SUPER CONNECTIVITY OF STAR GRAPHS, ALTERNATING GROUP GRAPHS AND SPLIT-STARS

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ABSTRACT. The star graph S_n and the alternating group graph A_n are two popular interconnection graph topologies. A_n has a higher connectivity while S_n has a lower degree, and the choice between the two graphs depends on the specific requirement of an application. The degree of S_n can be even or odd but the degree of A_n is always even. We present a new interconnection graph topology, split-star graph S_n^2, whose degree is always odd. S_n^2 contains two copies of A_n, and can be viewed as a companion graph for A_n. We demonstrate that this graph satisfies all the basic properties required for a good interconnection graph topology. In this paper, we also evaluate S_n, A_n and S_n^2 with respect to the notion of super connectivity and super edge-connectivity.

1. INTRODUCTION

Useful distributed processor architectures offer the advantage of improved connectivity and reliability. An important component of such a distributed system is the system topology, which defines the inter-processor communication architecture. Two popular system topologies are star graphs [1, 6] and alternating group graphs [7].

Both the star graphs and the alternating group graphs interconnect $O(n!)$ vertices using a degree of $O(n)$ to provide a diameter of $O(n)$. Moreover they are regular graphs and maximally connectivity, that is, the connectivity is equal to the regularity of the graph.

Table 1 contains the basic properties of the star graph S_n and the alternating group graph A_n. A_n has a higher connectivity than S_n, and naturally S_n has a smaller degree (regularity). This is a trade-off, and choices depend on whether a particular application prefers higher connectivity or smaller regularity.

The alternating group graphs have even degrees for every n whereas the degree of the star graphs has the opposite parity of n. In this paper, we introduce a class of graphs, namely, the split-stars S_n^2, related to the

First author's research partially supported by a faculty research fellowship from Oakland University Foundation.

TABLE 1. Summary of Basics Properties

	no. of vertices	degree	diameter
S_n	$n!$	$n-1$	$\lfloor \frac{3n-3}{2} \rfloor$
A_n	$n!/2$	$2n-4$	$\lfloor \frac{3n-6}{2} \rfloor$
S_n^2	$n!$	$2n-3$	$\lfloor \frac{3n-4}{2} \rfloor$

TABLE 2. Summary of Connectivity Properties

	connectivity (fault tolerance)	**tightly super connected**	**tightly super edge-connected**
S_n	$n-1$	yes unless $n = 3$	yes unless $n = 3$
A_n	$2n-4$	yes unless $n = 4$	yes
S_n^2	$2n-3$	yes	yes unless $n = 3$

alternating group graphs in which the degree is always odd. They can be seen as companion graphs of the alternating group graphs as S_n^2 contains two copies of A_n. However, like S_n, S_n^2 has $n!$ vertices and its construction is a variant of the construction for S_n. Hence the split-stars act as bridges between the star graphs and the alternating group graphs. In Section 3, we give an optimal routing algorithm on a split-star and determine its diameter to ensure it has all the basic properties of a good graph topology. The results are contained in Table 1.

Like most interconnection networks, split-stars are cayley graphs. However, they are not edge-transitive. This is not a deficiency since they are *almost* edge-transitive and retain all the attractive properties required for interconnection networks while providing some diversifications.

We assume the reader is familiar with the basic terminologies in graph theory. (See [9].) Let G be an r-regular graph. Then the graph has connectivity at most r. G is said to be *maximally connected* if its connectivity [1] is r. It is well-known that both the star graphs and the alternating group

[1] In distributed computing, it is more commonly known as *fault tolerance*. However, some authors distinguish the terms connectivity and fault tolerance by defining fault tolerance to be one less than the connectivity.

graphs are maximally connected. But given an r-regular graph, we can impose an additional condition that the graph be close to $(r+1)$-connected in that its only minimal disconnecting sets are those induced by the neighbours of a vertex. This is a much stronger property than maximal connectivity. This makes an r-regular graph almost $(r+1)$-connected and it is indeed a desirable characteristic. If a graph has this property, then it is said to be *(loosely) super connected.* If, in addition, the deletion of a minimum disconnecting set results in a graph with two components (one of which has only one vertex), then the graph is *tightly super connected.* Note that an r-regular graph can be loosely super connected but not tightly super connected; for example, the complete bipartite graph $K_{r,r}$ with $r \geq 3$. Another related notion is that of *super edge-connectivity.* It requires the only minimal edge-disconnecting sets are those induced by a vertex. (Obviously, the notion of loosely superness and tightly superness are the same in this case.) The notion of superness was first introduced in [3]. Although maximal connectedness is a stronger notion than maximal edge-connectedness, it is possible for a graph to be super connected and yet not super edge-connected. In section 4, we state that the star graphs, the alternating group graphs and the split-stars have these properties except for a couple of small cases. The result is summarized in Table 2. The proofs of all these results are similar. Since the star graphs and the alternating group graphs are familiar objects, we will only provide a proof of the result for split-stars.

