0	0	0	0	0	0	29	0	58
5I[0,1, 7,14,16]	5	5	0	0	0	5	10	0	5	0	0	0	5	5	5	5	0	0	30
15I[0,1, 8,14,18]	15	0	0	15	0	15	15	15	0	15	0	0	15	15	0	0	15	15	90
19I[0,2, 7,15,17]	0	38	0	0	19	0	19	19	0	19	0	0	19	0	38	0	19	0	114
11I[0,2, 8,14,18]	0	11	0	11	0	22	0	11	0	11	0	11	0	11	0	11	0	11	66
25I[0,2, 9,14,18]	0	25	0	25	25	0	25	0	50	0	0	25	0	25	0	25	0	25	150
21I[0,3, 6,16,17]	21	0	42	0	0	21	0	0	0	21	21	0	21	21	0	21	21	0	126
13I[0,3, 8,15,18]	0	0	26	0	13	0	13	13	0	13	0	13	0	0	26	0	0	13	78
10I[0,3, 9,14,18]	0	0	10	10	10	10	0	0	20	0	10	0	0	10	10	0	0	10	60
11I[0,4, 5,16,17]	22	0	0	11	11	0	0	0	0	0	11	22	11	0	0	11	11	0	66
9I[0,4, 7,15,18]	0	0	18	9	0	0	9	9	0	0	18	0	0	9	9	0	0	9	54
17I[0,5, 6,16,18]	17	17	0	0	17	17	0	0	0	17	17	17	17	0	0	17	0	17	102
8I[0,1, 2, 6,13,14,17]	24	8	8	16	8	8	8	8	0	0	16	16	16	8	8	8	8	0	96
	104	104	104	104	104	104	104	104	104	104	104	104	104	104	104	104	104	104	1128

$$c_{18} = \frac{47}{78}$$

$m = 19$ (1162 inequalities) $d = 14$

11I[0,6,17]	[0, 0, 0, 0, 0,11, 0, 0, 0, 0,11, 0, 0, 0, 0, 0,11, 0, 0].x≤	22
14I[0,7,18]	[0, 0, 0, 0, 0, 0,14, 0, 0, 0,14, 0, 0, 0, 0, 0, 0,14, 0].x≤	28
18I[0,8,18]	[0, 0, 0, 0, 0, 0, 0,18, 0,18, 0, 0, 0, 0, 0, 0, 0,18, 0].x≤	36
10I[0,9,18]	[0, 0, 0, 0, 0, 0, 0, 0,20, 0, 0, 0, 0, 0, 0, 0, 0,10, 0].x≤	20
I[0,1, 9,15,18]	[1, 0, 1, 0, 0, 1, 0, 1, 2, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0].x≤	6
9I[0,1, 9,15,19]	[9, 0, 0, 9, 0, 9, 0, 9, 9, 9, 0, 0, 0, 9, 9, 0, 0, 9, 9].x≤	54
5I[0,2, 7,16,18]	[0,10, 0, 0, 5, 0, 5, 0, 5, 0, 5, 0, 0, 5, 0,10, 0, 5, 0].x≤	30
2I[0,2, 8,14,19]	[0, 2, 0, 0, 2, 4, 0, 2, 0, 0, 2, 2, 0, 2, 0, 0, 2, 0, 2].x≤	12
6I[0,2, 8,15,19]	[0, 6, 0, 6, 0, 6, 6, 6, 0, 0, 6, 0, 6, 0, 6, 0, 6, 0, 6].x≤	36
4I[0,2, 8,16,19]	[0, 4, 4, 0, 0, 4, 0, 8, 0, 0, 4, 0, 0, 4, 0, 4, 4, 0, 4].x≤	24
2I[0,2, 9,14,19]	[0, 2, 0, 0, 4, 0, 2, 0, 2, 2, 0, 2, 0, 2, 0, 0, 2, 0, 2].x≤	12
10I[0,2, 9,15,19]	[0,10, 0,10, 0,10,10, 0,10,10, 0, 0,10, 0,10, 0,10, 0,10].x≤	60
16I[0,3, 8,16,19]	[0, 0,32, 0,16, 0, 0,32, 0, 0,16, 0,16, 0, 0,32, 0, 0,16].x≤	96
9I[0,3, 8,17,18]	[9, 0, 9, 0, 9, 0, 0, 9, 9, 9, 0, 0, 0, 9, 9, 0, 9, 9, 0].x≤	54
39I[0,4, 9,16,19]	[0, 0,39,39,39, 0,39, 0,39,39, 0,39, 0, 0,39,39, 0, 0,39].x≤	234
30I[0,4, 6,17,18]	[30,30, 0,30, 0,30, 0, 0, 0, 0,30,30,30,30, 0, 0,30,30, 0].x≤	180
10I[0,5, 6,17,19]	[10,10, 0, 0,10,10, 0, 0, 0, 0,10,10,10,10, 0, 0,10, 0,10].x≤	60
2I[0,1, 2, 6,14,15,18]	[6, 2, 2, 4, 2, 2, 0, 2, 2, 0, 0, 4, 4, 4, 2, 2, 2, 2, 0].x≤	24
11I[0,1, 2, 7,14,15,17]	[33,22,11, 0,11,11,22,11, 0,11, 0,11,22,22,22,11,11, 0, 0].x≤	132
	[98,98,98,98,98,98,98,98,98,98,98,98,98,98,98,98,98,98,98].x≤	1120

