An Inequality on Connected Domination Parameters[1]

HONGQUAN YU[2] TIANMING WANG
Institute of Mathematical Sciences
Dalian University of Technology
Dalian 116024, P.R. CHINA

Abstract. Let $G = (V, E)$ be a connected graph. Let $\gamma_c(G), d_c(G)$ denote the connected domination number, connected domatic number of G, respectively. We prove that $\gamma_c(G) \leq 3d_c(G^c)$ if the complement of G is also connected. This confirms a conjecture of Hedetniemi and Laskar(1984), and Sun(1992). Examples are given to show that equality may occur.

1. Introduction

All graphs under consideration are finite, undirected and loopless without multiedges. Let $G = (V, E)$ be a graph with vertex set V and edge set E. G^c denotes the complement of G, $w(G)$ denotes the number of connected components of G. For $u \in V$, the (open) neighborhood of u in G, denoted by $N_G(u)$, is the set of all vertices adjacent to u. The closed neighborhood of u in G, denoted by $N_G[u]$, is defined to be $N_G(u) \cup \{u\}$. For a set $S \subseteq V$, the (open) neighborhood and closed neighborhood of S in G is defined respectively by $N_G(S) = \cup_{u \in S} N_G(u)$, $N_G[S] = \cup_{u \in S} N_G[u]$. Moreover, for a set $S \subseteq V$, $G[S]$ denotes the subgraph of G induced by S.

A set $D \subseteq V$ is a dominating set of G if $V - D \subseteq N_G(D)$. A dominating set D is called a connected dominating set if $G[D]$ is connected. The domination (connected domination) number of G, denoted by $\gamma(G)$ $(\gamma_c(G))$, is the minimum cardinality of a dominating (connected dominating) set of G. The connected domatic number $d_c(G)$ of G, is defined to be the maximum number of pairwisely disjoint connected dominating sets contained in V. A dominating (connected dominating) set of G is called minimal if none of its proper subsets is also a dominating (connected dominating) set of G.

Since the concepts of dominations are closely related to optimization problems on networks design, numerous research has been done on this topic, see [8] for a survey. Some inequalities involving the domination number, connected domination number, domination independence number, irredundance number and upper irredundance number have been established

[1] Research supported by the National Natural Science Foundation of China.
[2] E-mail: jmreyu@gingko.dlut.edu.cn

by various authors, see [1,2,5,7,9]. For a recent and important reference, see [4].

In [9], Sun reproposed the conjecture of Hedetniemi and Laskar [7] that if G and G^c are both connected, then there holds the inequality $\gamma_c(G) \leq 3d_c(G^c)$. In this paper, we shall prove the conjecture and show that equality may also occur.

2. Main results

The main result of this paper is the following theorem.

Theorem 2.1 *If both G and G^c are connected. Then $\gamma_c(G) \leq 3d_c(G^c)$.*

The proof of the theorem is based on a series of lammas. The first one is trivial if one considers a spanning tree of G and two pendant vertices in the tree, see [3].

Lemma 2.2 *Let $G = (V, E)$ be a connected graph of order $n \geq 2$. Then there exist two non-cut vertices of G in V.*

The following two lemmas are used to estimate the connected domination number of a graph.

Lemma 2.3 *Let G be a connected graph. Let $G_1, G_2, \cdots, G_s (s \geq 2)$ be connected subgraphs of G with connected dominating sets $D_1, D_2, \cdots, D_s$, respectively, such that $\cup_{i=1}^s V(G_i) = V(G)$. Then there exists a connected dominating set D of G such that $D \supseteq \cup_{i=1}^s D_i$ and*

$$|D| \leq \sum_{i=1}^{s} |D_i| + 2s - 2.$$

In particular, if for some $i \neq j, 1 \leq i, j \leq s$, $N_G[D_i] \cap N_G[D_j] \neq \emptyset$, then D may satisfy that

$$|D| \leq \sum_{i=1}^{s} |D_i| + 2s - 3.$$

Proof We proceed by induction on s. Let $s = 2$. If $N_G[D_1] \cap N_G[D_2] \neq \emptyset$, then take $u \in N_G[D_1] \cap N_G[D_2]$. It is obvious that $D_1 \cup D_2 \cup \{u\}$ is a connected dominating set satisfying the assertion of the lemma. Assume that $N_G[D_1] \cap N_G[D_2] = \emptyset$. For any $u \in D_1, v \in D_2$, there exists a path as $ux_1x_2 \cdots x_r v$ in G by the connectedness of G, where $r \geq 2$, and $x_1 \in N_G[D_1]$, $x_r \in N_G[D_2]$. Let x_j be such that $x_j \in N_G[D_1]$, and $x_{j+1}, \cdots, x_r \notin N_G[D_1]$. Then $1 \leq j \leq r-1$. Let $D = D_1 \cup D_2 \cup \{x_j, x_{j+1}\}$. Then D is a connected dominating set of G and $|D| \leq |D_1| + |D_2| + 2$.