Throughout this paper, we consider permutations of $\mathcal{N} = \{1, 2, \ldots, n\}$. A permutation $[a_1, a_2, \ldots, a_n]$ corresponds to the bijection π with $\pi(i) = a_i$. If $a_i = i$, then i is in its *natural position.* The identity permutation $[1, 2, \ldots, n]$ is denoted by ι. It is well-known that every permutation can be written as a product of disjoint *cycles* known as the *disjoint cycle decomposition.* The *cycle* $(a_1, a_2, \ldots, a_m)$ denotes the bijection f on the set $\{a_1, a_2, \ldots, a_m\}$ where $f(a_i) = a_{i+1}$ and the subscript arithmetics are done in modulo m, and is referred to as an *m-cycle.* A permutation is *even* (*odd*) if it has a factorization as a product of an even (odd) number of, not necessarily disjoint, 2-cycles.

2. A SPLIT-STAR

[1] introduced the star graph through a puzzle. We consider a similar game: n checkers with labels $a_1, a_2, \ldots, a_n$, $n \geq 3$, are placed on the vertices of the graph in Figure 1. The objective of the game is to perform a sequence of *valid moves* so that each checker a_i is placed on the ith vertex. There are two types of moves: *2-exchange* and *3-rotation.* A 2-exchange interchanges the checkers on the vertices 1 and 2. A 3-rotation rotates the checkers on the vertices of a triangle, that is, the triangle with vertices 1, 2 and k for some $k \in \{3, 4, \ldots, n\}$. Each move corresponds to premultiplying the current permutation π by the appropriate cycle. A 2-exchange yields

the new permutation $\pi(1,2)$. Similarly, a 3-rotation of π through k yields either $\pi(1,2,k)$ or $\pi(2,1,k)$. These 3-cycles will be called *valid*. Note that $(1,2,k)$ and $(2,1,k)$ are inverses of each other, and that for any valid 3-cycle γ, $\gamma(1,2) = (1,2)\gamma^{-1}$.

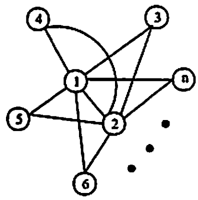

FIGURE 1. Generator-graph

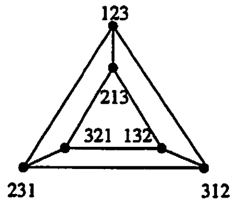

FIGURE 2. S_3^2

The number of possible instances of this game is $n!$. Two permutations are said to be *related* if one can be obtained from the other by a move. Let S_n^2 be the relation graph of these instances. It is called a *split-star*. Figure 2 gives S_3^2 and Figure 3 gives S_4^2. The following observation is obvious.