$c_{19} = \frac{80}{133}$

$m = 20$ (1288 inequalities) $d = 15$

I[0, 6, 18]	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0] .x≤	2
I[0, 8, 19]	[0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0] .x≤	2
I[0, 9, 20]	[0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1] .x≤	2
I[0, 2, 10, 15, 20]	[0, 1, 0, 0, 2, 0, 0, 1, 0, 2, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1] .x≤	6
I[0, 3, 7, 17, 19]	[0, 1, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 0] .x≤	6
I[0, 3, 9, 16, 20]	[0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1] .x≤	6
I[0, 1, 2, 7, 15, 16, 19]	[3, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 2, 2, 1, 1, 1, 1, 0] .x≤	12
	[3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3] .x≤	36

$c_{20} = \frac{3}{5}$

$m = 21$ (3554 inequalities) $d = 15$.

(Solution using intial basis of $B_{21} = I_{21}$.)

I[0,6,19]	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0].x≤ 2
3I[0,7,19]	[0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 3, 0, 0].x≤ 6
I[0,8,19]	[0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0].x≤ 2
9I[0,8,20]	[0, 0, 0, 0, 0, 0, 0, 9, 0, 0, 0, 9, 0, 0, 0, 0, 0, 0, 0, 9, 0].x≤ 18
3I[0,9,20]	[0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0].x≤ 6
I[0,9,21]	[0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1].x≤ 2
2I[0,1, 7,16,20]	[2, 0, 0, 2, 0, 2, 2, 0, 2, 0, 0, 0, 2, 0, 2, 2, 0, 0, 2, 2, 0].x≤ 12
I[0,1,10,16,20]	[1, 0, 0, 1, 0, 1, 0, 0, 1, 2, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0].x≤ 6
3I[0,1,10,16,21]	[3, 0, 0, 0, 3, 3, 0, 0, 3, 3, 3, 0, 0, 0, 3, 3, 0, 0, 0, 3, 3].x≤ 18
4I[0,2,10,16,21]	[0, 4, 0, 0, 4, 4, 0, 4, 0, 4, 4, 0, 0, 4, 0, 4, 0, 0, 4, 0, 4].x≤ 24
2I[0,3, 6,18,19]	[2, 0, 4, 0, 0, 2, 0, 0, 0, 0, 0, 2, 2, 0, 2, 2, 0, 2, 2, 0, 0].x≤ 12
I[0,3, 9,17,21]	[0, 0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1].x≤ 6
6I[0,3,10,17,21]	[0, 0, 6, 6, 0, 0,12, 0, 0, 6, 6, 0, 0, 6, 0, 0, 6, 6, 0, 0, 6].x≤ 36
3I[0,4, 9,18,21]	[0, 0, 3, 3, 3, 0, 0, 0, 6, 0, 0, 3, 0, 3, 0, 0, 3, 3, 0, 0, 3].x≤ 18
2I[0,5, 8,18,21]	[0, 0, 4, 0, 2, 0, 0, 2, 0, 2, 0, 0, 4, 0, 0, 2, 0, 2, 0, 0, 2].x≤ 12
3I[0,1, 2, 6,15,17,19]	[6, 9, 0, 6, 3, 3, 0, 0, 3, 0, 3, 0, 6, 3, 6, 3, 6, 3, 3, 0, 0].x≤ 36
I[0,1, 2, 7,15,17,19]	[2, 3, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 2, 1, 2, 1, 1, 0, 0].x≤ 12
2I[0,1, 2, 7,15,17,20]	[4, 4, 2, 0, 4, 2, 2, 2, 0, 2, 0, 0, 4, 2, 4, 2, 2, 2, 2, 2, 0].x≤ 24
	[20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20].x≤252