In general, assume that the result is true for $s = 2, \cdots, k$. Suppose now that the connected graph G has $k+1$ connected subgraphs $G_1, G_2, \cdots, G_{k+1}$ and each with a connected dominating sets $D_i, 1 \leq i \leq k$ such that $\cup_{i=1}^{k+1} V(G_i) = V(G)$.

Regard each G_i as a VERTEX, and G_i and G_j ($i \neq j$ is adjacent if $V(G_i) \cap V(G_j) \neq \emptyset$ or there exist $u \in V(G_i), v \in V(G_j)$ such that $uv \in E(G)$. We then get a connected graph **G** of order $k+1 \geq 3$. By Lemma 2.2, delete a non-cut VERTEX of **G**, say $V(G_{k+1})$ (the vertices in $\cup_{i=1}^{k} V(G_i) \cap V(G_{k+1})$ remain unchanged). We may obtain a new connected graph as $G[\cup_{i=1}^{k} V(G_i) - V(G_{k+1})]$.

By the induction hypothesis, there exists a connected dominating set D' of $G[\cup_{i=1}^{k} V(G_i) - V(G_{k+1})]$ such that $D' \supseteq \cup_{i=1}^{k} D_i$ and $|D'| \leq \sum_{i=1}^{k} |D_i| + 2k - 2$. Moreover, if there is a pair D_i, D_j, $1 \leq i \neq j \leq k$ with $N_G[D_i] \cap N_G[D_j] \neq \emptyset$, then $|D'| \leq \sum_{i=1}^{k} |D_i| + 2k - 3$. By the same argument used in the case of $s = 2$, we know that there exists a connected dominating set D of G with $D \supseteq D' \cup D_{k+1} \supseteq \cup_{i=1}^{k+1} D_i$, and $|D| \leq |D'| + |D_{k+1}| + 2 \leq \sum_{i=1}^{k+1} |D_i| + 2(k+1) - 2$. In particular, if there is some $D_i, 1 \leq i \leq k$ such that $N_G[D_i] \cap N_G[D_{k+1}] \neq \emptyset$, then $N_G[D'] \cap N_G[D_{k+1}] \neq \emptyset$. Thus $|D| \leq |D'| + |D_{k+1}| + 1 \leq \sum_{i=1}^{k+1} |D_i| + 2(s+1) - 3$. □

The following lemma is a natural extension of Lemma 2.3.

Lemma 2.4 *Let G be a connected graph. Let $G_1, G_2, \cdots, G_s$ be connected subgraphs of G with connected dominating sets $D_1, D_2, \cdots, D_s$, respectively. Let $V(G) - \cup_{i=1}^{s} V(G_i) = X$. Then there exists a connected dominating set D of G with*

$$|D| \leq \sum_{i=1}^{s} |D_i| + 2s - 2 + |X|.$$

Moreover, if there exists a pair $D_i, D_j (i \neq j)$ such that $N_G[D_i] \cap N_G[D_j] \neq \emptyset$, then D may be such that

$$|D| \leq \sum_{i=1}^{s} |D_i| + 2s - 3 + |X|.$$

Proof Without loss of generality, assume that $X \cap [\cup_{i=1}^{s} V(G_i)] = \emptyset$ (otherwise we may consider instead $X' = X - X \cap [\cup_{i=1}^{s} V(G_i)]$). Let the connected components of $G[X]$ be $W_1, W_2, \cdots, W_k$, where $1 \leq k \leq |X|$. For each component $W_j, 1 \leq j \leq k$, there exists $u_j \in V(W_j)$ and $v_j \in \cup_{i=1}^{s} V(G_i)$ such that $u_j v_j \in E(G)$ since G is connected. Let $v_j \in V(G_{j_i})$ and call that W_j is ADJACENT to G_{j_i}. By connecting each W_j to one of its ADJACENT subgraphs out of $G_1, G_2, \cdots, G_s$ and then expanding the connected dominating sets in a natural way, we may get subgraphs of G as $G'_1, G'_2, \cdots, G'_s$ such that $V(G) = \cup_{i=1}^{s} V(G'_i)$. It follows from Lemma 2.3 that there exists a connected dominating set D of G with $D \supseteq \cup_{i=1}^{s} D_i$ and $|D| \leq \sum_{i=1}^{s} |D_i| + 2s - 2 + |X|$. The other part of the result also follows from Lemma 2.3. □