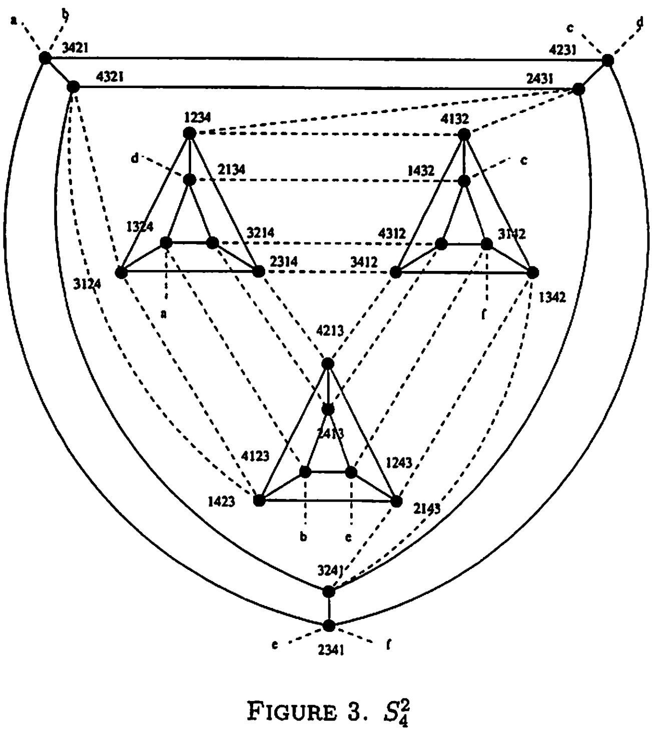

FIGURE 3. S_4^2

Proposition 2.1. *Let $n \geq 3$. Then S_n^2 is a $(2(n-2)+1)$-regular undirected graph on $n!$ vertices. Moreover, S_n^2 is vertex-transitive*[2] *and has two equivalence classes of edges*[3]*, namely, those induced by 2-exchanges and those induced by 3-rotations.*

Let $S_{n,E}^2$ be the subgraph of S_n^2 induced by the set of even permutations. This is precisely the alternating group graph, A_n, introduced in [7]. Let $S_{n,O}^2$ be the subgraph of S_n^2 induced by the set of odd permutations. Then $S_{n,O}^2$ is isomorphic to $S_{n,E}^2$ via $\phi([a_1, a_2, a_3, \ldots, a_n]) = [a_2, a_1, a_3, \ldots, a_n]$. Moreover, the edges corresponding to the 2-exchanges induced a perfect matching[4] between the set of even permutations and the set of odd permutations.

3. Diameter and Optimal Routing

This section will be brief. Complete proofs can be found in the technical report [4]. Since S_n^2 is vertex-transitive, it is enough to consider optimal routing from every other vertex to ι. This is a computationally tight reduction. Intuitively: If π is even, we use the routing for A_n; if π is odd, we route it to $\iota(1,2)$ and then use a 2-exchange. This works because of the following:

Proposition 3.1. *Let π be a permutation. Then there is at most one 2-exchange in an optimal routing of length q from π to ι. Moreover, such a 2-exchange can be made to occur as the last step in another optimal routing from the given optimal routing in $O(qn)$ time. This 2-exchange is present in an optimal routing if and only if π is odd.*

Proof. Any routing is a product of valid 3-cycles and cycles of the form $(1,2)$. Suppose this product contains two or more cycles of the form $(1,2)$. Consider the following subproduct of this routing, $(1,2)\gamma_1\gamma_2\cdots\gamma_k(1,2)$ where each γ_i is a valid 3-cycle. This is equivalent to $\gamma_1^{-1}\gamma_2^{-1}\cdots\gamma_k^{-1}$. This decreases the length of the routing, so the original routing is not optimal. Suppose the optimal routing is $\gamma_1\gamma_2\cdots\gamma_i(1,2)\gamma_{i+1}\cdots\gamma_k$ where each γ_j is a valid 3-cycle. This is equivalent to $(1,2)\gamma_1^{-1}\gamma_2^{-1}\cdots\gamma_i^{-1}\gamma_{i+1}\cdots\gamma_k$. Thus we may assume the 2-exchange occurs at the end. The last statement is obvious. □

A *greedy* strategy for a routing from $\pi_a = [a_1, a_2, \ldots, a_n]$ to ι is the following: Let π, the current permutation, be π_a; repeat the following until π is equal to ι.

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[2]Two vertices u and v are *equivalent* if there is an automorphism ϕ such that $\phi(u) = v$. A graph is *vertex-transitive* if every pair of vertices are equivalent

[3]Two edges (u, v) and (x, y) are *equivalent* if there is an automorphism ϕ such that $\phi(u) = x$ and $\phi(v) = y$, or $\phi(u) = y$ and $\phi(v) = x$.

[4]Given a graph $G = (V, E)$, $M \subseteq E$ is a *perfect matching* if every vertex of the graph $H = (V, M)$ has degree one.

- If there is a symbol in the first two positions of π that is neither 1 nor 2, then perform a 3-rotation to put this symbol, say i, to the ith position. Let π be the updated permutation.
- If the symbols in the first two positions of π are 1 and 2, and there exists another symbol j not in the jth position, then perform a 3-rotation to put this symbol in one the first two positions. Let π be the updated permutation.
- If the symbols in the first two positions of π are 1 and 2 and every other symbol i is in the ith position, then perform a 2-exchange move if 1 is in the 2nd position (and hence 2 is in the 1st position.) Let π be the updated permutation, which of course will be ι.

