But where are designs used?

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Students, funding bodies and others are often interested, not only in the structure of combinatorial designs, but in where and how such designs are used. This bibliography provides a partial answer to this question by giving some recent references, almost all of them from 1993 or later, to papers in a variety of subject areas where various designs have been used. We have concentrated on references in journals with which many mathematicians are unfamiliar. Consequently we have decided not to give references to the use of designs in cryptology and to relations of designs to error-correcting codes and Gray codes. To get a start into this literature, consult the **Handbook of Combinatorial Designs** [1]. Applications of designs to graph theory are also not listed; again the **Handbook** provides a natural starting point.

We have included a reference to Cox [2] although it is quite an old book, because it is still the very best description of planning issues and gives a general overview of the questions involved in the planning of experiments. Kuehl [6] provides a general introduction to the statistical issues of designing experiments and other authors provide descriptions of specialised areas. For example, Federer [3] discusses the design of intercropping trials. The volume edited by Kempton and Fox [5] deals with the design of plant breeding trials. Pearce [7] discusses designs for crop experimentation, and

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Williams and Matheson [9] the design of forestry experiments. Senn [8] deals with the design of cross-over trials in clinical medicine.

Finally we cite the book of Garey and Johnson [4] since computational complexity is of importance in finding and studying large designs.

Complete block designs are frequently used in practice because of their desirable statistical properties and a few references have been included for comparison with incomplete designs. Probably one of the earliest classes of combinatorially interesting designs are the incomplete block designs. Special types of incomplete block designs include α-designs, pairwise balanced designs and covering designs. Latin square designs and Youden square designs are special cases of row-column designs and are often, but not exclusively, used as change-over designs. Orthogonal arrays are often used in industrial experimentation and references to them are typically found by looking for the keyword 'Taguchi'. There are some references to neighbour designs, used in both agricultural field trials and in biochemical tests.

We have also included references which address various practical issues, such as the correct choice of plot size and the consequences to the design problem of having correlated errors. Some papers are listed which discuss the relative merits of various designs for particular application areas. Sometimes the treatments (or elements) in a design are in fact consumer goods or services, and the responses are the ranks assigned by survey participants to the elements within a block from best to worst. These ranks are then analysed. Some relevant references are given under 'ranking as response'; see Table 1.

There are some examples of algorithmic constructions in the statistics literature. Often the designs that are constructed are of practical use but not combinatorially interesting. We have tried to find some examples where there appear to be open combinatorial problems. We have also included some references to catalogues of designs. Perhaps some of the designs in these can be improved.

Finally, to be sure that we are all talking about the same designs, we have included a glossary with our definitions of the various types of designs.

Most of the references were obtained from Current Contents (1993 to Week 31 of 1998). In that time using the keywords 'block design' gave 112 references, using 'complete block designs' gave 18, 'incomplete block design' gave 34 and 'latin square' gave 88. In choosing references we have tried to cover a variety of subject areas and design types, and we apologise if your favourite reference is not here.

For many of the papers listed here, the original abstracts gave little or no information about the mathematics involved. We have instead provided in each case a brief indication of the relevant designs and the ways in which they are used.

It can be helpful to have some broad classification of the papers to get started in the right direction. Table 1 gives one possibility, classifying the papers by the types of designs that are used; Table 2 on the other hand indicates areas of application. Note that in Table 2 the papers listed as dealing with experimental design are those which discuss designs in general; the other papers discuss designs for use in some particular area.

Algorithmic constructions	11, 30, 47, 72
α-designs	5, 53, 75
Augmented designs	20, 50
Block designs	13, 20, 51, 73
Catalogues	62, 67, 68, 70
Change-over designs	1, 4, 16, 18, 23, 28, 31, 40, 42, 55, 56, 57, 63, 69, 71
Comparison of designs	3, 9, 20, 27, 34, 48, 51, 60, 62, 70, 75
Complete block designs	3, 5, 25, 37, 41, 48, 51, 75
Correlated errors	27, 68
Covering designs	11
Incomplete block designs	4, 9, 14, 21, 24, 31, 38, 39, 46, 50, 51, 53, 55, 58, 59, 62, 70, 76
Latin squares	1, 3, 12, 15, 17, 18, 19, 23, 26, 28, 29, 32, 40, 42, 45, 48, 54, 55, 56, 57, 63, 65, 66, 69, 70, 71
Neighbour designs	13, 27
Orthogonal arrays	7, 8, 10, 22, 33, 34, 35, 36, 43, 44, 47, 64, 74
Pairwise balanced designs	6
Plot size determination	49
Ranking as response	21, 31, 38
Row-column designs	19, 20, 27, 30, 50, 52, 60, 61, 67, 68, 72
Youden squares	1, 2

Table 1: A classification of the papers by design-type

Animal science	1, 16, 41, 48, 57, 58, 63, 71
Biomedical work	9, 12, 23, 28, 39, 40, 42, 55, 56, 69
Engineering	6, 7, 11, 22, 26, 32, 34, 35, 36, 43, 44, 54, 65, 66, 73, 74, 76
Experimental design	2, 8, 10, 13, 17, 19, 20, 25, 29, 31, 36, 47, 50, 67, 68
Games	61
Marketing	21, 33, 38
Plant science	3, 5, 14, 24, 27, 30, 45, 46, 49, 51, 52, 53, 59, 60, 62, 72, 75
Sensory perception	4, 15, 18, 64, 70
Soil conservation	37

Table 2: A classification of the papers by application

Standard Reference Books

1. Colbourn, C.J. and Dinitz, J.H. (editors) (1996). *The CRC Handbook of Combinatorial Designs*, Boca Raton: CRC Press.
   See also the home page at `http://www.emba.uvm.edu/~ dinitz/hcd.htm` for updates to this book.

