$\{K_{1,3}, Z_2\}$-FREE GRAPHS OF LOW CONNECTIVITY

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ABSTRACT. A graph G is $\{R, S\}$*-free* is G contains no induced subgraphs isomorphic to R or S. The graph Z_1 is a triangle with a path of length 1 off one vertex; the graph Z_2 is a triangle with a path of length 2 off one vertex. A graph that is $\{K_{1,3}, Z_1\}$-free is known to be either a cycle or a complete graph minus a matching. In this paper, we investigate the structure of $\{K_{1,3}, Z_2\}$-free graphs. In particular, we characterize $\{K_{1,3}, Z_2\}$-free graphs of connectivity 1 and connectivity 2.

1. INTRODUCTION

All graphs considered in this paper are simple graphs, no loops or multiple edges. For terms not defined here, see [3]. A graph G is $\{H_1, H_2, \dots, H_k\}$-*free* ($k \geq 1$) if G contains no induced subgraph isomorphic to an H_i, $1 \leq i \leq k$. The set of graphs $\{H_1, H_2, \dots, H_k\}$ is called a *forbidden family*. The *neighborhood of a vertex* v is the set of vertices that are adjacent to v and is denoted by $N(v)$. The *closed neighborhood of a vertex* v, $N[v]$, is $N(v) \cup \{v\}$.

Two special graphs will be discussed in this paper, Z_1 and Z_2. The graph Z_1 is a triangle with a path of length 1 off one vertex. The graph Z_2 is a triangle with a path of length 2 off one vertex.

In recent years the concept of forbidden families has played an important role in the study of hamiltonian-like properties ([1], [2]). Two of the forbidden families are $\{K_{1,3}, Z_1\}$ and $\{K_{1,3}, Z_2\}$. It is well known that a $\{K_{1,3}, Z_1\}$-free graph is either a cycle or a complete graph minus a matching. Thus, it is natural to ask about the structure of graphs free of the other forbidden families, especially $\{K_{1,3}, Z_2\}$.

In this paper, we characterize all $\{K_{1,3}, Z_2\}$-free graphs with connectivity 1 and connectivity 2. To characterize these graphs, we will use the following theorem by Shepherd [4].

Theorem 1 ([4]). A connected graph G is claw-free if and only if for every minimal cut set S and every $v \in S$, $\langle N(v) - S \rangle$ is the disjoint union of two complete graphs.

First, we use Theorem 1 to prove:

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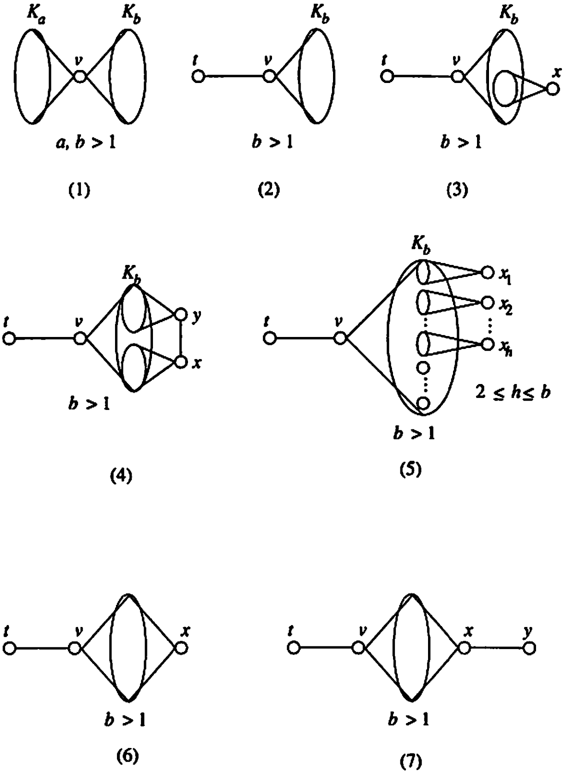


Figure 1: $\{K_{1,3}, Z_2\}$-free Families of Connectivity 1.

Theorem 2. Let G be a $\{K_{1,3}, Z_2\}$-free graph with $\kappa(G) = 1$. Then G is either a path or a member of one of the families of graphs shown in Figure 1.