Theorem 3.2. *Let π be a permutation. The greedy algorithm gives an optimal routing from π to ι.*

Proof. It follows from the fact that the greedy algorithm is optimal for the alternating graph (see [7]) and Proposition 3.1. □

Example 3.3. Let $\pi = [4, 5, 6, 2, 1, 7, 3]$. A greedy routing is $[\underline{4}, \underline{5}, 6, \underline{2}, 1, 7, 3]$, $[\underline{5}, \underline{2}, 6, 4, \underline{1}, 7, 3]$, $[\underline{2}, \underline{1}, \underline{6}, 4, 5, 7, 3]$, $[\underline{6}, \underline{2}, 1, 4, 5, \underline{7}, 3]$, $[\underline{2}, \underline{7}, 1, 4, 5, 6, \underline{3}]$, $[\underline{3}, \underline{2}, \underline{1}, 4, 5, 6, 7]$, $[2, 1, 3, 4, 5, 6]$ which gives ι by applying a 2-exchange. An alternate optimal routing is to start with $[\underline{4}, \underline{5}, \underline{6}, 2, 1, 7, 3]$ (a non-greedy step) to $[5, 6, 4, 2, 1, 7, 3]$ and then proceed with a greedy routing. This shows that there exist optimal routings that are not greedy.

In fact, the length of any greedy routing from π_a to π_b depends only on the cycle structure of $\pi_b^{-1}\pi_a$. (See [7] or [4].) It is straightforward to compute both the maximum distance between vertices (the *diameter*), and the average distance in S_n^2.

Theorem 3.4. *The diameter of S_n^2 is $\lfloor \frac{3n}{2} \rfloor - 2$. An optimal routing between any two vertices in S_n^2 can be found in $O(n^2)$ time.*

Theorem 3.5. *The average distance of S_n^2 is $\Omega = n + H_n - 5 + \frac{4}{n}$ where H_n is the nth harmonic number.*

4. Connectivity

It is well-known that A_n and S_n are maximally connected. In fact, this follows from the fact that they are regular, connected edge-transitive graphs. We now consider the connectivity of S_n^2 which is not edge-transitive. One can use the fact that A_n is maximally connected to show that S_n^2 is maximally connected but we will give a proof without this fact. The reason is a slight modification of the argument provides a proof for S_n^2 being tightly super connected.

Let H_n^i be the subgraph of S_n^2, $n \geq 4$, induced by the set of vertices corresponding to the permutations with i in the nth position. Then H_n^i

is isomorphic to S^2_{n-1}. (See Figure 3.) Suppose we know that S^2_{n-1} with $n \geq 5$ is $(2(n-3)+1)$-connected. An edge is a *solid edge* if both ends belong to H^i_n for some i; otherwise it is a *dashed edge.* Hence each vertex is incident to exactly two dashed edges. Therefore there are exactly $2(n-1)!$ dashed edges with exactly one end in H^i_n. (See Figure 3.) It is not difficult to see that the number of dashed edges between H^i_n and H^j_n is constant. Hence the number of dashed edges between H^i_n and H^j_n is $2(n-2)!$. In fact, these edges can be determined explicitly. The only vertices in H^i_n that are adjacent to H^j_n are the vertices with j in the 1st or 2nd position. Hence there are two choices for j and since i is in the last position, we have $(n-2)!$ ways to permute the remaining symbols. We claim that S^2_n is $(2(n-2)+1)$-connected. It is easy to check that S^2_3 and S^2_4 are maximally connected. We continue with an inductive argument. Suppose S^2_n is not $(2n-3)$-connected. Then there exists a set T of $2n-4$ vertices such that $S^2_n \setminus T$ is not connected. Let $T = T_1 \dot\cup T_2 \dot\cup T_3 \dot\cup \cdots \dot\cup T_n$[5] such that T_i is a set of vertices in H^i_n. Let $t_i = |T_i|$ for $1 \leq i \leq n$. So $\sum_{i=1}^n t_i = 2n-4$.