Let G and G^c be connected with $d_c(G^c) = k$. Then $V(G^c) = V(G)$ may be partitioned into k pairwisely disjoint connected dominating sets as $V(G^c) = \cup_{i=1}^{k} D_i \cup V_1$, where one may assume further that D_i's, $1 \leq i \leq k$, are all minimal connected dominating sets of G^c. We shall consider separately two cases according as $V_1 = \emptyset$ or not.

Lemma 2.5 *If $V_1 = \emptyset$, then $\gamma_c(G) \leq 3k - 2$.*

Proof It is easily seen that $|D_i| \geq 2$ for $1 \leq i \leq k$, since otherwise G would contain an isolated vertex. Since $G^c[D_i]$ is connected, then by Lemma 2.2 there exists a non-cut vertex $x_i \in D_i$ of $G^c[D_i]$. Since D_i is minimal, $N_{G^c}(x_i) \not\subseteq N_{G^c}[D_i - \{x_i\}]$. Let $y_i \in N_{G^c}(x_i) - N_{G^c}[D_i - \{x_i\}]$. Then $y_i \in \cup_{j \neq i} D_j$, and $y_i x \in E(G)$ for each $x \in D_i - \{x_i\}$. Set $S_i = (D_i - \{x_i\}) \cup \{y_i\}$. Then $G[S_i]$ is a connected subgraph of G with a connected dominating set $\{y_i\}$. Call y_i the CENTER of S_i. It is clear that $x_i \neq x_j$ for $i \neq j$ since $x_i \in D_i$ and D_i's are pairwisely disjoint.

Let $Y^* = \{y_1^*, y_2^*, \cdots, y_s^*\}$ be the set of distinct y_i's, where $1 \leq s \leq k$. Let the connected components of $G[Y^*]$ be $H_1, H_2, \cdots, H_t$, and combine the sets S_i's if their CENTERs coincide or are in the same component of $G[Y^*]$. We may finally obtain t sets as $S_1^*, S_2^*, \cdots, S_t^*$, where $S_i^*, 1 \leq i \leq t_1$ has a unique CENTER, and $S_i^*, t_1 + 1 \leq i \leq t_1 + t_2 = t$ has at least two CENTERs. Moreover, $t_1 + 2t_2 \leq s$.

For each set S_j^*, $1 \leq j \leq t_1$, that has a unique CENTER, say y_j^*, we have $y_j^* \in D_{i_0}$ for some i_0. Thus $y_j^* = x_{i_0}$ since otherwise we would have $y_i^* y_{i_0} \in E(G)$, a contradiction. Therefore at least t_1 elements out of $X = \{x_1, x_2, \cdots, x_k\}$ coincide with $y_1^*, \cdots, y_{t_1}^*$. Let $X' = X - X \cap Y^*$. Then $|X'| \leq k - t_1$. Since $\cup_{j=1}^{t} S_j^* \cup X' = V(G)$, it follows from Lemma 2.4 that there exists a connected dominating set D of G such that

$$|D| \leq |\{y_j^* | y_j^* \in Y^*\}| + 2t - 2 + |X'| \leq 2s + k - 2 \leq 3k - 2. \quad \square$$

In the case when $V_1 \neq \emptyset$, the discussion are divided into two subcases according as V_1 is a dominating set of G^c or not.

Lemma 2.6 *If $V_1 \neq \emptyset$ and V_1 is not a dominating set of G^c. Then $\gamma_c(G) \leq 3k$.*

Proof First, rephrase the first two paragraphs in the proof of Lemma 2.5. Since V_1 is not a dominating set of G^c, there exists $u \in V(G) - V_1$ such that $ux \notin E(G^c)$ for any $x \in V_1$. Thus $ux \in E(G)$. Assume that $u \in D_{i_0}$.

