Smallest weak and smallest totally weak critical sets in the latin squares of order at most seven

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ABSTRACT: A critical set in a latin square of order n is a set of entries in a latin square which can be embedded in precisely one latin square of order n. Also, if any element of the critical set is deleted, the remaining set can be embedded in more than one latin square of order n. In this paper we find smallest weak and smallest totally weak critical sets for all the latin squares of orders six and seven. Moreover, we computationally prove that there is no (totally) weak critical set in the back circulant latin square of order five and we find a totally weak critical set of size seven in the other main class of latin squares of order five.

1 Introduction

A *latin square* L of order n is an $n \times n$ array with entries chosen from a set N, of size n, such that each element of N occurs precisely once in each row and column. For convenience, a latin square will sometimes be represented as a set of ordered triples $(i, j; k)$, which is read to mean that element k occurs in cell (i, j) of the latin square L. For a latin square of order n we shall use $1, 2, 3, \ldots, n$ as the entries, and rows and columns will also be labelled from

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1 to n. A *partial latin square* P of order n is an $n \times n$ array with entries chosen from a set N, of size n, such that each element of N occurs at most once in each row and column. Thus P may contain a number of empty cells. We sometimes denote P by the set $\{(i,j;k)|i,j,k \in N\}$. A *critical set* in a latin square L (of order n) is a set $C = \{(i,j;k) \mid i,j,k \in N\}$, such that

(1) L is the only latin square of order n which has element k in cell (i,j) for each $(i,j;k) \in C$; and

(2) no proper subset of C satisfies (1).

A *smallest critical set* (SCS) in a latin square L is a critical set of minimum cardinality. A *uniquely completable set* (UC) in a latin square L of order n is a partial latin square in L which satisfies condition (1) above.

Let L be a latin square of order n and let $\{a,b,c\} = \{1,2,3\}$. The (a,b,c)-*conjugate* of L, $L_{(a,b,c)}$, is defined as follows:

$$L_{(a,b,c)} = \{(x_a, x_b; x_c) \mid (x_1, x_2; x_3) \in L\}.$$

Two latin squares L and L' of order n are *isotopic* if there are three bijections from the rows, columns, and symbols of L to the rows, columns, and symbols, respectively, of L', that map L to L'. Two latin squares L and L' of order n are *main class isotopic* if L is isotopic to any conjugate of L'. Table 1 shows the number of main and isotopy classes for latin squares of order $1 \leq n \leq 7$ (see Dénes and Keedwell [7]).

$n =$	1	2	3	4	5	6	7
Main classes	1	1	1	2	2	12	147
Isotopic classes	1	1	1	2	2	22	563

Table 1

In the process of completing the uniquely completable set U to the latin square L of order n which it characterizes, we say that adjunction of a triple $t = (r,c;s)$ is *forced* (see [10]) in the process of completion of a set T of triples ($|T| < n^2$, $U \subseteq T \subset L$) to the complete set of triples which represents L, if either

(i) $\forall r' \neq r$, $\exists z \neq c$ such that $(r',z;s) \in T$ or $\exists z \neq s$ such that $(r',c;z) \in T$ (that is, in the partial completion F of L, each cell of column c except that in row r is either in a row of F which already contains the symbol s or else is already filled with an element z distinct from s), or

(ii) $\forall c' \neq c$, $\exists z \neq r$ such that $(z, c'; s) \in T$ or $\exists z \neq s$ such that $(r, c'; z) \in T$ (that is, in the partial completion F of L, each cell of row r except that in column c is either in a column of F which already contains the symbol s or else is already filled with an element z distinct from s), or

(iii) $\forall s' \neq s$, $\exists z \neq r$ such that $(z, c; s') \in T$ or $\exists z \neq c$ such that $(r, z; s') \in T$ (that is, in the partial completion F of L, every symbol except s already occurs either in column c or else in row r of F).

The uniquely completable set U is called *strong* if we can define a sequence of sets of triples $U = F_1 \subset F_2 \subset F_3 \subset \ldots \subset F_r = L$ such that each triple $t \in F_{i+1} \setminus F_i$ is forced in F_i for $1 \leq i \leq r - 1$. A uniquely completable set is *super-strong* if each triple in this sequence is forced only by virtue of property (iii) in the above definition of forcing. (Note that the authors of [3] have used the terms *semi-strong* and strong in place of strong and super-strong as defined above.) A uniquely completable set which is not strong is called *weak*. In particular, a critical set may be weak or strong. Note that by this definition there might be some empty cells in the weak critical set which are forced.