2. Cox, D.R. (1958), *The Planning of Experiments*, New York: Wiley.

3. Federer, W.T. (1994). *Statistical Design and Analysis for Intercropping Experiments*, New York: Springer-Verlag.

4. Garey, M.R. and Johnson, D.S. (1979). *Computers and Intractability: a Guide to the Theory of NP-completeness*, San Francisco: W.H. Freeman.

5. Kempton, R.A. and Fox, P.N. (editors) (1997). *Statistical Methods for Plant Variety Evaluation*, London: Chapman & Hall.

6. Kuehl, R.O. (1994). *Statistical Principles of Research Design and Analysis*, Belmont, California: Duxbury Press.

7. Pearce, S.C. (1983). *The Agricultural Field Experiment: a Statistical Examination of Theory and Practice*, Chichester: Wiley.

8. Senn, S. (1993). *Cross-over Trials in Clinical Research*, Chichester: Wiley.

9. Williams, E.R. and Matheson, A.C. (1995) *Experimental Design and Analysis for Use in Tree Improvement*, East Melbourne: CSIRO.

Annotated Bibliography

1. Abel-Caines, S.F., Grant, R.J. and Morrison, M. (1998). Effect of soybean hulls, soy lecithin and soapstock mixtures on ruminal fermentation and milk composition in dairy cows. *Journal of Dairy Science* **81**, 462-470.
   A Latin square of order 5 and a 4 × 5 Youden square were used as change-over designs. The actual designs used are not given.

2. Altan, S. and Raghavarao, D. (1996). Nested Youden square designs. *Biometrika* **83**, 242-245.
   This paper considers the use of Youden square designs, a special class of row-column designs, for planning experiments with few experimental units and many treatments.

3. Babu, R., Agarwal, M.C., Kumar, N. and Puri, D.N. (1996). Relative efficiency of latin square design on natural grasslands in agra ravines. *Annals of Arid Zone* **35**, 137-142.
   Tables are given that show, for a range of plot sizes and plot proportions, the relative efficiency of Latin squares of orders 3, 4, 5 and 6 compared with a completely randomised design and a randomised complete block design. The final table gives the replication required in a completely randomised design and a randomised complete block design to give the same efficiency as a Latin square of order 3, 4, 5 and 6.

4. Ball, R.D. (1997). Incomplete block designs for the minimisation of order and carry-over effects in sensory analysis. *Food Quality and Preference* **8**, 111-118.
   The author shows how to construct incomplete block designs that are balanced for order effects and nearly-balanced for residual effects, and so how to make suitable change-over designs for sensory experiments.

5. Bauske, E., Hewings, A.D., Kolb, F.L. and Carmer, S.G. (1994). Variability in enzyme-linked immunosorbent assays and control of experimental error by use of experimental designs. *Plant Diseases* **78**, 1206-1210.
   This paper investigates the use of randomised complete block designs and a certain family of α-(0,1) designs in controlling the effects of

plate variability in enzyme-linked immunosorbent assays. Both such designs allow more precision than the completely randomised design.

6. Berkovich, E. and Berkovich, S. (1998). A combinatorial architecture for instruction-level parallelism. *Microprocessors & Microsystems* **22**, 23-31.
   The application of pairwise balanced designs to organising the interconnections in a multiprocessor system is described.

7. Bounou, M., Lefebvre, S. and Do, X.D. (1995). Improving the quality of an optimal power flow solution by Taguchi method. *International Journal of Electrical Power & Energy Systems* **17**, 113-118.
   An orthogonal array with 8 rows, each with 3 symbols, and 18 columns is used to achieve the objective of the title.

8. Box, G. and Tyssedal, J. (1996). Projective properties of certain orthogonal arrays. *Biometrika* **83**, 950-955.
   This paper looks at orthogonal arrays with k rows, each with 2 symbols, and asks for what values of P can every set of P rows, chosen from the k, have each of the 2^P column vectors at least once? This projective property is useful when the designs are used as screening designs. If at most P of the factors are active then the orthogonal array can be treated as a complete factorial in the active factors.

9. Brittain, E., Palensky, J., Blood, J. and Wittes, J. (1997). Blinded subjective rankings as a method of assessing treatment effect - a large sample example from the systolic hypertension in the elderly program. (SHEP) *Statistics in Medicine* **16**, 681-693.
   In clinical trials there is often more than one response variable which can make it difficult to determine whether treatment has been successful. Hence some studies have combined these various response variables by using one rating given independently by several experienced raters. For large trials it is not possible for the same raters to rate everyone in the study. The authors show that incomplete block designs are an effective way to implement the rating.

10. Cheng, C.S. (1998). Some hidden projection properties of orthogonal arrays with strength three. *Biometrika* **85**, 491-495.
   The projection properties are considered for 2-level orthogonal arrays of strength 3 with the number of runs not a multiple of 16. It is shown that projecting such an array onto any 5 rows gives an array in which main effects and two-factor interactions can be estimated if interaction effects of higher order can be ignored.

11. Cohen, D.M., Dalal, S.R., Fredman, M.L. and Patton, G.C. (1997). The AETG system: an approach to testing based on combinatorial design. *IEEE Transactions on Software Engineering* **23**, 437-444.
Covering designs can be used in the construction of test cases for the testing of software. The designs are obtained using an algorithm which is briefly described, as are two test cases of the use of the algorithm.

12. Costa, P., Bressolle, F., Sarrazin, B., Mosser, J. and Galtier, M. (1993). Pharmacokinetics of moxisylyte in healthy volunteers after intravenous and intracavernous administration. *Journal of Pharmaceutical Sciences* **82**, 729-733.
Two drugs for treating erectile impotence were administered to twelve male subjects. Each subject received one drug at a time with at least two weeks between drug applications. The authors describe this as a 'randomised Latin squares study' from which we assume that the Latin square of order 2 provided the treatment sequences and so six subjects received one drug first and six the other drug first.