Case 1. $Y^* \cap V_1 = \emptyset$. If $u \neq x_{i_0}$, then $uy_{i_0} \in E(G)$. For each set S_j^*, $1 \leq j \leq t_1$, that has a unique CENTER, say y_j^*, we have by the preceeding discussion that $y_j^* \in X = \{x_1, x_2, \cdots, x_k\}$. Let $X' = X - \{y_1^*, \cdots, y_{t_1}^*\}$. Then $|X'| \leq k - t_1$. Combine V_1 with the set S_j^* having y_{i_0} as a CENTER. The subgraph of G induced by this new set has a connected dominating set $\{u\} \cup \{y^* | y^* \in S_j^*\}$. Then by Lemma 2.4, there exists a connected dominating set D of G such that

$$|D| \le s + 1 + 2(t_1 + t_2) - 2 + k - t_1 \le 3k - 1.$$

If $u = x_{i_0}$ and $x_{i_0} \notin X'$. Then $u = x_{i_0} = y'_{j_0}$ for some j_0. Combine X with the set having a unique CENTER y'_{j_0}. Then by Lemma 2.4, there exists a connected dominating set D of G such that

$$|D| \le s + 2(t_1 + t_2) - 2 + k - t_1 \le 3k - 2.$$

If $u = x_{i_0}$ and $x_{i_0} \in X'$. Let $X'' = X' - \{x_{i_0}\}$, and $V_1 \cup \{x_{i_0}\} = \overline{V_1}$. It is clear that $|X''| = |X'| - 1 \le k - t_1 - 1$ and $G[\overline{V_1}]$ has a connected dominating set as $\{x_{i_0}\}$. Thus by Lemma 2.4, there exists a connected dominating set D of G such that

$$|D| \le s + 1 + 2(t_1 + t_2 + 1) - 2 + k - t_1 - 1 \le 3k.$$

Case 2. $Y^* \cap V_1 \ne \emptyset$. Forming a new set by combining the sets S_j^*'s with V_1 if the CENTERs of S_j^*'s are contained in V_1. The subgraph of G induced by this new set has a connected dominating set as $\{u\} \cup \{y^* \mid y^* \in V_1\}$. Let now the remaining sets with a unique CENTER be I: $S_1^*, S_2^*, \cdots, S_{\overline{t_1}}^*$, $\overline{t_1} \le t_1$, and the sets with at least two CENTERs be II: $S_{\overline{t_1}+1}^*, \cdots, S_{\overline{t_1}+\overline{t_2}}^*$, $\overline{t_1}+\overline{t_2} = \overline{t} \le t$. For each CENTER y'_j of the set in class I, $1 \le j \le \overline{t_1}$, if $y'_j \notin V_1$, then $y'_j \in D_{k_0}$ for some k_0 and thus $y'_j = x_{k_0}$ since otherwise $y'_j y_{k_0} \in E(G)$, a contradiction. Thus each CENTER of sets of class I is contained in V_1 or in $X = \{x_1, x_2, \cdots, x_k\}$. Assume that t'_1 of the $\overline{t_1}$ CENTERs are contained in V_1 and the remaining t''_1 of them are contained in X. Then $t'_1 + t''_1 = \overline{t_1}$. On the other hand, assume that t'_2 of the CENTERs of sets of class II are contained in V_1, and the remaining $t''_2 = \overline{t_2} - t'_2$ are contained in X. Let $X' = X - X \cap Y^*$. Then $|X'| \le k - t''_1 - t''_2 \le k - t''_1$. Thus by Lemma 2.4, there exists a connected dominating set D of G such that

$$|D| \le s + 1 + 2(1 + t''_1 + t''_2) - 2 + k - t''_1 \le s + 1 + t''_1 + 2t''_2 + k.$$

Since $Y^* \cap V_1 \ne \emptyset$, then t'_1 and t'_2 can not be all zero, so the equalities in $t''_1 \le t_1$ and $t''_2 \le t_2$ can not occur simultaneously. Thus $|D| \le s + t_1 + 2t_2 + k \le 2s + k \le 3k$. □

Lemma 2.7 *If V_1 is a dominating set of G^c. Then $\gamma_c(G) \le 3k$.*

Proof It is obvious from the assumption that $G^c[V_1]$ is not connected. Let the connected components of $G^c[V_1]$ be $W_1, W_2, \cdots, W_s$, $s \ge 2$.