Example 1.1 Consider the following critical set C and uniquely completable set U of order six. Since $C \subseteq U$ and no cell of U is forced it follows that C is a weak critical set of order six.

1	*	3	*	*	6
*	1	*	*	*	*
3	*	*	*	*	*
*	*	*	5	2	*
*	6	*	2	*	*
*	*	2	*	4	*

Critical set C

1	2	3	4	5	6
2	1	*	*	*	*
3	*	*	*	*	2
*	*	*	5	2	*
*	6	*	2	*	*
*	*	2	*	4	*

Uniquely comp. set U

A *totally weak critical set* is a critical set in which no cell is forced. Note that by this definition the critical set C, given in Example 1.1, is not a totally weak critical set. Obviously, a totally weak critical set is also a weak critical set. A *smallest weak critical set* (SWCS) in a latin square L is a weak critical set of minimum cardinality. Similarly, a *smallest totally weak critical set* (STWCS) in a latin square L is a totally weak critical set of minimum cardinality.

The sizes of smallest critical sets for the latin squares of orders four and five were determined in [6, 8]. Howse in [9] found smallest critical sets for all the latin squares of order six. The authors in [2] gave smallest critical sets for all the latin squares of order seven. They also found (see [1]) smallest critical sets for the latin squares of order eight which are based on group of order eight. Earlier it was shown by Bate and van Rees [3] that the siz of a strong or super-strong critical set in a latin square of order n canno be less than $\lfloor n^2/4 \rfloor$.

This paper deals with smallest weak critical sets and smallest totall weak critical sets in the latin squares of order at most seven. Keedwel in [10] proved that the smallest order for which there exists a latin squar which has a weak critical set is order five. Burgess in [4] found a weakl completable set of order n for each $n \geq 5$. Sittampalam and Keedwel found a weak critical set of size twelve in the dihedral group of order si (see [11]). Adams and Khodkar in [2] found smallest critical sets for th latin squares 6.3, 6.7, 6.8, 7.3 and 7.42 in [5] which are also weak critica sets. In particular, they found a weak critical set of size twelve for th dihedral group of order six (see the weak critical set 6.7 in Section 2).

In this paper we find a smallest weak critical set and a smallest totall weak critical set for each main class of latin squares of orders six and seven An exhaustive computer search shows that there is no weak critical se (therefore, no totally weak critical set) in the back circulant latin square o order five (latin square 5.1 in [5]). Moreover, we find a totally weak critica set of size seven for latin square 5.2 in [5] (see Section 2).

The authors in [2] ask the following question:

> For a given latin square of order $n \geq 6$, does there exist a weak critical set among the smallest critical sets in this latin square?

Our results in this paper show that the answer to this question is neg ative.

All the results in this paper are obtained by computer searches. T reduce the search size we make use of the latin interchanges which wer generated and used in [2].

We summarize the results of this paper in the following tables.

Main class	Size of SCSs	Size of SWCSs	Size of STWCSs
5.1	6	No WCS	No WCS
5.2	7	7	7

Table 2: Latin squares of order 5

#of Main classes	Size of SCSs	Size of SWCSs	Size of STWCSs
1	9	11	12
5	10	11	11
5	11	11	11 for 2 classes
			12 for 3 classes
1	12	12	13

Table 3: Latin squares of order 6

#of Main classes	Size of SCSs	Size of SWCSs	Size of STWCSs
1	12	15	16
2	13	15	15
116	14	14 for 17 classes	15 for 17 classes
		15 for 99 classes	15 for 98 classes
			16 for 1 class
26	15	15	15 for 22 classes
			16 for 4 classes
1	16	16	16
1	17	17	18

Table 4: Latin squares of order 7

2 Table of results

In this section we give a smallest totally weak critical set for each main class of latin squares of orders six and seven as well as the main class 5.2. Recall that a totally weak critical set of order n is also a weak critical set of order n. Now if the size of a smallest totally weak critical set is larger than the size of a smallest weak critical set in a latin square then we first list a smallest weak critical set. For example, in the latin square 6.1 of [5] the size of a smallest weak critical set is 11 and the size of a smallest totally weak critical set is 12: we give an example for each.