13. David, O. and Kempton, R.A (1996). Designs for interference. *Biometrics* **52**, 597-606.
Interference refers to the situation when the experimental units used to assess a set of treatments are smaller than the units used in practice and so the experimental situation inadvertently allows for treatments to interfere with each other in ways that can not usually happen. This paper proposes both block designs, related to group divisible designs, and neighbour designs that avoid this problem, as well as giving a list of situations when the interference may be on adjacent plots or on larger groups.

14. Dey, A. and Midha, C.K. (1996) Optimal block designs for diallel crosses. *Biometrika* **83**, 484-489.
The use of triangular partially balanced incomplete block designs in the construction of optimal designs for diallel crosses is discussed.

15. Dijksterhuis, G. (1996). On triangle tests and Latin squares - a method of producing test plans. *Food Science & Technology-Lebensmittel-Wissenschaft & Technologie* **29**, 146-150.
A triangle test is used to test for differences between the taste of products. In such a test, a taster is asked to taste three products in a specified order, and to nominate which of the three is the odd-one-out. Here a procedure is described, based on Latin squares, for calculating triangles for any number of tasters and tests, in which the tests are balanced with respect to order effects.

16. Dougherty, C.T. and Cornelius, P.L. (1997). Estimation of direct, residual and total effects of canopy treatments affecting intake by grazing cattle. *Agronomy Journal* **89**, 840-845.
This paper describes the results of a change-over experiment for three treatments in which there was complete balance for first-order residual effects. Each of twelve animals received only two of the three treatments (and so the design is described as a balanced incomplete block design in the paper) and received these in four sequences balanced for first order residual effects.

17. Draper, N.R., Lewis, S.M., John P.J.M., Prescott, P., Dean, A.M. and Tuck M.G. (1993). Mixture designs for 4 components in orthogonal blocks. *Technometrics* **35**, 268-276.
The use of a 4×4 Latin square to design a mixture experiment is described. The properties of the various Latin squares of order 4 in this context are investigated.

18. Durier, C., Monod, H. and Bruetschy, A. (1997). Design and analysis of factorial sensory experiments with carry-over effects. *Food Quality & Preference* **8**, 141-149.
The authors discuss various practical considerations in the choice of design for sensory experiments in which the treatments have a factorial structure. They conclude that complete Latin squares used as change-over designs have various desirable properties.

19. Edmondson, R.N. (1993). Systematic row-and-column designs balanced for low order polynomial interactions between rows and columns. *Journal of the Royal Statistical Series B-Methodological* **55**, 707-723.
This paper considers the construction of row-column designs and Latin square designs when there may be polynomial trend along the rows and columns. Constructions are given for Latin squares of orders 5, 6, 7, 9 and 2^n, and row-column designs of size $2^n \times 2^m$.

20. Federer, W.T. (1998). Recovery of interblock, intergradient and intervariety information in incomplete block and lattice rectangle designed experiments. *Biometrics* **54**, 471-481.
This paper discusses the use of augmented designs, laid out as block designs or as row-column designs, in the recovery of the additional information mentioned in the title.

21. Finn, A. and Louviere, J.J. (1992). Determining the appropriate resonse to evidence of public concern: the case of food safety. *Journal of Public Policy and Marketing* **11**, 12-25.
This paper describes the use of Hadamard designs to obtain ranked responses on food safety issues.

22. Fortanbary, M.W., Mansager, B.K. and Newberry, C.F. (1996). Supporting acquisition decisions through effective experimental design. *Mathematical & Computer Modelling* **23**, 81-90.
Two orthogonal arrays, one with 7 rows, each row with 2 symbols, and 8 columns, and one with 4 rows, each row with 3 symbols, and 9 columns, were used to see the effect on a weapon system of changes in weapon parameters.

23. Frape, D.L., Williams, N.R., Scriven, A.J., Palmer, C.R., Osullivan, K. and Fletcher, R.J. (1997). Diurnal trends in responses of blood plasma concentrations of glucose, insulin and c-peptide following high- and low-fat meals and their relation to fat metabolism in healthy middle-aged volunteers. *British Journal of Nutrition* **77**, 523-535
A complete set of three mutually orthogonal Latin squares of order 4 is used to investigate the topic of the title.

24. Ghosh, D.K. and Divecha, J. (1997). Two associate class partially balanced incomplete block designs and partial diallel crosses. *Biometrika* **84**, 245-248.
This paper discusses the application of partially balanced incomplete block designs to diallel cross experiments.

25. Ghosh, S.P. (1993). Statistical databases - design of experiment structures. *Information Systems* **18**, 233-247.
The author shows how data can be extracted from a relational database so that it forms a randomised complete block design. The data can then be analysed using standard techniques. Incomplete block designs are not explored but could probably be used in the same way.

26. Gipser, T., Jager, H.A. and Rapp, L. (1998). Broadcasting, scalability and reconfigurability aspects in an all-optical network architecture. *Fiber & Integrated Optics* **17**, 21-40.
The number of different wavelength assignment schemes in an optical network is shown to be the same as the number of distinct Latin squares of given order.

27. Grondona, M.O., Crossa, J., Fox, P.N. and Pfeiffer, W.H. (1996). Analysis of variety yield trials using two-dimensional separable ARIMA processes. *Biometrics* **52**, 763-770.
Thirty-five cereal yield trials were analysed using published methods and the results provide some justification for the construction of neighbour-balanced row-column designs.

28. Helewa, A., Goldsmith, C.H. and Smythe, H.A. (1993). Measuring abdominal muscle weakness in patients with low back pain and matched controls - a comparison of 3 devices. *Journal of Rheumatology* **20**, 1539-1543.
A 3×3 Latin square was used as a change-over design to decide the order of testing of three devices to assess abdominal muscle strength.