We assume first that $\gamma_c(G^c) \ge 3$. It follows from Lemma 2.2 that for each D_i, $1 \le k$, there exist two non-cut vertices $x_i, y_i \in D_i$. Since D_i is a minimal connected dominating set of G^c, there exist $\overline{x_i}, \overline{y_i} \in V(G^c) - D_i$, such that $\overline{x_i} x_i \in E(G^c)$, and $\overline{x_i} u \in E(G)$ for any $u \in D_i - \{x_i\}$; $\overline{y_i} y_i \in$

$E(G^c)$, and $\overline{y_i}v \in E(G)$ for any $v \in D_i - \{y_i\}$. Let $S = \{x_i, y_i, 1 \le i \le k\}$, and $\overline{S} = \{\overline{x_i}, \overline{y_i}, 1 \le k\}$.

Then $|S| = 2k, |\overline{S}| \le 2k$. For any $u \in D_1 \cup \cdots D_k, u \in D_i$, it is easy to see that u is adjacent to at least one vertex of $\overline{S}$ in G.

We consider two cases as follows.

Case 1. $V_1 \cap \overline{S} \ne \emptyset$. Without loss of generality, let $u \in V(W_1) \cap D_i$. Then for any $v \in V(W_2)$, the subgraph $G[\overline{S} \cup \{v\}]$ has no isolated vertices. In fact, the vertices of $G[\overline{S} \cup \{v\}]$ contained in V_1 are adjacent to u or v in G. And for any $x \in \overline{S} - \{v\} - V_1 \subseteq \cup_{i=1}^{k} D_i - V_1$, there exists j_0 such that $x \in D_{j_0}$, so that $x\overline{x_{j_0}} \in E(G)$ or $x\overline{y_{j_0}} \in E(G)$. Thus $q = w(G[\overline{S} \cup \{v\}]) \le k$. For any $v_i \in C_i$, where $C_1, C_2, \cdots, C_q$ are connected components of $G[\overline{S} \cup \{v\}]$, since $\gamma_c(G^c) \ge 3$ and $v_j v_{j+1} \in E(G^c)$, $1 \le j \le q-1$, there exists w_j such that $w_j v_j \notin E(G^c)$ and $w_j v_{j+1} \notin E(G^c)$, $1 \le j \le q-1$. Thus $w_j v_j \in E(G), w_j v_{j+1} \in E(G)$. It is clear that $\overline{S} \cup \{v\} \cup \{w_1, w_2, \cdots, w_{q-1}\}$ is a connected dominating set of G. So that $\gamma_c(G) \le |\overline{S}| + q \le 2k + q \le 3k$.

Case 2. $V_1 \cap \overline{S} = \emptyset$. Then $\overline{S} \subseteq \cup_{i=1}^{k} D_i$. For any $x \in \overline{S}$, assume that $x \in D_{i_0}$. Then $x\overline{x_{i_0}} \in E(G)$ or $x\overline{y_{i_0}} \in E(G)$. So that $G[\overline{S}]$ has no isolated vertices and $G[\overline{S}]$ has $q \le k$ connected components. Since G is connected, there exists $u \in \cup_{i=1}^{k} D_i$ and $v \in V_1$ such that $xy \in E(G)$. Without loss of generality, assume that $u \in V(W_1)$. Let $w \in V(W_2)$.

Subcase 2.1. If some D_i contains three elements of B, say $\overline{x}, \overline{y}, \overline{z} \in D_i \cap \overline{S}$. Let $T = \{x_i^*, y_i^*, x^*, y^*, z^*\}$. It is clear that $G[T]$ is connected, thus $q = w(G[\overline{S}]) \le k-2$. By choosing $q-1$ vertices, say $w_1, w_2, \cdots, w_{q-1}$, connecting these components as done in Case 1, we that $\overline{S} \cup \{w_1, w_2, \cdots, w_{k-1}\} \cup \{u, v, w\}$ is a connected dominating set of G. Therefore $\gamma_c(G) \le |\overline{S}| + q - 1 + 3 \le 3k$.

Subcase 2.2. Each D_i has at most two vertices of $\overline{S}$. If $w(G[\overline{S}]) \le k-2$, the result follows in a like way from the discussion in Subcase 2.1.