$c_{21} = \frac{3}{5}$

inequalities generated = 148491, number of pivots = 1280, inequalities generated per pivot ≈ 116

$m = 21$ (3554 inequalities) $d = 15$

(Solution generated by starting with the basis matrix B_{20} for the case $m = 20$ and using $B_{21} = \begin{pmatrix} B_{20} & 0 \\ 0 & 1 \end{pmatrix}$ as the initial basis matrix.)

2I[0,6,19]	[0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0].x≤ 4
2I[0,8,19]	[0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0].x≤ 4
9I[0,8,20]	[0, 0, 0, 0, 0, 0, 0, 9, 0, 0, 0, 9, 0, 0, 0, 0, 0, 0, 0, 9, 0] x≤ 18
3I[0,9,20]	[0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0].x≤ 6
2I[0,9,21]	[0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 2].x≤ 4
2I[0,1, 7,16,20]	[2, 0, 0, 2, 0, 2, 2, 0, 2, 0, 0, 0, 2, 0, 2, 2, 0, 0, 2, 2, 0].x≤ 12
2I[0,1,10,16,21]	[2, 0, 0, 0, 2, 2, 0, 0, 2, 2, 2, 0, 0, 0, 2, 2, 0, 0, 0, 2, 2].x≤ 12
I[0,1,10,18,20]	[1, 0, 0, 1, 0, 1, 0, 0, 1, 2, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0].x≤ 6
4I[0,2,10,16,21]	[0, 4, 0, 0, 4, 4, 0, 4, 0, 4, 4, 0, 0, 4, 0, 4, 0, 0, 4, 0, 4].x≤ 24
2I[0,3, 6,18,19]	[2, 0, 4, 0, 0, 2, 0, 0, 0, 0, 0, 2, 2, 0, 2, 2, 0, 2, 2, 0, 0].x≤ 12
I[0,3,10,16,21]	[0, 0, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1].x≤ 6
6I[0,3,10,17,21]	[0, 0, 6, 6, 0, 0,12, 0, 0, 6, 6, 0, 0, 6, 0, 0, 6, 6, 0, 0, 6].x≤ 36
I[0,4, 7,19,20]	[1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 1, 0].x≤ 6
4I[0,4, 9,18,21]	[0, 0, 4, 4, 4, 0, 0, 0, 8, 0, 0, 4, 0, 4, 0, 0, 4, 4, 0, 0, 4].x≤ 24
I[0,5, 8,18,21]	[0, 0, 2, 0, 1, 0, 0, 1, 0, 1, 0, 0, 2, 0, 0, 1, 0, 1, 0, 0, 1].x≤ 6
2I[0,1, 2, 6,15,17,19]	[4, 6, 0, 4, 2, 2, 0, 0, 2, 0, 2, 0, 4, 2, 4, 2, 4, 2, 2, 0, 0].x≤ 24
2I[0,1, 2, 7,15,17,19]	[4, 6, 0, 2, 2, 2, 2, 2, 0, 2, 0, 2, 2, 2, 4, 2, 4, 2, 2, 0, 0].x≤ 24
2I[0,1, 2, 7,15,17,20]	[4, 4, 2, 0, 4, 2, 2, 2, 0, 2, 0, 0, 4, 2, 4, 2, 2, 2, 2, 2, 0].x≤ 24
	[20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20,20].x≤252

$c_{21} = \frac{3}{5}$

inequalities generated = 108865, number of pivots = 996, inequalities generated per pivot ≈ 109