Having the result in Theorem 2, we use this result and Theorem 1 to prove:

Theorem 3. Let G be a $\{K_{1,3}, Z_2\}$-free graph of order at least 10 that is not a cycle. Further, let $\kappa(G) = 2$. Then G is a member of one of the families of graphs in Figures 2, 3, and 4.

2. Proof of Theorem 2

First, we suppose that G is a tree. We note that if a tree has a vertex v of degree 3 or more, then v and three of its neighbors form a claw. Hence, a

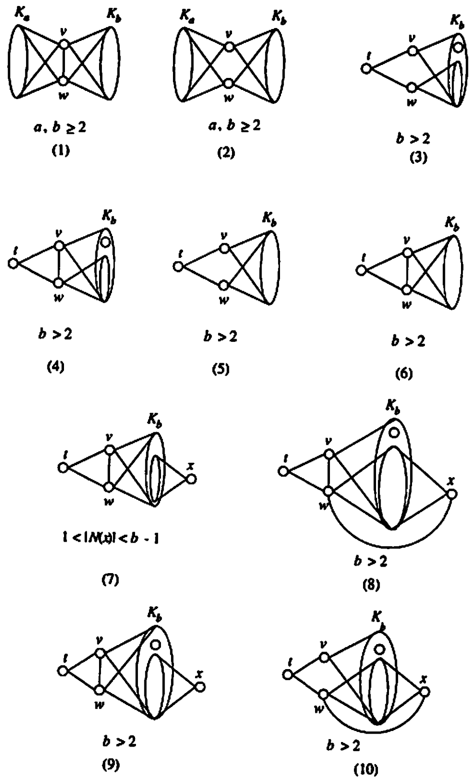


Figure 2: $\{K_{1,3}, Z_2\}$-free Families of Connectivity 2.

tree can only be $\{K_{1,3}, Z_2\}$-free if it has vertices of only degrees 2 or 1. Since paths are clearly $\{K_{1,3}, Z_2\}$-free the only trees that are $\{K_{1,3}, Z_2\}$-free are paths.

We now suppose that G is not a tree. Let v be a cut vertex in G. By Theorem 1, $\langle N(v) \rangle = K_a \cup K_b$ where K_a and K_b are two disjoint cliques of orders a and b, respectively. We will divide remainder of the proof into three parts.

PART 1: For this part, suppose that $a > 1$ and $b > 1$. Thus, we have the family of graphs in (1) of Figure 1. Suppose that $\langle N[v] \rangle \neq G$. Then, since G is connected, there is a vertex x in $V(G)$ that is not in $N[v]$ such that x is adjacent to either K_a or K_b. Without loss of generality, assume that x is adjacent to a vertex, say u, in $V(K_a)$. Since $b > 1$, there are two vertices, say p and q, in K_b that form a Z_2 with x, u, and v. Since v is a cut vertex, x cannot be adjacent to p or q. Also, notice that x cannot be

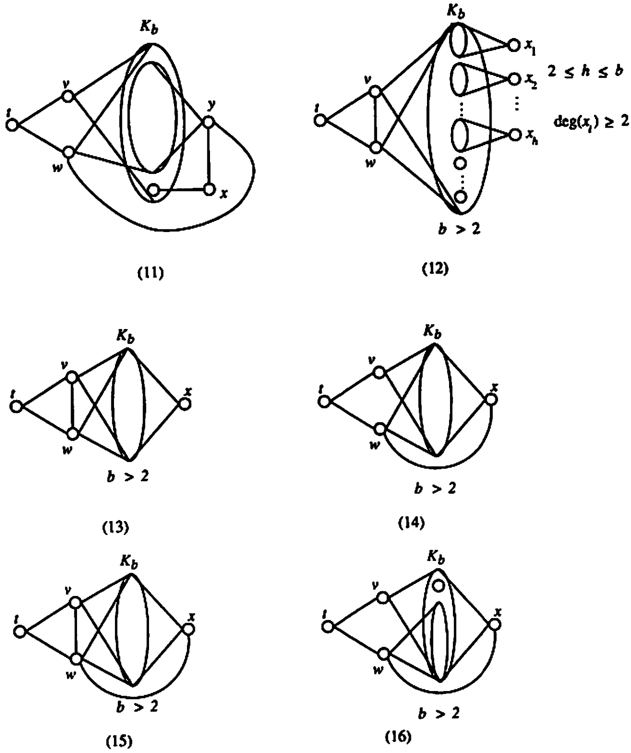
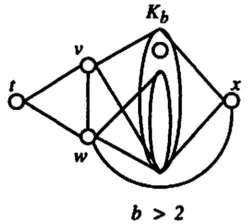