29. Irving, R.W. and Jerrum, M.R. (1994). 3-dimensional statistical data security problems. *SIAM Journal on Computing* **23**, 170-184.
Consider a three-dimensional array containing cross-tabulated integer statistics. Suppose that the row, column and 'file' sums of this table (that is, the sums of all lines parallel to the co-ordinate axes) are revealed, together with the entries in some of the individual cells in the table. This paper shows that to determine whether there are any tables with the given sums is intractable in a particular way; see Garey and Johnson [4] for the definition of NP-completeness.

30. John, J.A. and Williams, E.R. (1997). The construction of efficient two-replicate row-column designs for use in field trials. *Applied Statistics - Journal of the Royal Statistics Society Series C* **46**, 207-214.
An algorithm is given to generate row-column designs with two-replicates. Such designs are said to be used in multisite tree or plant breeding programmes but no subject references are given. Executable code is available from the authors.

31. Kepner, J.L. and Wackerly, D.D. (1996). On rank transformation techniques for balanced incomplete repeated-measures designs. *Journal of the American Statistical Association* **91**, 1619-1625.
This paper discussed the use of rank transformation methods for detecting treatment effects in a balanced incomplete block design in which the blocks correspond to subjects who receive the treatments in a block in turn, an example of a change-over design.

32. Kim, K. and Prasanna, V.K. (1993). Latin squares for parallel array access. *IEEE Transactions on Parallel & Distributed Systems* **4**, 361-370.
If more than one processing element tries to access the same memory module at the same time then there is said to be *memory conflict*. The authors describe the use of Latin squares to provide access to memory modules in a computer in such a way that the conflict is kept to a minimum. They also show that a new class of Latin squares are necessary and give some constructions for them.

33. Kuhfeld, W.F., Tobias, R.D. and Garratt. M, (1994). Efficient experimental design with marketing research applications. *Journal of*

Marketing Research **31**, 545-557.
Design considerations for market research are discussed, in particular the question of how orthogonal arrays and orthogonal arrays with variable numbers of symbols are used in the design of choice experiments and conjoint analysis studies.

34. Lautenschlager, U., Erikstad, S.O., Allen, J.K. and Mistree, F. (1995). Experimental satellite trajectory analysis using decision-based robust design. *Journal of Guidance Control & Dynamics* **18**, 1126-1132.
This paper discusses the use of orthogonal arrays to analyse trajectories of space vehicles and compares the results with those obtained using Monte Carlo simulations. The results are comparable but the orthogonal array uses only 27 simulations compared to the 1000 used for the Monte Carlo method.

35. Lim, J.M., Kim, S., Yum, B.J., Hwang, H. (1996). Determination of an optimal configuration of operating policies for direct-input-output manufacturing systems using the Taguchi method. *Computers & Industrial Engineering* **31**, 555-560.
One orthogonal array with 4 rows, each with 3 symbols, and 27 columns, and one with 6 rows, each with 2 symbols, and 8 columns, are used to achieve the objective of the title.

36. Lin, D.K.J. (1994). Making full use of Taguchi's orthogonal arrays. *Quality & Reliability Engineering International* **10**, 117-121.
This paper shows how the columns of an orthogonal array that are not assigned factors in the experiment can be used to estimate interaction effects and the conclusions that can be drawn as a consequence.

37. Logsdon, S.D. and Kaspar, T. C. (1995). Tillage influences as measured by ponded and tension infiltration. *Journal of Soil & Water Conservation* **50**, 571-575.
This paper describes the results of an experiment in which some measurements were made on whole farms, and some measurements were made on researcher-managed fields laid out as a randomized complete block design. There is a good description of the different treatments (tillage systems) and discussion of the difficulties with earlier work in the area.

38. Louviere, J.J., Finn, A. and Timmermans, H. (to appear). Retailing research, in *McGraw-Hill Handbook of Marketing Research*, New York: McGraw-Hill Book Company.
Ranked responses are used for investigating aspects of retail images.

39. Macnab, A.J., Levine, M., Glick, N., Phillips, N., Susak, L. and Elliott, M. (1994). The Vancouver Sedative Recovery Scale for children: validation and reliability of scoring based on videotaped instruction. *Canadian Journal of Anaesthesia* **41**, 913-918.
A balanced incomplete block design with $v = 16$, $k = 6$ and $\lambda = 2$ is used to study the effectiveness of videotaped instruction of inexperienced raters in the use of a standard rating scale for recovery of children from sedation.

40. Mcnair, P.J., Depledge, J., Brettkelly, M. and Stanley, S.N. (1996). Verbal encouragement - effects on maximum effort voluntary muscle action. *British Journal of Sports Medicine* **30**, 243-245.
Latin squares of order 2 are used as change-over designs in this paper.

41. Mahan, D.C., Lepine, A.J. and Dabrowski, K. (1994). Efficacy of Magnesium-L-Ascorbyl-2-Phosphate as a vitamin C source for weanling and growing-finishing swine. *Journal of Animal Science* **72**, 2354-2361.
A randomised complete block design is used in a feeding trial on pigs to investigate the problem outlined in the title.

42. Matthews, J.N.S. and Hoenich, N.A. (1994). Statistical aspects of the design and analysis of studies to compare haemodialysis membranes. *Nephrology, Dialysis, Transplantation* **9**, 176-183.
The authors discuss various practical considerations in the choice of design for the comparison of haemodialysis membranes. They conclude that complete Latin squares used as change-over designs have various desirable properties.

43. Mazumdar, S.K. and Hoa, S.V. (1995). Application of Taguchi method for process enhancement of on-line consolidation technique. *Composites* **26**, 669-673.
This paper gives the results of an experiment to look at the effects of laser power, tape speed and consolidation pressure on interply bond strength of a thermoplastic composite. An orthogonal array with 3 rows, each with 3 symbols, and containing 9 runs was used.