$m = 22$ (4101 inequalities) $d = 16$

71I[0, 7,20]	[0, 0, 0, 0, 0, 0, 71, 0, 0, 0, 0, 0, 71, 0, 0, 0, 0, 0, 0, 71, 0, 0] .x≤	142
87I[0, 9,21]	[0, 0, 0, 0, 0, 0, 0, 0, 87, 0, 0, 87, 0, 0, 0, 0, 0, 0, 0, 0, 87, 0] .x≤	174
122I[0,10,21]	[0, 0, 0, 0, 0, 0, 0, 0, 0,122,122, 0, 0, 0, 0, 0, 0, 0, 0, 0,122, 0] .x≤	244
I[0,10,22]	[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1] .x≤	2
70I[0, 2, 7,18,20]	[0,140, 0, 0, 70, 0, 70, 0, 0, 0, 70, 0, 70, 0, 0, 70, 0,140, 0, 70, 0, 0] .x≤	420
I[0, 2, 8,18,20]	[0, 2, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 2, 0, 1, 0, 0] .x≤	6
13I[0, 3, 6,18,20]	[0, 13, 26, 0, 0, 13, 0, 0, 0, 0, 0, 13, 0, 13, 13, 0, 13, 13, 0, 13, 0, 0] .x≤	78
6I[0, 3, 6,19,20]	[6, 0, 12, 0, 0, 6, 0, 0, 0, 0, 0, 0, 6, 6, 0, 6, 6, 0, 6, 6, 0, 0] .x≤	36
43I[0, 3, 7,19,20]	[43, 0, 43, 43, 0, 0, 43, 0, 0, 0, 0, 43, 43, 0, 0, 43, 43, 0, 43, 43, 0, 0] .x≤	258
23I[0, 3, 9,19,21]	[0, 23, 23, 0, 0, 23, 0, 0, 23, 23, 0, 23, 0, 0, 0, 23, 0, 23, 23, 0, 23, 0] .x≤	138
75I[0, 3,11,17,22]	[0, 0, 75, 0, 75, 75, 0, 75, 0, 0,150, 0, 0, 75, 0, 0, 75, 0, 75, 0, 0, 75].x ≤	450
19I[0, 4, 7,19,21]	[0, 19, 19, 19, 0, 0, 19, 0, 0, 0, 0, 19, 0, 19, 19, 0, 19, 0, 19, 0, 19, 0] .x≤	114
48I[0, 4,10,18,22]	[0, 0, 0, 96, 0, 48, 0, 48, 0, 48, 0, 48, 0, 48, 0, 0, 0, 96, 0, 0, 0, 48].x ≤	288
78I[0, 4,10,19,22]	[0, 0, 78, 78, 0, 78, 0, 0, 78, 78, 0, 78, 0, 0, 78, 0, 0, 78, 78, 0, 0, 78].x ≤	468
21I[0, 5, 8,20,22]	[0, 21, 21, 0, 21, 0, 0, 21, 0, 0, 0, 21, 0, 21, 21, 0, 21, 0, 0, 21, 0, 21] .x≤	126
55I[0, 5, 9,19,22]	[0, 0, 55, 55, 55, 0, 0, 0, 55, 55, 0, 0, 55, 55, 0, 0, 55, 0, 55, 0, 0, 55] .x≤	330
8I[0, 5, 9,20,22]	[0, 8, 0, 8, 8, 0, 0, 0, 8, 0, 8, 0, 8, 0, 8, 0, 8, 0, 0, 8, 0, 8] .x≤	48
11I[0, 5,10,20,22]	[0, 11, 0, 0, 22, 0, 0, 0, 0, 22, 0, 11, 0, 0, 11, 0, 11, 0, 0, 11, 0, 11].x ≤	66
7I[0, 6, 8,20,22]	[0, 14, 0, 0, 0, 7, 0, 7, 0, 0, 0, 7, 0, 14, 0, 7, 0, 0, 0, 7, 0, 7] .x≤	42
51I[0, 1, 2, 8,16,17,21]	[153, 51, 0, 51, 51, 51, 51,102, 51, 0, 0, 0, 51, 51,102,102, 51, 0, 51, 51, 51, 0] .x≤	612
48I[0, 1, 2, 9,16,17,22]	[144, 48, 0, 0, 48, 48, 96, 96, 48, 0, 0, 0, 48, 48, 96, 96, 48, 0, 0, 48, 48, 48] .x≤	576
2I[0, 1, 2,10,16,17,21]	[6, 2, 0, 2, 2, 2, 2, 2, 2, 2, 2, 0, 0, 2, 4, 4, 2, 0, 2, 2, 2, 0] .x≤	24
	[352,352,352,352,352,352,352,352,352,352,352,352,352,352,352,352,352,352,352,352,352,352] .x≤	4642