Figure 3: $\{K_{1,3}, Z_2\}$-free Families of Connectivity 2 continued.

adjacent to v by Theorem 1. Since G is $\{K_{1,3}, Z_2\}$-free, $G = N[v]$. Thus, no such x exists, and G is a member of only family (1) in Figure 1.

PART 2: Suppose without loss of generality that $a = 1$, $b > 1$, and $K_a = \{t\}$. (See (2) in Figure 1.) If $N[v] = V(G)$, G is a member of family (2) and we are finished. So suppose that $\langle N[v] \rangle \neq V(G)$. Then since G is connected, there is a vertex $x \in V(G) - N[v]$ that is adjacent to some subset of $N(v)$. If x is adjacent to t then by the same arguments in Part 1, there is a Z_2 in G. Thus, x must be adjacent to some vertex (or vertices) in $V(K_b)$.

Case2.1: Suppose $N_{K_b}(x) \neq V(K_b)$. Either $\langle N[v] \cup \{x\} \rangle$ is all of G or it is not. If it is all of G, we are done since G will be a member of family (3) in Figure 1. So, assume that $\langle N[v] \cup \{x\} \rangle \neq V(G)$. Since G is connected, pick y to be a vertex of G that is not in $\langle N[v] \cup \{x\} \rangle$ such that y is adjacent to something in $\langle N[v] \cup \{x\} \rangle$. Then either y is adjacent to x or it is not.



(17)

Figure 4: $\{K_{1,3}, Z_2\}$-free Families of Connectivity 2 continued.

Subcase 2.1.1: First, suppose that $xy \in E(G)$. Note, x cannot be a cut vertex, since if x were a cut vertex, then y would be adjacent to x, but to no vertices in $N[v]$. A Z_2 would be formed by v, a vertex in $N_{K_b}(x)$, a vertex in $V(K_b) - N_{K_b}(x)$, x, and y; hence y must be adjacent to some vertices in $N[v]$.

If y were adjacent to some vertex, say w, in $N_{K_b}(x)$, a Z_2 would be formed by $\{t, v, w, x, y\}$. However, we now note that y is adjacent to *all* the vertices of $V(K_b) - N_{K_b}(x)$, since if there were a vertex w in $V(K_b) - N_{K_b}(x)$ that was not adjacent to y, then a Z_2 would be formed by v, w, x, y and a vertex in $N_{K_b}(x)$. Hence, we see that $N_{K_b}(x)$ and $N_{K_b}(y)$ partition the vertex set of K_b. Thus, G is a member of family (4) of Figure 1.

Suppose graph (4) in Figure 1 is not all of G. Then, since G is connected, there is a vertex z adjacent to a vertex in $V(K_b) \cup \{x, y\}$.

Suppose first that z is adjacent to s, a vertex of K_b. Then, s is either in $N_{K_b}(x)$ or $N_{K_b}(y)$. Without loss of generality assume the $s \in N_{K_b}(y)$. Then z is adjacent to y or else $\langle\{s, v, y, z\}\rangle$ induces a $K_{1,3}$ centered at s. But then $\langle\{s, t, v, y, z\}$ induces a Z_2. Hence, z cannot be adjacent to a vertex in K_b.

Now suppose, without loss of generality, that $yz \in E(G)$. Then v, y, z, a vertex of $N_{K_b}(x)$, and a vertex of $N_{K_b}(y)$ induce a Z_2. Since z is not adjacent to any vertices of K_b, graph (4) in Figure 1 must be all of G.