Here we use the numbering system used in [5] but for the entries in a latin square of order n we always use the symbols $1, 2, 3 \ldots, n$. For each critical set, an identifying number is given, followed by four integers. The first three integers (see also [5]) give the number of transversals, the number of 2×2 subsquares, and the number of 3×3 subsquares, respectively, in the completed latin square. The fourth integer (see also [2, 9]) gives the size of the smallest critical set for the corresponding latin square. The last integer is the size of the smallest weak (totally weak) critical set in the corresponding latin square. In the following critical sets, $*$ indicates an empty cell.

2.1 SWCSs and STWCSs of order five

There is no weak critical set in the back circulant latin square of order five (latin square 5.1 in [5]). The following is a smallest totally weak critical set for the main class 5.2 in [5].

*	*	*	*	5
*	1	4	*	*
3	*	*	*	*
4	*	2	*	*
*	*	*	2	*

5.2:3,4,0,7,7

2.2 SWCSs and STWCSs of order six

1	*	3	*	*	6
*	1	*	*	*	*
3	*	*	*	*	*
*	*	*	5	2	*
*	6	*	2	*	*
*	*	2	*	4	*

6.1:0,9,4,9,11

1	*	3	*	5	*
*	*	*	*	*	*
3	*	5	6	*	*
*	3	*	*	2	*
*	*	*	*	*	4
6	5	*	1	*	*

6.1:0,9,4,9,12

1	*	3	*	*	*
2	*	*	*	6	*
*	4	*	6	*	*
*	*	*	5	2	*
*	*	1	*	*	*
*	5	*	*	*	4

6.2:32,9,0,11,11

1	*	3	*	*	*
*	1	*	*	*	3
*	*	*	6	2	*
*	*	*	1	*	*
*	6	*	*	*	4
*	*	5	*	4	*

6.3:24,15,0,11,11

1	*	3	4	*	*
*	*	*	5	*	*
3	4	1	*	*	*
*	*	*	*	*	2
*	6	*	*	1	*
6	3	*	*	*	*

6.3:24,15,0,11,12

1	*	*	*	5	*
*	*	*	5	*	3
*	4	1	*	*	*
*	*	6	*	*	2
*	*	*	*	4	*
6	*	*	2	*	*

6.4:8,7,0,10,11

1	2	*	*	*	*
*	*	*	5	*	3
*	*	2	*	1	*
*	5	*	*	*	*
*	*	*	3	4	*
6	*	*	*	*	4

6.5:8,5,0,10,11

*	*	*	*	5	*
*	1	*	*	6	*
3	*	*	*	*	2
*	*	*	3	*	1
5	6	*	*	*	*
*	*	2	1	*	*

6.6:0,9,4,11,11

1	*	*	4	*	*
*	*	4	5	*	3
*	4	5	*	*	*
*	*	*	3	2	*
*	6	*	*	*	*
*	*	2	*	*	5

6.6:0,9,4,11,12

*	2	3	*	*	*
2	*	*	*	*	5
*	*	*	6	*	*
4	*	*	5	1	*
*	3	*	*	4	*
*	*	5	*	*	1

6.7:0,27,4,12,12

1	*	3	*	5	*
*	1	*	*	6	*
3	*	*	6	*	*
4	*	*	5	1	*
*	*	*	*	*	2
*	4	5	*	*	*

6.7:0,27,4,12,13

1	*	3	*	*	*
*	*	*	*	6	5
*	5	*	*	*	4
*	*	2	*	*	*
*	*	*	2	4	*
6	*	*	1	*	*

6.8:0,19,0,11,11

*	2	3	*	*	*
*	1	*	*	6	*
*	*	*	6	*	*
4	*	2	*	*	*
5	*	*	*	*	3
*	*	*	1	4	*

6.9:0,15,0,10,11

1	*	3	*	*	*
*	*	4	*	6	*
3	5	*	*	*	*
*	*	*	*	2	*
*	*	6	2	*	*
*	4	*	*	*	1

6.10:8,11,0,10,11

1	*	3	*	*	*
*	*	*	5	*	3
*	4	*	6	*	*
4	6	*	*	*	*
*	*	*	*	2	*
*	*	1	*	*	2

6.11:8,4,0,10,11

1	*	*	*	*	*
*	*	*	*	6	4
*	*	2	*	4	*
*	*	*	2	*	3
5	*	*	*	*	*
6	5	*	1	*	*

6.12:0,0,4,11,11

1	2	*	4	*	*
*	*	*	*	6	*
3	*	2	*	*	*
4	*	*	2	*	*
*	4	*	*	*	1
*	5	*	1	*	*

6.12:0,0,4,11,12

2.3 SWCSs and STWCSs of order seven

1	*	*	4	*	6	*
*	3	*	*	*	7	6
*	*	*	*	*	*	5
4	*	*	*	1	2	*
*	*	*	1	6	*	*
*	7	*	2	*	*	*
*	*	5	*	*	*	*