$c_{22} = \frac{211}{352}$

inequalities generated = 169058 number of pivots = 1278 inequalities generated per pivot ≈ 132

$m = 23$ (4283 inequalities) $d = 17$

Using slightly incorrect version of $T_{23}^{\text{stretched}}$.

316I[0, 7,21]	[0, 0, 0, 0, 0, 0,316, 0, 0, 0, 0, 0, 0,316, 0, 0, 0, 0, 0, 0,316, 0, 0].x≤	632
551I[0, 8,21]	[0, 0, 0, 0, 0, 0, 0,551, 0, 0, 0, 0,551, 0, 0, 0, 0, 0, 0, 0,551, 0, 0].x≤	1102
258I[0, 9,21]	[0, 0, 0, 0, 0, 0, 0, 0,258, 0, 0,258, 0, 0, 0, 0, 0, 0, 0, 0,258, 0, 0].x≤	516
272I[0,10,22]	[0, 0, 0, 0, 0, 0, 0, 0, 0,272, 0,272, 0, 0, 0, 0, 0, 0, 0, 0, 0,272, 0].x≤	544
268I[0,11,22]	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0,536, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,268, 0].x≤	536
148I[0, 2, 7,19,21]	[0,296, 0, 0,148, 0,148, 0, 0, 0, 0,148, 0,148, 0, 0,148, 0,296, 0,148, 0, 0].x≤	888
17I[0, 2, 8,19,21]	[0, 34, 0, 0, 0, 17, 0, 17, 0, 0, 17, 0, 17, 0, 0, 0, 17, 0, 34, 0, 17, 0, 0].x≤	102
100I[0, 3, 8,19,22]	[0, 0,200, 0,100, 0, 0,100, 0, 0,100, 0, 0,100, 0,100, 0, 0,200, 0, 0,100, 0].x≤	600
40I[0, 4, 7,20,22]	[0, 40, 40, 40, 0, 0, 40, 0, 0, 0, 0, 0, 40, 0, 40, 40, 0, 40, 0, 40, 0, 40, 0].x≤	240
205I[0, 4, 8,20,22]	[0,205, 0,410, 0, 0, 0,205, 0, 0, 0,205, 0,205, 0,205, 0,205, 0,205, 0,205, 0].x≤	1230
97I[0, 4, 9,19,23]	[0, 0, 0,194, 97, 0, 0, 0, 97, 97, 0, 0, 0, 97, 97, 0, 0, 0,194, 0, 0, 0, 97].x≤	582
185I[0, 4,10,19,23]	[0, 0, 0,370, 0,185, 0, 0,185,185, 0, 0,185, 0,185, 0, 0, 0,370, 0, 0, 0,185].x≤	1110
150I[0, 4,10,20,23]	[0, 0,150,150, 0,150, 0, 0, 0,300, 0, 0,150, 0, 0,150, 0, 0,150,150, 0, 0,150].x≤	900
58I[0, 4,11,19,23]	[0, 0, 0,116, 0, 0, 58, 58, 0, 0, 58, 58, 0, 0, 58, 0, 0, 0,116, 0, 0, 0, 58].x≤	348
10I[0, 5, 8,21,23]	[0, 10, 10, 0, 10, 0, 0, 10, 0, 0, 0, 0, 10, 0, 10, 10, 0, 10, 0, 0, 10, 0, 10].x≤	60
60I[0, 5,10,21,23]	[0, 60, 0, 0,120, 0, 0, 0, 0, 60, 60, 0, 60, 0, 0, 60, 0, 60, 0, 0, 60, 0, 60].x≤	360
410I[0, 5,11,20,23]	[0, 0,410, 0,410,410, 0, 0,410, 0,410,410, 0, 0,410, 0, 0,410, 0,410, 0, 0,410].x≤	2460
75I[0, 1, 2, 4,17,18,20]	[225,225,150, 75, 0, 0, 0, 0, 0, 0, 0, 0, 75, 75, 75,225,150,150, 75, 75, 0, 0, 0].x≤	900
5I[0, 1, 2, 4,17,19,20]	[15, 15, 10, 5, 0, 0, 0, 0, 0, 0, 0, 0, 5, 0, 10, 10, 10, 10, 10, 5, 0, 0, 0].x≤	60
85I[0, 1, 2,11,17,18,22]	[255, 85, 0, 85, 85, 85, 85, 0, 85, 85,170, 0, 0, 0, 85,170,170, 85, 0, 85, 85, 85, 0].x≤	1020
29I[0, 1, 3, 9,17,18,23]	[58, 29, 29, 0, 29, 58, 0, 58, 58, 0, 0, 0, 0, 58, 29, 29, 58, 29, 0, 29, 0, 29, 29].x≤	348
352I[0, 1, 3,10,17,18,23]	[704,352,352, 0,352,352,704,352,352,352, 0, 0,352,352,352,352,704,352, 0,352, 0,352,352].x≤	4224
94I[0, 1, 3,11,17,18,23]	[188, 94, 94, 0, 94,188, 94, 94, 0, 94, 94, 94, 0, 94, 94, 94,188, 94, 0, 94, 0, 94, 94].x≤	1128
	[s, s, s, s, s, s, s, s, s, s, s, s, s, s, s, s, s, s, s, s, s, s, s].x≤	19890