44. Miyamoto, Y., Ogawa, S., Miyajima, M., Sato, H., Takayama, K. and Nagai, T. (1995). An evaluation of process variables in wet granulation. *Drug Development & Industrial Pharmacy* **21**, 2213-2225.
The authors discuss the use of an orthogonal array with five factors, each with two levels, and containing 16 runs to try to improve the flowability of powders used in the manufacture of pharmaceuticals.

45. Morton, J.R. (1997). Use of certain 5x5 latin squares for increasing information from agroforestry experiments. *Journal of Agricultural Science* **128**, 1-4.
The use of Latin squares of order 5 for studying tree crops is considered. The author shows how to include border plots with mixed tree species, so that data from border plots need not be wasted. A discussion of the analysis of such an experiment is included.

46. Mukerjee, R. (1997). Optimal partial diallel crosses. *Biometrika* **84**, 939-948.
This paper gives a definition of optimality for use when designing partial diallel crosses and shows that a class of group divisible designs are in fact optimal.

47. Nguyen, N.K. (1996). A note on the construction of near-orthogonal arrays with mixed levels and economic run size. *Technometrics* **38**, 279-283.
An algorithm is given for constructing near-orthgonal arrays, designs that are useful as screening designs when, because of the number of symbols to be used in each row, the smallest orthogonal array is too large to be practicable.

48. Odulaja, A. and Abuzinid, I.M. (1997). The relative efficiencies of latin square and randomized complete block designs for insect trapping experiments - an investigation using field data on tsetse flies. *Ecological Entomology* **22**, 184-188.
This paper gives a clear discussion of the topic of the title, together with other related references.

49. Ortiz, R. (1995). Plot techniques for assessment of bunch weight in banana trials under two systems of crop management. *Agronomy Journal* **879**, 63-69.
This paper describes a uniformity trial for bananas where the results are then analysed with plots of different sizes. Here 'plot size' refers to the number of plants per plot. This allows for recommendations of the smallest experiments that will enable effects of a given magnitude to be detected with reasonable certainty.

50. Pearce, S.C. (1994). Reinforced lattices. *Journal of the Royal Statistical Society B* **56**, 469-476.
Suppose that a control treatment is adjoined to every block in a lattice design, laid out either as a block design or as a row-column design. The statistical consequences are discussed.

51. Pearce, S.C. (1995). Some design problems in crop experimentation .1. the use of blocks. *Experimental Agriculture* **31**, 191-203.
This paper discusses the role of blocks, for instance the control of extraneous variation, in crop experimentation.

52. Pearce, S.C. (1995). Some design problems in crop experimentation .2. multiple blocking systems. *Experimental Agriculture* **31**, 279-290.
This paper gives a good discussion of the situations in which a row-column design might be used to improve the situation and circumstances in which the use of such a design would make things worse. Similar discussions of strip-plot and split-plot designs are also presented.

53. Pearce, S.C. (1995). Some design problems in crop experimentation .3. non orthogonality. *Experimental Agriculture* **31**, 409-422.
Various aspects of blocking for crop experiments are discussed, including balanced incomplete block designs, α-designs and lattice designs, and generalisations of the concept of balance which make designs more useful.

54. Pottie, G.J. and Calderbank, A.R. (1995). Channel coding strategies for cellular radio. *IEEE Transactions on Vehicular Technology* **44**, 763-770.
This paper shows how Latin squares and sets of MOLS can be used in the construction of a synchronous cellular radio system. The roles that rows, columns and elements of the Latin squares play in the radio system are explicitly given.

55. Prescott, P., Ryan, J. and Shuttleworth, P. (1994). A comparative study of several antibiotic formulations using a design based on a combination of balanced incomplete blocks and Latin squares. *Statistics in Medicine* **13**, 11-21.
An experiment to assess the palatibility of six antibiotic formulations was based on a balanced incomplete block design with $v = 6$ and $k = 3$. Because each subject in the experiment had to assess three of the six formulations and it was believed that order of presentation might affect the results, a pair of quasi-complete Latin squares was used to balance the order of presentation. This is an example of a change-over design.

56. Robergs, R.A., Wagner, D.R. and Skemp, K.M. (1997). Oxygen consumption and energy expenditure of level versus downhill running. *Journal of Sports Medicine & Physical Fitness* **37**, 168-174.
A Latin square of order four was used to determine the order of assignment of four 'slopes' to 13 runners. Each runner did the first

assigned slope, had a 15 minute break and did the second assigned slope. The third and fourth slopes were done in a similar way 3 to 10 days later. The actual square used is not given.

57. Rossi, J.E., Goetsch, A.L. and Galloway, D.L. (1998). Intake and digestion by Holstein steers consuming different particle size fractions of broiler litter. *Animal Feed Science & Technology* **71**, 145-156.
Broiler litters from two sources were each tested in a change-over design based on Latin squares of order four. One treatment in each square was the control. The other three treatments were the broiler litter, the broiler litter that passed through a 1mm screen aperture and the broiler litter retained by the sieve. No mention of residual effects is made and the actual squares used are not given.

58. Sanchez, W.K., Beede, D.K. and Delorenzo, M.A. (1994). Macromineral element interrelationships and lactational performance - empirical models from a large data set. *Journal of Dairy Science* **77**, 3096-3110.
This paper describes the combined analysis of a series of ten experiments to model lactational performance. Each experiment 'utilized partially balanced or balanced incomplete block designs' but no further details or references are given.

59. Singh, M. and Hinkelmann, K. (1995). Partial diallel crosses in incomplete blocks. *Biometrics* **51**, 1302-1314.
This paper discusses the use of partially balanced incomplete block designs to perform diallel crosses.

60. Seeger, P. (1994). Row-column designs for sugar beet variety trials. *Swedish Journal of Agricultural Research* **24**, 45-48.
Two types of row-column designs are compared. The first type has no restrictions on the layout within subsets of rows; in the second type, the sets of rows are partitioned into sets of four, and each treatment has to appear in each set of four rows. These designs are termed *resolvable* and appear to be slightly more precise.