Subcase 2.1.2: Finally, suppose the vertex $xy \notin E(G)$. Note that y cannot be adjacent to vertex s in $N_{K_b}(x)$ or else a claw centered at s will be induced by $\{s, v, x, y\}$. Thus, y must be adjacent to some vertex (or vertices) in $V(K_b) - N_{K_b}(x)$. Assume $\langle N[v] \cup \{x, y\}$ is not all of G. Then, since G is connected, there is a vertex z in G that is not in $N[v] \cup \{x, y\}$ which is adjacent to $N(v) \cup \{x, y\}$. Note that neither x nor y can be a cut vertex or else a Z_2 would be induced by v, y, a vertex in $N_{K_b}(x)$, and a vertex in $N_{K_b}(y)$ (assuming, without loss of generality, that y is the cut vertex). Also, $\{x, y\}$ cannot be a minimal cut set with respect to z or a Z_2 would be induced in exactly the same manner as above.

Now, suppose that z is adjacent to both x and y. Then z must be adjacent to some vertex (or vertices) of K_b. Observe that z cannot be adjacent to (1) a vertex in $N_{K_b}(x)$ or $N_{K_b}(y)$ or else a Z_2 would be induced; or (2) vertex in $V(K_b) - \{N(x) \cup N(y)\}$ or else a Z_2 would be. Thus, z cannot be adjacent to both x and y.

Next, suppose z is adjacent to only one of x or y. Suppose, without loss of generality, z is adjacent to x. Then z cannot be adjacent to (1) a vertex in $N(x)$ or a Z_2 is induced; (2) a vertex in $N(y)$ or a claw is induced; or (3) a vertex in $V(K_b) - \{N(x) \cup N(y)\}$ or a Z_2 is induced. Hence, z cannot be adjacent to one of x or y.

Therefore, z can only be adjacent to some vertex (or vertices) in $V(K_b) - \{N(x) \cup N(y)\}$. By continuing the preceding arguments, we see that G must be a member of family (5) in Figure 1.

Case 2.2: Suppose that $N_{K_b}(x) = V(K_b)$. If this is all of G , then G is a member of family (6) in Figure 1. Hence, suppose that it is not all of G. Then, since G is connected, there is a vertex y that is adjacent to either x or a vertex in K_b.

If y is adjacent to a vertex, say w, of K_b and x, then, by the same arguments as in Subcase 2.1.1, a Z_2 is formed by t, v, w, x and y. Now, if y is adjacent to a vertex, say w, in K_b but not adjacent to x, then a claw is formed by v, w, x and y.

Consequently, y can only be adjacent to x and we have the situation depicted in graph (7) in Figure 1. If $N[v] \cup \{x, y\}$ is all of G, then we are done. Thus, assume that $N[v] \cup \{x, y\} \neq V(G)$. Then there is a vertex z which is adjacent to x or y since G is connected. (Note that z cannot be adjacent to a vertex of K_b by the argument above.) Suppose that z is adjacent to only vertex x. Then a claw is formed. Thus, if z is adjacent to x, z must also be adjacent to y. Suppose that z is adjacent to x and y, then a Z_2 is formed by t, v, x, y, z and a vertex in K_b. Hence, z can only be adjacent to y. However, a Z_2 still exists induced by x, y, z and two vertices in K_b. Thus, G is either a member of family (6) or (7) in Figure 1.

CASE 3: Suppose that $a = b = 1$. The situation thus far is a path on three vertices, say u, v and w. Recall that we assumed that G was not a tree (path). Hence, there is a vertex of degree at least 3. If deg(w) ≥ 3, then there are vertices x and y that are adjacent to w. Note that neither x nor y can be adjacent to u since v is a cut vertex. Also, x and y are not adjacent to v since $b = 1$. Thus, x must be adjacent to y or else a $K_{1,3}$ would be formed. But then we have a Z_2 formed by the set of vertices $\{u, v, w, x, y\}$. Now, if deg(w) $= 2$, then by proceeding along the path in the direction from u to v we will find a vertex of degree at least 3. Hence, by the preceding argument, we will form a Z_2 that cannot be removed. Thus, we see that a and b cannot both be equal to 1.

So, if G is a $\{K_{1,3}, Z_2\}$-free graph of connectivity 1, then G is a path or a member of one of the seven families in Figure 1. □

3. Proof of Theorem 3

Since G has connectivity 2, let $T = \{v, w\}$ be a cutset of G and let $H = G - w$. So, H is a $\{K_{1,3}, Z_2\}$-free graph with $\kappa(H) = 1$. Hence, we know that H is a graph of Figure 1 or H is a path. We will also consider the graph $H' = G - v$.