Here, column sums are $s = 1445$.

$c_{23} = \frac{234}{391}$

inequalities generated = 86904, number of pivots = 747, inequalities generated per pivot ≈ 116

$m = 23$ (4283 inequalities) $d = 17$

Using corrected version of $T_{23}^{\text{stretched}}$.

138I[0, 7,21]	[0, 0, 0, 0, 0, 0,138, 0, 0, 0, 0, 0,138, 0, 0, 0, 0, 0,138, 0, 0].x≤ 276
203I[0, 8,21]	[0, 0, 0, 0, 0, 0, 0,203, 0, 0, 0, 0,203, 0, 0, 0, 0, 0, 0,203, 0, 0].x≤ 406
109I[0, 9,21]	[0, 0, 0, 0, 0, 0, 0, 0,109, 0, 0,109, 0, 0, 0, 0, 0, 0, 0,109, 0, 0].x≤ 218
74I[0,10,22]	[0, 0, 0, 0, 0, 0, 0, 0, 0, 74, 0, 74, 0, 0, 0, 0, 0, 0, 0, 0, 0, 74, 0].x≤ 148
142I[0,11,22]	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0,284, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,142, 0].x≤ 284
65I[0, 2, 7,19,21]	[0,130, 0, 0, 65, 0, 65, 0, 0, 0, 0, 65, 0, 65, 0, 0, 65, 0,130, 0, 65, 0, 0].x≤ 390
I[0, 2, 8,19,21]	[0, 2, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 2, 0, 1, 0, 0].x≤ 6
40I[0, 3, 8,19,22]	[0, 0, 80, 0, 40, 0, 0, 40, 0, 0, 40, 0, 0, 40, 0, 40, 0, 0, 80, 0, 0, 40, 0].x≤ 240
16I[0, 4, 7,20,22]	[0, 16, 16, 16, 0, 0, 16, 0, 0, 0, 0, 0, 16, 0, 16, 16, 0, 16, 0, 16, 0, 16, 0].x≤ 96
82I[0, 4, 8,20,22]	[0, 82, 0,164, 0, 0, 0, 82, 0, 0, 0, 82, 0, 82, 0, 82, 0, 82, 0, 82, 0, 82, 0].x≤ 492
33I[0, 4, 9,19,23]	[0, 0, 0, 66, 33, 0, 0, 0, 33, 33, 0, 0, 0, 33, 33, 0, 0, 0, 66, 0, 0, 0, 33].x≤ 198
103I[0, 4,10,19,23]	[0, 0, 0,206, 0,103, 0, 0,103,103, 0, 0,103, 0,103, 0, 0, 0,206, 0, 0, 0,103].x≤ 618
60I[0, 4,10,20,23]	[0, 0, 60, 60, 0, 60, 0, 0, 0,120, 0, 0, 60, 0, 0, 60, 0, 0, 60, 60, 0, 0, 60].x≤ 360
4I[0, 5, 8,21,23]	[0, 4, 4, 0, 4, 0, 0, 4, 0, 0, 0, 0, 4, 0, 4, 4, 0, 4, 0, 0, 4, 0, 4].x≤ 24
24I[0, 5,10,21,23]	[0, 24, 0, 0, 48, 0, 0, 0, 0, 24, 24, 0, 24, 0, 0, 24, 0, 24, 0, 0, 24, 0, 24].x≤ 144
164I[0, 5,11,20,23]	[0, 0,164, 0,164,164, 0, 0,164, 0,164,164, 0, 0,164, 0, 0,164, 0,164, 0, 0,164].x≤ 984
30I[0, 1, 2, 4,17,18,20]	[90, 90, 60, 30, 0, 0, 0, 0, 0, 0, 0, 0, 30, 30, 30, 90, 60, 60, 30, 30, 0, 0, 0].x≤ 360
2I[0, 1, 2, 4,17,19,20]	[6, 6, 4, 2, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 4, 4, 4, 4, 4, 2, 0, 0, 0].x≤ 24
29I[0, 1, 2,10,17,18,22]	[87, 29, 0, 29, 29, 0, 29, 58, 29, 29, 0, 29, 0, 0, 29, 58, 58, 29, 0, 29, 29, 29, 0].x≤ 348
5I[0, 1, 2,11,17,18,22]	[15, 5, 0, 5, 5, 5, 5, 0, 5, 5, 10, 0, 0, 0, 5, 10, 10, 5, 0, 5, 5, 5, 0].x≤ 60
135I[0, 1, 3,10,17,18,23]	[270,135,135, 0,135,135,270,135,135,135, 0, 0,135,135,135,135,270,135, 0,135, 0,135,135].x≤1620
55I[0, 1, 3,11,17,18,23]	[110, 55, 55, 0, 55,110, 55, 55, 0, 55, 55, 55, 0, 55, 55, 55,110, 55, 0, 55, 0, 55, 55].x≤ 660
	[578,578,578,578,578,578,578,578,578,578,578,578,578,578,578,578,578,578,578,578,578,578,578].x≤7956

$c_{23} = \frac{234}{391}$

inequalities generated = 31796, number of pivots = 255, inequalities generated per pivot ≈ 125
I[0,5,9,20,23] was in the basis of solution, but with value zero.