If $w(G[\overline{S}]) = k$. Then each component is isomorphic to K_2, and each D_i contains exactly two vertices of $\overline{S}$. Let $\overline{x}, \overline{y} \in \overline{S} \cap D_i$. Then we have $\{\overline{x}, \overline{y} = \{x_i, y_i\}$, i.e., $S = \overline{S}$, and $x_i, y_i \notin E(G), \bar{x}\bar{y} \notin E(G)$, since otherwise $G[\overline{S}]$ would contain a connected subgraph of order ≥ 4. Moreover, the k components of $G[\overline{S}]$ are $x_i\overline{y_i} \in E(G), y_i\overline{x_i} \in E(G), 1 \le i \le k$. Let $D = (\overline{S} - \{\overline{x_1}, x_1\}) \cup \{w_1, w_2, \cdots, w_{k-1}\} \cup \{u, v, w\}$, where $w_1, w_2, \cdots, w_{k-1}$ are taken as above. Then D is a connected dominating set of G, and $\gamma_c(G) \le |D| \le 2k - 2 + k - 1 + 3 = 3k$.

Assume now that $w(G[\overline{S}]) = k-1$. If $S = \overline{S}$, then each D_i contains exactly two vertices of $\overline{S}$ as $\{x_i, y_i\}$. $G[\overline{S}]$ has $k-2$ components as K_2 and one component as $G[\{x_i, y_i, \overline{x_i}, \overline{y_i}\}]$ for some i, $1 \le i \le k$. Take a non-cut vertex of $G[\{x_i, y_i, \overline{x_i}, \overline{y_i}\}]$, say x_i, and then take $k-2$ vertices $w_1, w_2, \cdots, w_{k-2}$ connecting the $k-2$ components and $G[\{y_i, \overline{x_i}, \overline{y_i}\}]$. It is clear that $D = (\overline{S} - \{x_i\}) \cup \{w_1, w_2, \cdots, w_{k-2}\} \cup \{u, v, w\}$ is a connected dominating set of G, and $\gamma_c(G) \le |D| \le 2k - 1 + k - 2 + 3 = 3k$. If $S \ne \overline{S}$, then $S \not\subseteq \overline{S}$. Thus $|\overline{S}| \leq 2k - 1 = |S| - 1$. Taking $k - 2$ vertices connecting the $k - 1$ components of $G[\overline{S}]$ as done above, we get that $\gamma_c(G) \leq |D| \leq 2k - 1 + k - 2 + 3 = 3k$.

Finally, if $\gamma_c(G^c) = 2$. Then $\gamma(G) = 2$. Thus $|V_1| = 2$ and $G[V_1]$ has exactly two isolated vertices. The case for $k = 1$ is trivial. If $k \geq 2$, let $D = \overline{S} \cup V_1$. It is clear that D is a connected dominating set of G and thus $\gamma_c(G) \leq |D| \leq 2k + 2 \leq 3k$. □

The proof of Theorem 2.1 follows from Lemmas 2.5–2.7.

As a final remark, we note that for the graph $G = G^c = C_5$, one has $D_c(G^c) = 1$ and $\gamma_c(G) = 3$.

REFERENCES

1. R.B. Allan, R. Laskar, On domination and some related topics in graph theory, Proc. 9th S.E. Conf. on Combinatorics, Graph Theory and Computing, *Utilitas Math.*, 1978, 43-48.
2. B. Bollobás, E.J. Cockayne, Graph-theoretic parameters, concerning domination, independence and irredundance, *J. Graph Theory*, **3**(1979), 241–249.
3. J.A. Bondy, U.S.R. Murty, *Graph Theory with Applications,* Macmillan, London and Elsevier, New York, 1976.
4. E.J. Cockayne, J.H. Hattingh, S.M. Hedetniemi, S.T. Hedetniemi, A.A. McRae, Using maximality and minimality conditions to construct inequality chains, *Discrete Math.*, **176**(1997), 43–61.
5. E.J. Cockayne, S.T. Hedetniemi, Independence graphs, in: Proc. 5th S.E. Conf. on Combinatorics, Graph Theory and Computing, *Utilitas Math.*, 1974, 471–491.
6. S.T. Hedetniemi, R. Laskar, Connected domination in graphs, in: B. Bollobás ed., *Graph Theory and Combinatorics*, Academic Press, London, 1984, 209–218.
7. P. Duchet, H. Meyniel, On Hadwiger's number and stability number, *Ann. Discrete Math.*, **13**(1982), 71–74. Press, London, 1984, 209–218.
8. S.T. Hedetniemi, R.C. Laskar, Bibliography on domination in graphs and some basic definitions of domination parameters, *Discrete Math.*, **86**(1990), 257–277.
9. Sun Liang, Some results on connected domination in graphs, *Mathematica Applicata*, **5**(1992), 29–34.