7.1:3,18,1,15,15

1	*	*	4	*	6	*
*	*	*	*	4	*	*
*	*	2	*	*	*	5
4	*	*	7	1	*	*
*	*	*	1	*	3	*
6	7	*	*	*	*	*
*	*	5	*	2	*	4

7.1:3,18,1,15,16

*	*	3	*	5	*	*
2	*	*	*	*	7	*
*	*	*	6	*	*	*
4	*	*	7	1	*	*
*	*	*	*	*	*	3
*	*	4	*	*	5	1
7	6	*	2	*	*	*

7.2:23,26,3,15,15

*	2	3	*	5	*	*
*	3	*	5	*	*	6
*	*	*	*	7	4	*
4	*	*	*	*	*	2
5	*	7	*	*	*	*
*	*	*	3	*	*	*
*	6	*	2	*	1	*

7.2:23,26,3,15,16

*	*	*	*	5	*	*
*	3	*	*	*	7	*
*	*	2	6	7	*	*
4	*	6	*	*	*	1
5	4	*	*	*	*	3
6	7	4	*	*	*	*
*	*	*	1	*	2	*

7.3:63,42,7,17,**17**

1	2	3	*	*	6	*
2	3	*	5	*	*	*
*	*	*	*	7	4	*
*	*	6	*	*	*	1
5	*	7	*	*	*	*
*	*	*	3	1	*	*
*	6	*	*	*	2	4

7.3:63,42,7,17,**18**

*	*	3	4	*	6	*
2	*	*	*	*	*	6
*	*	*	*	*	5	4
*	5	*	*	*	2	*
*	*	7	1	*	*	*
*	7	*	*	3	*	*
*	*	4	3	*	*	*

7.4:23,14,1,14,**15**

*	2	*	*	5	*	*
*	*	1	*	*	7	4
*	*	4	6	*	*	*
*	*	6	7	*	1	*
5	*	*	*	2	*	*
6	*	*	1	*	*	*
*	4	*	*	*	*	*

7.5:19,6,0,14,**15**

*	2	3	*	5	*	*
2	3	4	*	*	*	*
*	*	*	6	*	1	*
*	*	*	*	2	*	*
*	*	*	*	*	*	1
*	7	*	5	3	*	*
*	*	*	*	*	4	6

7.6:25,0,0,14,**15**

1	2	3	4	*	*	*
*	*	*	*	6	7	*
*	*	5	*	7	*	*
*	*	6	*	*	*	*
*	*	*	1	*	*	4
*	*	*	2	*	4	*
*	1	*	3	*	*	*

7.7:133,0,0,12,**15**

1	2	3	4	*	*	*
*	3	4	5	*	*	*
*	*	*	*	7	1	*
*	5	6	*	*	*	3
*	*	*	*	2	*	*
*	*	*	2	*	*	*
7	*	*	*	*	*	6

7.7:133,0,0,12,**16**

1	*	*	4	*	6	*
2	*	*	*	*	*	*
*	*	1	*	7	*	*
4	*	*	*	*	2	*
*	*	*	6	*	1	4
*	7	*	5	*	*	*
*	*	5	*	3	*	*

7.8:21,18,1,14,**15**

*	2	*	*	*	*	*
*	*	4	*	6	*	5
3	*	*	2	*	5	*
*	*	6	7	1	*	*
*	*	*	6	*	*	1
*	7	*	*	*	3	*
*	*	*	*	*	*	4

7.9:30,16,1,14,**15**

1	*	3	*	*	*	7
*	*	*	*	6	*	*
3	4	*	*	*	*	*
*	*	6	7	*	*	*
*	*	*	*	2	4	*
*	7	*	5	*	*	3
*	6	5	*	*	2	*

7.10:43,30,3,16,**16**

1	*	*	4	*	*	7
*	*	*	*	6	7	*
3	4	*	*	*	*	*
*	5	6	*	*	*	*
*	*	*	*	*	4	1
*	*	5	*	*	*	*
7	*	*	*	*	1	3