CASE 1: Suppose that H is a path. Since $N_{H'}(w)$ is the disjoint union of two complete graphs, w is adjacent to at most two vertices in each component of $G - \{v, w\}$; otherwise, a claw would be formed. Observe that w is adjacent to the end vertices of H since the degree of the end vertices is 1 and G has connectivity 2.

Subcase 1.1: Suppose that v and w are adjacent. Then $G - \{v, w\}$ has one component that is an isolated vertex otherwise a claw would be created that could not be removed as H is a path. The other component can only have at most two vertices or a $K_{1,3}$ or Z_2 would be formed. Thus, G would have order less than 10, a contradiction.

Subcase 1.2: Suppose now that v and w are not adjacent. Observe first that since $G \neq C_n$, one component of $G - \{v, w\}$ must have at least two vertices, and w must be adjacent to two vertices in one component.

Now, if a component of $H - v$ has three or more vertices, then w must be adjacent to only the end vertex or else a Z_2 or $K_{1,3}$ would be formed. Since $G \neq C_n$, each component of $H - v$ has two or fewer vertices. Thus, G has at most 6 vertices, a contradiction, since G has order at least 10.

CASE 2: Suppose that $H = G - w$ is in family (1) in Figure 1. Note that Case 1 above takes care of the case when $a = b = 1$. We assume that $a, b \geq 2$. Since $N_{H'}(v)$ is the disjoint union of two complete graphs, $N_{H'}(v) \subseteq V(K_a) \cup V(K_b)$ since $V(G) = V(H) \cup \{w\}$.

Subcase 2.1: If $vw \in E(G)$, then w is adjacent to all the vertices in either K_a or K_b. Suppose instead there are vertices s and t in K_a and K_b, respectively, that are not adjacent to w. But now, $\{v, w, s, t\}$ forms a claw. Assume, without loss of generality, that w is adjacent to all the vertices in K_a. If w is not adjacent to all the vertices of K_b, a Z_2 would be formed by two vertices in K_a, w, a vertex in $N_{K_b}(w)$, and a vertex in $V(K_b) - N_{K_b}(w)$. Thus, G is a member of the first family in Figure 2.

Subcase 2.2: Next, suppose $vw \notin E(G)$ and $a \geq 3$ or $b \geq 3$. (Since G has order at least 10, both a and b cannot be less than 3.) Then w is adjacent to $N(v)$, for if not then, w must be adjacent to all but one vertex of K_b or a Z_2 is formed. Note that w must be adjacent to all the vertices of K_a or a Z_2 will be formed by two vertices in K_b, w, a vertex in $N_{K_a}(w)$, and a vertex in $V(K_a) - N_{K_a}(w)$. Now, since $a \geq 2$, w must be adjacent to all of K_b or a Z_2 is present. Thus, G is a member of family (2) in Figure 2.

CASE 3: Suppose that H is a member of family (2) in Figure 1. As $N_{H'}(w)$ is the disjoint union of two complete graphs, w is adjacent to t and a subset of K_b. Since $|V(G)| \geq 10$ and $b \geq 3$, w must be adjacent to all but one vertex of K_b or a Z_2 would exists. Hence, G is in one of families (3) through (6) in Figure 2.

CASE 4: Suppose that H is a member of family (3) in Figure 1. Since $N_{H'}(w)$ is the disjoint union of two complete graphs, w is adjacent to t and to some clique included in $V(K_b \cup \{x\})$. Note that if $wx \in E(G)$, then w is adjacent to at most $N[x]$, or $N_{H'}(w)$ would not be the disjoint union of two complete graphs. Since $|V(G)| \geq 10$, we know $|V(K_b)| \geq 6$. We now consider three subcases.

Subcase 4.1: Suppose that $2 \leq |N_{K_b}(x)| \leq b-2$. Then $wx \notin E(G)$, for otherwise w and v must be adjacent or a Z_2 is formed by two vertices in $K_b - N(x)$ and $\{t, v, w\}$. But there is still a Z_2 formed by two vertices in $K_b - N(x)$ and $\{v, w, x\}$. Thus, x cannot be adjacent to w. Also, observe that if w is adjacent to a vertex in $N(x)$, then it is adjacent to all the vertices of $N(x)$ or else there are Z_2's.