61. Seeger, P. (1996). Nested row-column and generalised Howell designs used in bridge competitions. *Scandinavian Journal of Statistics* **23**, 243-255.
A statistical model to analyse bridge games is considered, and designs good for home competitions are found.

62. Solorzano, V., Gilmour, S.G., Phelps, K. and Kennedy, R. (1997). Assessment of suitable designs for field experiments involving airborne diseases. *Journal of Agricultural Science* **129**, 249-256.

This paper considers the best designs for experiments to investigate plant diseases caused by airborne pathogens. In such an investigation there is likely to be interplot interference. The authors review the results about good experiments for that situation, and then use the relevant principles to design small experiments for up to seven treatments. The enumeration for some numbers of treatments was not exhaustive.

63. Sporndly, E. The effect of fouling on herbage intake of dairy cows on late season pasture. *Acta Agriculturae Scandinavica A - Animal Science* **46**, 144- 153.
A 3 × 3 Latin square is used as a change-over design to investigate the topic of the title.

64. Takatsuji, T. and Tanaka, K. (1996). A procedure for scaling sensory attributes based on multidimensional measurements - application to sensory sharpness of knives. *Measurement Science & Technology* **7**, 869-875.
An orthogonal array was used to determine which factors affect the perception of the sharpness of a knife.

65. Trivedi, Y.C. and Kurz, L. (1994). A class of robust image processors. *Pattern Recognition* **27**, 1111-1125.
This paper describes the use of 5 × 5 mutually orthogonal Latin squares in the construction of robust image restorers.

66. Trivedi, Y.C. and Kurz, L. (1995). Image restoration using recursive estimators. *IEEE Transactions on Systems, Man & Cybernetics* **25**, 1470-1482.
This paper describes the use of 5 × 5 mutually orthogonal Latin squares for edge detection when restoring images that have been corrupted by noise.

67. Ture, T.E. (1994). Optimal row-column designs for multiple comparisons with a control - a complete catalog. *Technometrics* **36**, 292-299.
This paper gives a motivating example for considering designs containing a standard, or control, treatment against which all the other test treatments must be compared with equal precision. A catalogue of optimal row-column designs for this purpose is given.

68. Uddin, N. and Morgan, J.P. (1997). Efficient block designs for settings with spatially correlated errors. *Biometrika* **4**, 443-454.
The structure of optimal and near-optimal designs is considered, when the blocks are viewed as $p \times 2$ arrays, and errors on some plots are

assumed to be correlated, using either a doubly geometric or an autonormal process. Thus the designs here are examples of neighbour designs. A table of designs with $v < 30$ is given.

69. Van Dokkum, W., Delagueronniere, V., Schaafsma, G., Bouley, C., Luten, J. and Latge, C. (1996). Bioavailability of calcium of fresh cheeses, enteral food and mineral water - a study with stable calcium isotopes in young adult women. *British Journal of Nutrition* **75**, 893-903.
This paper discusses the results of an experiment that used 'two completely randomised Latin square designs' to allocate six treatments to each of 12 healthy women as a change-over experiment.

70. Wakeling, I.N. and Macfie, H.J.H. (1995). Designing consumer trials balanced for first and higher orders of carry-over effect when only a subset of k samples from t may be tested. *Food Quality & Preference* **6**, 299-308.
Tables of designs are given, based on MOLS of orders 4, 5, 7, 8, 9 and 11 and a few designs based on incomplete block designs. SAS code is given to construct the MOLS and to replace the blocks of an incomplete block design with a Williams design. (SAS is a common statistical package.)

71. Whitehair, K.J., Steffey, E.P., Willits, N.H. and Woliner, M.J. (1993). Recovery of horses from inhalation anesthesia. *American Journal of Veterinary Research* **54**, 1693-1702.
A change-over experiment is described, based on a pair of Williams squares of order three, utilising six horses to investigate the recovery from the effects of three anesthetics.

72. Williams, E.R. and John, J.A. (1996). Row-column designs and factorial designs for use in agricultural field trials. *Applied Statistics* **45**, 39-46.
An algorithm is given for the construction of factorial experiments in row-column layouts.

73. Wu, X.F., Li, J. and Kameda, H. (1998). Reliability analysis of disk array organizations by considering uncorrectable bit errors. *IEICE Transactions on Information & Systems E* **81**, 73-80.
An analytic model is presented for studying the reliability of some disk array organizations discussed in the literature. Basing the organization of a disk array on block designs allows gracefully degraded performance after disk failure.

74. Yang, G.C.C. and Kao, K.L. (1994). Feasibility of using a mixture of an electroplating sludge and a calcium carbonate sludge as a binder for sludge solidification. *Journal of Hazardous Materials* **36** 81-88.
This paper describes the use of an orthogonal array with nine columns and four rows, each row with three levels.

75. Yau, S.K. (1997). Efficiency of alpha-lattice designs in international variety yield trials of barley and wheat. *Journal of Agricultural Science* **128**, 5-9.
Using the results of 714 trials, classified by yield levels and by irrigation conditions among other things, the author gives information on the efficiency of α designs relative to randomised complete block designs.

76. Yener, B., Ofek, Y. and Yung, M.T. (1997). Combinatorial design of congestion-free networks. *IEEE/ACM Transactions on Networking* **5**, 989-1000.
The use of balanced incomplete block designs in the construction of congestion-free networks is described.