Suppose $t_i \leq 2n-6$ for all $1 \leq i \leq n$. Since S^2_{n-1} is $(2n-5)$-connected, $H^i_n \setminus T_i$ is connected. The number of dashed edges between $H^i_n \setminus T_i$ and $H^j_n \setminus T_j$ in $S^2_n \setminus T$ is at least $2(n-2)! - 2(2n-6) = 2(n-2)! - 4n + 12$ which is positive if $n \geq 5$.

Suppose there exists a T_i with $t_i = 2n-4$. Without loss of generality, we may assume $t_1 = 2n-4$ and $t_i = 0$ for $2 \leq i \leq n$. Suppose $H^1_n \setminus T_1$ is not connected. Let v_1 and v_2 be two vertices in different components of $H^1_n \setminus T_1$. Since $t_i = 0$ for $2 \leq i \leq n$, v_1 (v_2, respectively) is still adjacent to a vertex in H^i_n for some i. Hence $S^2_n \setminus T$ is connected.

Suppose there exists a T_i with $t_i = 2n-5$. Without loss of generality, we may assume $t_1 = 2n-5$, $t_2 = 1$ and $t_i = 0$ for $3 \leq i \leq n$. Suppose $H^1_n \setminus T_1$ is not connected. Let v_1 and v_2 be two vertices in different components of $H^1_n \setminus T_1$. Since it is clear that $(S^2_n \setminus H^2_n) \setminus T_2$ is connected, it is enough to show that v_1 (v_2, respectively) is still adjacent to a vertex in H^i_n for some i. This is true since v_1 (v_2, respectively) is incident to two dashed edges but $t_2 = 1$ and $t_i = 0$ for $3 \leq i \leq n$. Hence we have the following theorem.

Theorem 4.1. *Let $n \geq 3$. Then S^2_n is maximally connected.*

Now that we know S^2_n is maximally connected, it is still interesting to determine exactly which sets of size $2(n-2)+1$ are minimum disconnecting sets. Since S^2_n is $(2(n-2)+1)$-regular, S^2_n has at least $n!$ disconnecting sets.

Theorem 4.2. *Let $n \geq 3$. Then S^2_n is tightly super connected.*

Proof. It is easy to see that the statement is true for $n = 3, 4$. For $n \geq 5$, we follow the type of analysis in the proof of Theorem 4.1. We only have

[5]This notation denotes disjoint unions.

to make the analysis slightly tighter. Using the same terminology, we have $\sum_{i=1}^{n} t_i = 2n-3$. As before, if $t_i \leq 2n-6$ for all $1 \leq i \leq n$, then T is not a disconnecting set. Without loss of generality, the remaining cases are $t_1 = 2n-5$, $t_1 = 2n-4$ and $t_1 = 2n-3$. Suppose $t_1 = 2n-4$ or $t_1 = 2n-3$. Then $H_n^1 \setminus T_1$ may be disconnected. Let Y be a component of $H_n^1 \setminus T_1$. If we can show that there is a dashed edge having exactly one end in Y, then T is not a disconnecting set. This is obvious since each vertex in Y is incident to exactly two dashed edges and $|T \setminus T_1| \leq 1$. Now suppose $t_1 = 2n-5$. It is clear that the only case in which the above condition is not satisfied is when $|Y| = 1$ and $T \setminus T_1$ are the two other ends of the two dashed edges incident with the vertex in Y. Hence T is the neighbour set of the vertex in Y and $S_n \setminus T$ has exactly two components, one of which has only one vertex. □

In some situations, the vertices are highly reliable but the edges are not. In this case, one can look at the *edge-connectivity*. Since S_n^2 has connectivity $2(n-2)+1$, the edge-connectivity is $2(n-2)+1$, that is, *maximally connected*. (It is well-known and indeed easy to see that the connectivity is less than or equal to the edge-connectivity in any graph.) In fact, since S_n^2 is vertex-transitive, one can conclude it is maximally edge-connected by using a theorem of Watkins [8].