If w is not adjacent to all of K_b, then there is a claw whose center is a vertex in $N(x)$ with neighbors x, w, and any vertex in $K_b - N(x)$ that is not adjacent to w. If w is adjacent to a vertex in $K_b - N(x)$ and w is adjacent to v, then w is adjacent to all of K_b. Otherwise a Z_2 would be formed by t, v, w, x and a neighbor of x not adjacent to w. By the above argument if $vw \in E(G)$ and w is adjacent to a vertex in $K_b - N(x)$ and $|N(x)| \geq 2$, then w is adjacent to a vertex in $N(x)$ and hence to all of K_b. Assume $vw \notin E(G)$ and that w is only adjacent to part of $K_b - N(x)$. Then a Z_2 is formed by two vertices in $N(x)$, v, w, and t. Thus, w must be adjacent to all of $N(x)$ and hence to all of K_b. Now, note that if $vw \notin E(G)$, a $K_{1,3}$ is formed by x, v, w, and a vertex in $N(x)$. Therefore, G is a member of family (7) in Figure 2.

Subcase 4.2: Suppose that $|N_{K_b}(x)| = 1$. Note that $N_{H'}(w)$ is the disjoint union of two complete graphs. Observe that w is adjacent to both t and x or else G would have connectivity 1. Let $N_{K_b}(x) = \{y\}$. Notice that there are two Z_2's in G. The only way to eliminate both is to add the edges vw and wy. However, there is still a Z_2 induced by v, w, x and two vertices in $K_b - y$. This Z_2 cannot be eliminated since $N(w)$ is the disjoint union of two complete graphs. Thus, no graphs are possible in this case.

Subcase 4.3: Suppose that $|N_{K_b}(x)| = b - 1$. By the arguments of Subcase 4.1, if $wx \notin E(G)$, then w is adjacent to v and to all of K_b. If $wx \in E(G)$, then w is adjacent to at least $b - 1$ vertices in $N(x)$ or else a Z_2 would be formed. If $wx \in E(G)$ but $vw \notin E(G)$, then w is adjacent to all of $N[x]$ or else a Z_2 is formed. Thus, G is a member of one of families (8) through (10) in Figure 2

CASE 5: Suppose that H is in family (4) of Figure 1. Since $N_{H'}(w)$ is the disjoint union of two complete graphs, w is adjacent to t and a subset of $N[x] \cup N[y]$ that is a clique. Note that if w is adjacent to x (or y), then w can only be adjacent to at most $N[x]$ (or $N[y]$). As in Case 4, since $|V(G) \geq 10$, $b \geq 3$.

Subcase 5.1: Suppose that $2 \leq |N_{K_b}(x)| \leq b-2$ and $2 \leq |N_{K_b}(x)| \leq b-2$. Then w is not adjacent to x (or y). If it were, then since $|N_{K_b}(x)| \geq 2$, a Z_2 would be exist unless $wy \in E(G)$. But if $wy \in E(G)$, then a Z_2 would exist since w could not be adjacent to any vertices in K_b (as $N(w)$ is the disjoint union of two complete graphs). Hence, suppose, without loss of generality, that w is adjacent to a vertex in $N_{K_b}(x)$. Then w must be adjacent to all vertices in $N_{K_b}(x)$ or Z_2's would exist. But this implies that w must be adjacent to all vertices in $N_{K_b}(y)$ or claws would exist. Observe now that $vw \in E(G)$ or else a claw will be induced by x, v, w and a vertex in K_b adjacent to x. However, a claw is still induced by t, v, w, x and a vertex in $N_{K_b}(x)$. Thus, $|N_{K_b}(x)| = 1$, a contradiction.