$m = 24$ (12302 inequalities) $d = 17$

38I[0, 7,21]	[0, 0, 0, 0, 0, 0, 38, 0, 0, 0, 0, 0, 0, 38, 0, 0, 0, 0, 0, 0, 38, 0, 0, 0].x≤ 76
19I[0, 8,22]	[0, 0, 0, 0, 0, 0, 0, 19, 0, 0, 0, 0, 0, 19, 0, 0, 0, 0, 0, 0, 0, 19, 0, 0].x≤ 38
99I[0, 9,22]	[0, 0, 0, 0, 0, 0, 0, 0, 99, 0, 0, 0, 99, 0, 0, 0, 0, 0, 0, 0, 0, 99, 0, 0].x≤ 198
15I[0,10,22]	[0, 0, 0, 0, 0, 0, 0, 0, 0, 15, 0, 15, 0, 0, 0, 0, 0, 0, 0, 0, 0, 15, 0, 0].x≤ 30
17I[0,11,22]	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 34, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 17, 0, 0].x≤ 34
99I[0,11,23]	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 99, 99, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 99, 0].x≤ 198
36I[0, 3, 7,20,22]	[0, 36, 36, 36, 0, 0, 36, 0, 0, 0, 0, 0, 36, 0, 36, 0, 36, 0, 36, 36, 0, 36, 0, 0].x≤ 216
14I[0, 3, 8,20,22]	[0, 14, 14, 0, 14, 0, 0, 14, 0, 0, 0, 14, 0, 14, 0, 0, 14, 0, 14, 14, 0, 14, 0, 0].x≤ 84
19I[0, 3, 9,20,23]	[0, 0, 38, 0, 0, 19, 0, 0, 19, 0, 19, 0, 0, 19, 0, 0, 19, 0, 0, 38, 0, 0, 19, 0].x≤ 114
23I[0, 3,12,18,24]	[0, 0, 23, 0, 0, 46, 0, 0, 23, 0, 0, 46, 0, 0, 23, 0, 0, 23, 0, 0, 23, 0, 0, 23].x≤ 138
30I[0, 3,12,19,24]	[0, 0, 30, 0, 30, 0, 30, 0, 30, 0, 0, 60, 0, 0, 0, 30, 0, 0, 30, 0, 30, 0, 0, 30].x≤ 180
15I[0, 4, 8,20,22]	[0, 15, 0, 30, 0, 0, 0, 15, 0, 0, 0, 15, 0, 15, 0, 15, 0, 15, 0, 15, 0, 15, 0, 0].x≤ 90
21I[0, 4, 8,21,23]	[0, 21, 0, 42, 0, 0, 0, 21, 0, 0, 0, 0, 21, 0, 21, 0, 21, 0, 21, 0, 21, 0, 21, 0].x≤ 126
