AN ISOMORPHIC DECOMPOSITION OF K_{24}

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ABSTRACT. A decomposition of a graph H is a family of subgraphs of H such that each edge of H is contained in exactly one member of the family. For a graph G, a G-decomposition of the graph H is a decomposition of H into subgraphs isomorphic to G. If H has a G-decomposition, H is said to be G-decomposable; this is denoted by $H \to G$. In this paper we prove by construction that the complete graph K_{24} is G-decomposable where G is the complementary graph of the path P_5.

1. Introduction

Let G denotes a graph. A graph H is G-decomposable if there exists a family of subgraphs of H such that the following conditions are satisfied:

1/ Each member of the family is isomorphic to G.

2/ Each edge of H is contained in exactly one of the members. If such a family exists it is called a G-decomposition of H and H is said to be G-decomposable; this is denoted by $H \to G$. [1] discusses the necessary and sufficient conditions for $K_n \to G$, where K_n is the complete graph on n vertices and G denotes a graph on five vertices none of which is isolated. Even though most of these decompositions are present in [1], some cases remain open. Here we turn our attention to one of the unsolved cases.

2. The Main Result

Proposition 1. *If G is the graph complementary to the path P_5 and K_{24} is the complete graph on 24 vertices then K_{24} is G-decomposable.*

Following the terminology of [1], Proposition 1 reads $K_{24} \to G_{13}$.

Proof. The graph G is shown on Fig. 1. It is referred to as G_{13} in [1] and Fig. 2 shows the corresponding notation.

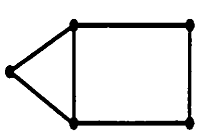

FIGURE 1

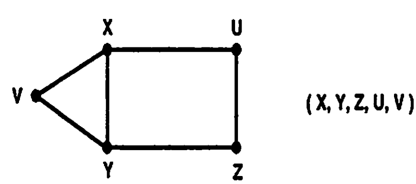

FIGURE 2

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TABLE 1

X	Y	Z	U	V	X	Y	Z	U	V
(1,	7,	11,	19,	21)	(1,	8,	12,	20,	22)
(1,	9,	13,	17,	23)	(1,	10,	14,	18,	24)
(2,	8,	11,	6,	21)	(2,	9,	12,	19,	22)
(2,	10,	13,	20,	23)	(2,	7,	14,	16,	24)
(3,	9,	11,	16,	21)	(3,	10,	12,	18,	22)
(3,	7,	13,	19,	23)	(3,	8,	14,	20,	24)
(4,	10,	11,	20,	21)	(4,	7,	17,	12,	22)
(4,	8,	13,	18,	23)	(4,	9,	14,	19,	24)
(12,	14,	22,	21,	1)	(13,	12,	23,	21,	2)
(14,	13,	24,	21,	3)	(11,	14,	23,	22,	2)
(13,	11,	24,	22,	1)	(11,	12,	24,	23,	3)
(21,	17,	14,	15,	11)	(22,	15,	7,	20,	16)
(23,	16,	12,	5,	13)	(24,	5,	9,	18,	14)
(21,	6,	10,	19,	5)	(22,	19,	15,	12,	6)
(23,	18,	11,	15,	19)	(24,	19,	8,	17,	16)
(21,	20,	15,	18,	16)	(22,	17,	16,	5,	18)
(23,	20,	9,	6,	17)	(24,	6,	8,	15,	20)
(6,	16,	7,	12,	4)	(6,	3,	4,	14,	15)
(4,	2,	3,	17,	5)	(5,	1,	4,	11,	3)
(5,	8,	10,	18,	7)	(16,	9,	15,	1,	8)
(7,	10,	17,	19,	9)	(5,	15,	4,	13,	10)
(18,	6,	1,	2,	7)	(20,	18,	16,	10,	8)
(19,	5,	17,	9,	20)	(15,	17,	6,	13,	2)

The G-decomposition of K_{24} must have 46 subgraphs of K_{24} isomorphic to G. Since efforts to apply the composition method or the method of differences to this case are seemingly futile, we present a direct G-decomposition of K_{24}. Let the vertices of K_{24} be denoted by the integers 1,2,...,24. Using the notation from Fig. 2 for G, Table 1 contains the G-decomposition of K_{24}. □

The present result completes the decomposition of K_{24} into isomorphic subgraphs with five vertices started in [1]. The supposition of the authors of [1] that such a decomposition is possible as well as the great number of results obtained by them with the help of the composition method and the method of differences show that these methods are not applicable in this case. They are used only partially in our investigation. The existence of other decompositions as well as the group of automorphisms can be the subject of a future study.

REFERENCES

1. J.-C. Bermond, C. Huang, A. Rosa, and D. Sotteau, Decomposition of Complete Graphs into Isomorphic Subgraphs with Five Vertices, ARS COMBINATORIA, vol. 10 (1908), pp. 211-254

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