Subcase 5.2: Suppose without loss of generality that $|N_{K_b}(x)| = 1$ and $|N_{K_b}(y)| = b-1$. By a similar argument used in Subcase 5.1, $wx \notin E(G)$ (but wy can be in $E(G)$). However, if $vw \in E(G)$, then $wy \notin E(G)$ since if it were, then $\{t, w, v, x, y\}$ would induce a Z_2. Thus, $xw \in E(G)$, a contradiction. Hence, w cannot be adjacent to y. Next, if $vw \notin E(G)$ but $wy \in E(G)$, then w is adjacent to $N_{K_b}[y]$; for if not, there is a vertex s in $N_{K_b}(y)$ that is not adjacent to w which together with $N_{K_b}(x), v, w$, and t would form a Z_2. If w is not adjacent to v or y, then by arguments similar to those in Subcase 5.1, we can see that w is adjacent to all of K_b. Hence, G must be a member of family (11) in Figure 3.

CASE 6: Suppose that H is a member of family (5) in Figure 1. Since $N_{H'}(w)$ is the disjoint union of two complete graphs, w is adjacent to t and a subset of K_b and/or a subset of $N[x_i]$ for $i = 1, 2, \ldots, h$. (Note w is adjacent to at most one x_i or else $N_{H'}(w)$ is not the disjoint union of two complete graphs.)

Subcase 6.1: Suppose that $wx_i \notin E(G)$ for any $2 \leq i \leq h$. Observe that $|N(x_i)| \geq 2$ for $2 \leq i \leq h$ or else G will have connectivity 1. If w is adjacent to a vertex in $N(x_i)$ for some i, then w must be adjacent to all of K_b or a claw or Z_2 exists. If there is a vertex y in $V(K_b) - N(x_i)$ not adjacent to w, then a $K_{1,3}$ will be formed by w, x_i, y, and the vertex in $N(x_i)$ that is adjacent to w. If there is a vertex y in $N(x_i)$ not adjacent to w, then a Z_2 is formed by t, w, x_i, y, and the vertex in $N(x_i)$ that is adjacent to w. So, suppose w is not adjacent to vertices in $N(x_i)$ for $2 \leq i \leq h$; that is, w is only adjacent to vertices in $V(K_b) - \bigcup_{i=2}^{h} N(x_i)$. Then a Z_2 is formed by two vertices that are not adjacent to w, a vertex to which w is adjacent, w, and t. Hence, w must be adjacent to all of K_b. Now, observe that vw must be an edge in G or else one x_i, one of its neighbors, v, and w for a $K_{1,3}$. Hence, G must be a member of family (12) in Figure 3.

Subcase 6.2: Suppose w is adjacent to x_i for some i, say x_s. Now, w can be adjacent to at most $N_{K_b}[x_s]$ since $N(w)$ is the disjoint union of two complete graphs. Observe that w must be adjacent to all vertices in $N_{K_b}(x_s)$ or else a Z_2 would be formed by two vertices not in $N_{K_b}(x_s)$, a vertex in $N_{K_b}(x_s)$ that is not adjacent to w, x_s, and w. However, some x_i ($i \neq s$), one of x_i's neighbors, a neighbor of x_s, x_s, and w still form a Z_2. Thus, w cannot be adjacent to x_i for any i, and G can only be a member of family (12) in Figure 3.

CASE 7: Suppose that H is a member of family (6) in Figure 1. Since $N_{H'}(w)$ is the disjoint union of two complete graphs, w is adjacent to t and a clique contained in $N_{K_b}[x]$. If w is adjacent to x, then w must be adjacent to at least $b-1$ vertices of K_b or else a Z_2 would exist. However, if w is not adjacent to x, then w is adjacent to all of K_b or else a Z_2 is present. Observe, that these Z_2's exist whether or not vw is an edge in G. Thus, G is a member of one of families (13) through (16) in Figure 3 or family (17) in Figure 4.

CASE 8: Suppose that H is a member of family (7) in Figure 1. As $N_{H'}(w)$ is the disjoint union of two complete graphs, w must be adjacent to t and y or else G would have connectivity 1 (since deg(t) or deg(y) would be equal to 1). Observe, however, that there are two Z_2's that are formed. The first is formed by two vertices in K_b, x, y, and w; the second by two vertices in K_b, v, t, and w. These can only be eliminated by adding the edges vw and vx since w cannot be adjacent to vertices in K_b. But, the graph formed by adding these two edges has a Z_2 that cannot be removed; namely, the one formed by two vertices in K_b, v, w, and y. Hence, no such graph is possible. □

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