7.11:43,18,3,14,**15**

1	*	3	*	*	6	*
*	*	*	*	*	*	5
3	4	1	*	*	*	*
*	5	*	7	*	*	2
*	3	*	*	*	4	*
6	*	*	*	*	*	*
*	*	*	5	3	*	*

7.12:55,22,3,14,**15**

1	*	3	*	*	*	*
*	*	*	*	6	*	*
3	4	1	*	*	5	*
*	*	*	7	1	*	2
5	*	7	*	*	4	*
*	*	*	*	*	*	3
*	6	*	5	*	*	*

7.12:55,22,3,14,**16**

1	2	*	*	*	*	*
*	*	4	*	*	7	*
3	*	*	*	*	*	6
*	*	*	*	*	3	*
*	*	7	*	2	4	*
*	7	2	*	3	*	*
*	*	*	6	*	*	1

7.13:13,18,1,15,**15**

1	*	*	*	*	6	*
*	*	4	3	*	*	*
*	4	*	*	7	*	6
*	*	*	*	*	*	2
5	*	*	1	*	*	*
*	7	2	*	4	*	*
*	3	*	*	2	*	*

7.14:33,22,1,15,**15**

1	*	*	*	*	6	*
*	*	4	3	*	*	5
*	4	*	*	*	*	*
*	*	6	*	1	*	*
5	*	*	*	*	2	3
*	7	*	5	*	*	4
*	*	*	*	2	*	*

7.15:15,22,1,15,**15**

1	*	3	*	5	*	*
*	1	*	*	*	7	*
3	*	1	*	*	*	*
*	*	*	7	*	*	2
5	6	*	*	*	*	*
*	*	*	2	*	4	*
*	5	*	*	*	*	3

7.16:30,8,0,14,**15**

1	*	3	*	*	6	*
*	*	4	*	*	*	*
3	*	1	*	*	*	6
*	*	*	7	*	5	*
5	*	*	*	2	*	*
*	7	*	2	*	*	*
*	*	*	*	3	*	1

7.17:14,15,0,14,**15**

1	*	3	*	*	*	*
*	*	*	3	*	*	5
3	*	1	*	7	*	*
*	*	6	7	1	*	*
*	6	*	*	*	4	*
6	*	*	*	*	*	*
*	5	*	*	*	*	4

7.18:20,12,0,14,**15**

1	*	3	*	*	*	*
*	1	*	*	*	7	5
*	4	*	5	*	*	6
*	*	*	*	*	*	*
5	6	*	*	*	*	1
*	*	*	*	2	3	*
*	*	2	*	4	*	*

7.19:22,11,0,14,**15**

1	2	3	*	*	*	*
2	1	*	*	*	7	*
*	*	*	5	*	*	6
4	*	*	*	*	*	*
*	*	7	*	*	1	*
*	7	*	*	*	*	4
*	*	*	6	3	*	*

7.20:15,14,0,14,**15**

1	*	*	4	*	*	*
*	1	*	*	6	*	*
3	4	*	5	*	*	*
*	*	6	*	*	*	2
5	*	*	*	*	*	*
*	7	*	1	*	4	*
*	*	2	*	3	*	*

7.21:24,11,0,14,**15**

1	*	*	4	*	*	*
*	1	4	*	*	*	5
*	4	1	*	*	*	6
*	*	6	*	*	5	*
*	*	*	*	3	*	*
*	*	*	*	2	3	*
7	*	*	2	*	*	*

7.22:18,11,0,13,**15**

1	*	3	*	5	*	*
*	*	*	*	*	7	*
3	*	1	*	*	*	6
*	*	6	*	*	5	*
*	7	*	*	4	*	*
6	*	*	2	*	*	1
*	*	*	*	*	*	4

7.23:22,13,0,14,**15**

1	*	*	4	5	*	*
*	1	*	3	*	*	*
*	4	*	5	7	*	*
*	*	6	*	*	*	*
5	*	*	*	1	*	*
6	*	*	*	*	*	2
*	*	2	*	*	*	3

7.24:19,14,0,15,15

1	*	3	*	*	6	*
*	*	*	*	*	*	5
3	*	1	*	*	2	*
4	*	*	*	*	*	*
*	*	7	*	1	*	*
*	7	*	*	*	*	2
*	5	*	6	*	*	4