Glossary

active factor An *active factor* is one which produces a change in the observed responses when its level is changed.

alpha design An $\alpha(0,1)$ *design* is a design on $v = sk$ treatments with r replicates of each treatment and with blocks of size k arranged in r resolution classes of s blocks each. Any pair of treatments appears in 0 or 1 blocks.

association scheme An *association scheme* is a way of describing the relations between the members of a set X of v treatments. If an association scheme with m associate classes is defined on X, then (i) any two distinct treatments are ith associates for exactly one value of i, $1 \leq i \leq m$; (ii) each treatment has exactly n_i ith associates, $1 \leq i \leq m$; (iii) for any pair of ith associates, x and y, say, there are a fixed number of treatments which are both jth associates of x and hth associates of y, and this number is independent of the particular pair of ith associates chosen. A number of different association schemes are in common use; for two associate classes these include the group divisible, triangular, Latin-square-type and cyclic schemes.

augmented design An *augmented design* is a block or row-column design to which additional treatments are adjoined at a later time.

balanced incomplete block design A *balanced incomplete block design* based on a given v-set, X, is a set of b blocks, each of size k, chosen from X, so that each treatment appears in r blocks and each pair of treatments appears in λ blocks.

block design Any arrangement of elements from a set of v treatments into (not necessarily disjoint) subsets of size k is called a *block design*. Blocks are used in experiments when it is not possible to get a homogeneous set of v experimental units on which to conduct the experiment, but it is possible to get smaller sets of homogeneous units. The statistical analysis of the results can then be used to compare the treatments independently of the blocks. Usually some additional requirements on the composition of the subsets are imposed: see balanced incomplete block designs; partially balanced incomplete block designs; pairwise balanced designs; randomised complete block design.

change-over design A *change-over design* has n subjects and t treatments, and lasts for p periods. Each subject receives one treatment in each period and a response is observed at the end of each period. Hence the design may be represented as a $p \times n$ array with entries from a set of t elements.

choice experiment A *choice experiment* is an experiment in which a group of subjects are presented with one, or more, choice sets describing a product such as, say, laundry detergent or holiday destinations, and are asked which option in the choice set they would choose. Incomplete block designs can be used to construct the choice sets.

complete block A *complete block* is one in which all the treatments appear. Compare to incomplete blocks.

complete factorial A *complete factorial* design is one in each possible combination of the factor levels appears at least once.

complete Latin square A *complete Latin square* is a Latin square in which each ordered pair of distinct treatments appears in adjacent positions exactly once in rows and exactly once in columns.

completely randomised design A *completely randomised design* is one conducted on a homogeneous set of plots, so there is no need for any blocks to help control variability and thus produce more accurate answers.

conjoint analysis In a *conjoint analysis* study, one or more subjects rank the elements of a choice set describing a product such as, say, laundry detergent or holiday destinations. These ranks are then analysed to try to determine the weighting that subjects have given to various attributes of the products.

control treatment A *control treatment* is one used as a benchmark treatment against which to compare a set of experimental treatments. It may represent the *placebo* in a medical trial, or standard practice in a trial of farming methods.

correlated errors In many applications the responses on experimental units in an experiment can be assumed to be independent of each other. In some situations, however, notably in change-over designs, each experimental unit receives several treatments in the course of the experiment and responses on the same unit may be assumed to be correlated. Thus we speak of *correlated errors.* Sometimes the phrase is used in generality; sometimes specific error processes are considered.

covering design A *covering design* is a block design in which every t-subset of treatments appears in at least λ blocks for given t and λ.

cyclic association scheme A *cyclic association scheme* is a two-associate class scheme related to a difference set. Let $\{1,2,\ldots,v-1\} = \{d_1,\ldots,d_{n_1}\} \cup \{e_1,\ldots,e_{n_2}\} = D \cup E$. Then a non-group divisible association scheme designed on Z_v is *cyclic* if $D = -D$ and if the non-zero differences of distinct elements of D contain each element of D an equal number of times and each element of A an equal number of times. The first associates of i are the elements of $i + D$.

diallel cross A diallel cross occurs in plant or animal breeding. We assume there are n parental genotypes. In a *complete diallel cross,* all possible crosses among the genotypes are made. Thus there are n^2 families and a rectangular association scheme arises naturally here. A *half-diallel cross* is one with no self-crosses but all other possible crosses, and the gender of the parental lines is assumed to be immaterial. Thus a triangular association scheme arises naturally here. A *partial diallel cross* uses only some of the $n(n-1)/2$ crosses and typically a cyclic association scheme is used in this case.

factor A *factor* is any attribute of the experimental units which may affect the responses observed in the experiment. A factor may be a *treatment* factor, allocated by the experimenter to the units, or a *blocking* factor, an inherent attribute of the units that may affect the response.

factorial design A *factorial design* is one in which the treatments are combinations of levels of two or more factors.

group divisible association scheme A *group divisible association scheme* has $v = mn$ treatments and this set of treatments is partitioned into m subsets (or groups, in a non-algebraic sense) of n treaments each. Treatments in the same group are first associates and those in different groups are second associates.

group divisible design A *group divisible* design is one in which the set of treatments has a group divisible association scheme.

Hadamard design A *Hadamard design* is a balanced incomplete block design in which the parameters (v, k, λ) are of the form $(4n-1, 2n-1, n-1)$ for some positive integer n.

incomplete block design An *incomplete block design* is one in which not all treatments appear in each block of the design. Sometimes balance properties are imposed on the sets of treatments in the blocks. See balanced incomplete block designs, partially balanced incomplete block designs, pairwise balanced designs.

interaction effect In a factorial design, the k-tuples of levels of a set of k factors, with $s_1, s_2, \ldots, s_k$ levels respectively can be used to partitions the set of responses into $\prod_i s_i$ subsets. The **interaction effect** of a set of k factors compares the responses among the $\prod_i s_i$ sets.

interference In field trials the yield in one plot may be either enhanced or reduced, depending on the treatments in neighbouring plots. This is termed *interference* and in some circumstances information about known properties of the treatments such as, say, expected height, is used when designing the experiment.

Latin square A *Latin square of order* n is an $n \times n$ array with n symbols arranged so that each symbol appears once in each row and once in each column of the array.