8I[0, 4, 9,20,23]	[0, 0, 8, 8, 8, 0, 0, 0, 8, 0, 8, 0, 0, 8, 0, 8, 0, 0, 8, 8, 0, 0, 8, 0].x≤ 48
5I[0, 4, 9,21,23]	[0, 5, 0, 5, 5, 0, 0, 0, 5, 0, 0, 5, 0, 5, 0, 0, 5, 0, 5, 0, 5, 0, 5, 0].x≤ 30
64I[0, 4,10,20,24]	[0, 0, 0,128, 0, 64, 0, 0, 0,128, 0, 0, 0, 64, 0, 64, 0, 0, 0,128, 0, 0, 0, 64].x≤ 384
7I[0, 5, 7,20,22]	[0, 14, 0, 0, 7, 0, 7, 0, 0, 0, 0, 0, 7, 0, 14, 0, 7, 0, 0, 7, 0, 7, 0, 0].x≤ 42
38I[0, 5, 7,21,22]	[38, 38, 0, 0, 38, 0, 38, 0, 0, 0, 0, 0, 0, 38, 38, 38, 38, 0, 0, 0, 38, 38, 0, 0].x≤ 228
12I[0, 5,10,22,24]	[0, 12, 0, 0, 24, 0, 0, 0, 0, 12, 0, 12, 0, 12, 0, 0, 12, 0, 12, 0, 0, 12, 0, 12].x≤ 72
23I[0, 5,11,20,24]	[0, 0, 0, 23, 23, 23, 0, 0, 23, 0, 23, 0, 23, 0, 23, 0, 0, 0, 23, 23, 0, 0, 0, 23].x≤ 138
3I[0, 5,12,19,24]	[0, 0, 0, 0, 6, 0, 6, 0, 0, 0, 0, 6, 0, 3, 0, 0, 0, 0, 6, 0, 0, 0, 0, 3].x≤ 18
3I[0, 6, 9,20,23]	[0, 0, 6, 0, 0, 3, 0, 0, 3, 0, 3, 0, 0, 6, 0, 0, 3, 0, 0, 3, 0, 0, 3, 0].x≤ 18
31I[0, 1, 3,10,18,19,24]	[62, 31, 31, 0, 31, 31, 31, 31, 62, 31, 0, 0, 0, 31, 31, 31, 31, 62, 31, 0, 31, 0, 31, 31].x≤ 372
86I[0, 1, 3,11,18,19,24]	[172, 86, 86, 0, 86, 86, 86,172, 0, 86, 86, 0, 86, 0, 86, 86, 86,172, 86, 0, 86, 0, 86, 86].x≤1032
	[272,272,272,272,272,272,272,272,272,272,272,272,272,272,272,272,272,272,272,272,272,272,272,272].x≤3904

$c_{24} = \frac{61}{102}$

inequalities generated = 70835, number of pivots = 350, inequalities generated per pivot ≈ 202