Theorem 4.3. [6] *Let $H = (V, E)$ be a κ-regular tightly super connected graph with $\kappa \geq 1$. If H has more than $2\kappa + 2$ vertices, then it is super edge-connected.*

Proof. Note that the assumption of H being a κ-regular tightly super connected graph implies H is simple, that is, H has neither multiple edges nor loops. Recall that the edge-connectivity of H is κ. If $\kappa = 1$, then H must be the graph with two vertices and one edge; so there is nothing to prove. Any minimum edge-disconnecting set is of the form $(X, \overline{X})$ for some $\emptyset \neq X \subset V$, that is, edges having exactly one end in X. Note that $|(X, \overline{X})| = d_H(X)$. Choose $\emptyset \neq X \subset V$ such that $d_H(X) = \kappa$ and $(X, \overline{X})$ is a (minimum) edge-disconnecting set. Our objective is to show either $|X| = 1$ or $|\overline{X}| = 1$. Let V_1 (V_2, respectively) be the set of vertices in X ($\overline{X}$, respectively) that are incident with an edge in $(X, \overline{X})$. We note that $|V_1| \leq \kappa$ and $|V_2| \leq \kappa$. If $V_1 = X$ and $V_2 = \overline{X}$, then $|X| \leq \kappa$ and $|\overline{X}| \leq \kappa$. Hence H has at most 2κ vertices, a contradiction. Without lost of generality, we may assume $V_1 \neq X$.

Since $V_1 \neq X$, V_1 is a disconnecting set. Since $d_H(X) = \kappa$ and H has edge-connectivity κ, V_1 is in fact of size κ, that is, V_1 is a minimum disconnecting set. Since H is tightly super connected, either $\overline{X}$ has only

[6]This is a weaker version of a more general result. For our purpose, this weaker version is enough. See [4] for a proof of the stronger result.

one vertex or $X \setminus V_1$ has only one vertex. If $|\overline{X}| = 1$, then we are done. So we assume $X \setminus V_1$ has only one vertex; call this vertex a. If $\overline{X} = V_2$, then the vertex-set of H is $V_1 \dot{\cup} V_2 \dot{\cup} \{a\}$; hence H has at most $2\kappa + 1$ vertices, a contradiction. Therefore $\overline{X} \setminus V_2$ is not empty. Then it is easy to see (as above) that V_2 is a minimum disconnecting set (so $|V_2| = \kappa$) and that we may assume $\overline{X} \setminus V_2$ is a singleton (or we are done); call this vertex b. So the vertex-set of H is $V_1 \dot{\cup} V_2 \dot{\cup} \{a, b\}$. This implies that H has $2\kappa + 2$ vertices, a contradiction. □

Corollary 4.4. *Let $n \geq 3$. Then S_n^2 is super edge-connected unless $n = 3$.*

Proof. Since S_n^2 is a $(2n-3)$-regular tightly super connected graph and it has $n!$ vertices which is greater than $4n - 6$ if $n \geq 4$, the result follows from Theorem 4.3. It is clear that S_3^2 is not super edge-connected. □

The corresponding result for S_n and A_n can be proved in a similar way, and we state it here without proof. (In fact, the proofs are somewhat simpler.)

Theorem 4.5. *Let $n \geq 3$. Then A_n is tightly super connected unless $n = 4$, and A_n is super edge-connected.*

Theorem 4.6. *Let $n \geq 3$. Then S_n is tightly super connected and super edge-connected unless $n = 3$,*

5. Concluding Remarks

In this paper, we introduced split-stars, a new class of interconnection networks. This compliments two existing classes, namely, the star graphs and the alternating group graphs. S_n^2 can be viewed as a companion graph of A_n: The degree of S_n^2 is always odd and the degree of A_n is always even, and both have higher connectivity than S_n. S_n^2 is constructed in a way similar to the construction of S_n, and therefore, can be seen as a variant of S_n. This gives a valuable addition to the collection of good interconnection networks. We also applied super connectivity and super edge-connectivity as measures of reliability of these interconnection networks. These are stronger measures than maximal connectedness and maximal edge-connectedness. These are desirable properties as they imply little degradation of structural integrity in the presence of a tolerable or even an intolerable number of faults. We showed that S_n, A_n and S_n^2 have these properties. In fact, there are other measures of reliability that are stronger than maximal connectedness and maximal edge-connectedness. Most of these measures are $\mathcal{NP}$-Hard to compute. One such existing measure is the toughness [5]. It follows from a theorem of [2] that S_n has toughness one as it is vertex-transitive and bipartite. It also follows from a theorem in [2] that A_n and S_n have toughness strictly greater than one. We have also computed this parameter for A_n and S_n. We discovered that they, having toughness 2, are much tougher than S_n.

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