Latin-square-type association scheme A *Latin-square-type* association scheme has $v = n^2$ treatments arranged in a square array, and $i - 2$ MOLS of order n. The first associates are those that are in the same row or the same column, or in a cell occupied by the same symbol when any of the Latin squares is superimposed in the array of treatments. All other treatments are second associates. If $i = n$ then the scheme is a group divisible one; if $i = n + 1$ then all the treatments are first associates. Lattice designs are examples of L_2-type designs.

lattice design A *lattice design* is an arrangement of k^2 treatments into blocks of size k such that the set of blocks can be partitioned into $t \leq$

$k+1$ resolution classes of k blocks each such that any two treatments appear together in at most one block.

main effect In a factorial design the levels of a factor with s_i levels can be used to partition the set of responses into s_i subsets. The *main effect* of a factor compares the responses among the s_i subsets.

mixture experiment In a *mixture experiment* a treatment consists of a mixture of ingredients, so the proportion of each ingredient must lie between 0 and 1.

MOLS Two Latin squares are said to be *orthogonal* if, when the two squares are superimposed, each of the ordered pairs of treatments appears exactly once. A set of *MOLS* is a set of Latin squares which are pairwise (or mutually) orthogonal.

neighbour design *Neighbour designs* are used in situations where it is thought that the responses on a particular plot may be affected by the treatments on adjacent plots. The adjacency may be in time, for example, when using change-over designs, or in space, either in field trials, or when using tests on agar plates.

optimal design An *optimal design* is a design which makes the best use of the available resources. Typically such designs minimise some function of the variance of the treatment estimators. The most commonly used functions are the determinant and trace of the variance-covariance matrix of the estimators.

orthogonal array An *orthogonal array* $OA(N, k, s, t)$ is a $k \times N$ array with entries from a set of s distinct symbols arranged so that, for any t rows of the array, each of the s^t column vectors appears equally often. Thus we see that $s^t|N$. We call N the *number of runs* in the OA, k the *number of factors*, s the *number of levels* for each factor and t the *strength* of the array. Sometimes $N/s^t = \lambda$ is called the *index* of the array. A *near-orthogonal array* is one in which some pairs of rows no longer have an equal occurrence of all possible ordered pairs of levels.

pairwise balanced design A *pairwise balanced design* is a design in which each pair of treatments appear in λ blocks. Not all the blocks need be the same size.

partially balanced incomplete block design A *partially balanced incomplete block design* is a design based on a v-set X, with b blocks of size k and with each treatment replicated r times, such that there is an association scheme with m associate classes defined on X, where, if elements x and y are ith associates, $1 \leq i \leq m$, then they occur together in precisely λ_i blocks.

quasi-complete Latin square A *quasi-complete Latin square* is a Latin square in which each unordered pair of distinct treatments appears exactly once in rows and exactly once in columns.

randomisation scheme A *randomisation scheme* is a scheme for randomly allocating the treatments to the experimental units in the design. The scheme must take account of the blocks of the design and any other restrictions that may apply.

randomised complete block design A *randomised complete block design* is one in which each treatment appears exactly once in each block and treatments are allocated at random to the plots in each block.

ranking as response Sometimes the treatments (or elements) in a design are in fact consumer goods or services, and the responses are the *ranks* assigned by survey participants to the elements in a block from best to worst. These ranks are then analysed.

rectangular association scheme A *rectangular association scheme* has $v = n^2$ treatments. If the treatments are arranged in an $n \times n$ array, then first associates are those in the same row, second associates are those in the same column and all other treatments are third associates.

residual effect In a change-over design the *residual effect* of a treatment is the effect of a treatment applied in one period on the response observed in the next period. It is sometimes called the *carry-over* effect.

resolvable design A *resolvable design* is one in which the blocks can be collected into *resolution classes* such that each treatment appears once in each resolution class. Such designs are useful when not all the blocks can be processed on the same day. Then one or more resolution classes are processed on a day.

response The *response* is the characteristic of the experimental unit that is measured in order to assess the effect of a treatment.

row-column design A *row-column design* has $v = pq$ treatments arranged in r replicates, each of p rows and q columns.

screening design An array on 2 symbols with k rows and N columns is an (N, k, p) *screening design* if for each choice of p rows, each of the 2^p column vectors appears at least once. We say that p is the *projectivity* of the screening design.

split-plot design In a *split-plot* design, plots are of two sizes, called the whole plots and the split-plots. Each whole plot is divided into several split-plots. For instance, the whole plots might be of an appropriate size for a tractor to plough, where the split-plots are the width of the seed drill.

strip-plot design A *strip-plot* design can be used to investigate the effects of two factors, each of which needs large plots. A rectangular plot is divided into strips both vertically and horizontally; then levels of one factor are applied to the vertical strips and levels of the other factor are applied to the horizontal strips.

treatment A *treatment* is one of several conditions that are being compared in an experiment. A treatment may refer to one attribute, such as the variety of wheat or the temperature, or it may be used to refer to a combination of levels of several factors.

trend In some field experiments there may be an increase or decrease in fertility going across the field. This is called a *trend* and is usually represented by a polynomial term in the model for the experiment. Similar problems can arise when results are collected over time, say in an industrial experiment in which the responses are measured sequentially before the settings are changed.

triangular association scheme A *triangular association scheme* has $v = n(n-1)/2$ treatments, $n \geq 5$. If the treatments are arranged in a symmetrical $n \times n$ array with the diagonal entries empty, then first associates are those in the same row or column and all other treatments are second associates.

uniformity trial A *uniformity trial* is one in which the same treatment is applied to each experimental unit but the results of analyses assuming various block structures can be compared.

Williams design A Williams design is a $t \times n$ array on t treatments in which each treatment appears equally often in each row and once in each column, and each pair of distinct treatments appears in adjacent positions equally often.

Youden square A *Youden square* is a symmetrical balanced incomplete block design in which the blocks can be arranged in a $k \times v$ array such that each treatment appears once in each row.