7.25:13,18,0,15,15

1	*	3	*	5	*	*
*	*	*	*	*	7	*
3	*	1	*	7	*	*
4	*	*	*	*	*	1
*	6	*	2	*	*	*
*	7	5	*	*	3	*
*	*	*	6	*	*	4

7.25:13,18,0,15,16

1	*	3	*	*	*	*
*	1	*	*	*	7	*
*	*	*	*	7	2	*
4	5	6	*	*	*	*
*	*	*	*	2	*	4
6	*	*	1	*	*	*
*	6	5	*	*	*	*

7.26:11,16,0,14,15

1	*	3	4	*	*	*
*	1	*	*	*	7	5
*	*	*	*	*	*	*
*	*	6	*	*	*	2
5	*	*	*	*	1	*
*	*	*	2	3	*	*
7	6	*	*	*	5	*

7.27:13,10,0,14,15

1	*	3	*	5	*	*
*	*	*	*	*	7	*
3	*	1	*	*	*	*
*	5	*	7	*	*	*
*	6	*	*	*	4	*
*	*	5	*	3	*	4
*	*	*	6	*	*	1

7.28:16,16,0,15,15

*	2	*	*	*	6	*
*	*	*	3	*	*	*
3	*	1	*	7	*	*
4	*	*	*	1	*	*
*	6	7	2	*	*	*
*	*	*	*	4	*	*
7	*	5	*	*	*	1

7.29:21,16,0,15,15

1	*	3	*	*	*	*
*	1	*	*	*	7	5
*	*	*	5	*	*	*
*	5	*	7	*	*	*
*	*	*	*	3	*	4
6	7	*	1	*	*	*
*	*	2	*	4	*	*

7.30:32,14,0,14,15

1	2	3	*	*	*	*
2	1	*	*	*	7	*
*	*	*	5	*	*	6
*	*	*	*	*	*	*
*	*	7	*	*	4	*
6	7	2	*	*	*	*
*	*	*	6	*	*	4

7.31:15,12,0,14,15

1	2	*	*	*	6	*
2	*	*	3	*	*	*
*	4	*	5	7	*	*
*	5	*	7	*	*	*
*	*	*	*	*	1	*
6	*	*	*	*	*	1
*	*	5	*	*	*	*

7.32:24,14,0,14,14

1	*	*	*	5	6	*
*	*	4	*	6	7	*
*	*	1	*	7	*	*
*	5	*	*	*	*	2
*	*	*	*	*	1	*
6	3	*	2	*	*	*
*	*	*	*	*	*	3

7.32:24,14,0,14,15

1	*	3	*	*	6	*
*	*	*	3	6	7	*
3	*	1	*	*	*	*
*	5	*	*	*	*	2
*	*	*	*	*	*	*
*	*	*	*	2	*	4
7	*	5	*	*	1	*

7.33:15,12,0,14,15

1	2	3	*	*	*	*
2	*	*	*	*	7	*
*	4	*	5	*	*	6
*	*	*	*	*	1	*
*	*	7	6	*	*	*
*	*	*	*	4	*	*
*	6	*	*	3	*	4

7.34:19,16,1,14,15

1	*	3	*	*	*	7
*	*	*	*	*	*	5
*	4	*	*	*	2	*
*	5	*	*	2	*	*
*	*	7	1	*	4	*
6	*	*	*	*	*	*
7	*	*	6	*	*	1

7.35:24,12,0,14,15

1	*	3	*	*	*	*
*	1	*	3	*	*	5
*	*	*	*	7	2	*
*	*	*	7	2	*	*
5	*	*	*	*	4	*
6	*	5	*	*	*	1
*	3	*	*	*	*	*

7.36:29,14,1,14,15

1	*	3	*	*	*	*
*	*	4	*	*	*	5
*	*	*	5	*	2	6
4	*	*	*	2	*	*
*	6	*	1	*	*	2
*	*	*	*	3	*	*
*	*	*	6	*	5	*

7.37:14,10,0,14,15

1	*	3	*	*	*	*
*	*	*	3	*	7	5
*	4	*	5	*	*	*
*	*	6	*	2	*	*
*	6	*	*	1	*	*
*	7	*	*	*	4	*
*	*	2	*	*	5	*

7.38:13,16,0,14,15

1	*	3	*	*	6	*
2	*	*	*	6	7	*
*	4	*	5	*	*	*
*	*	*	7	2	1	*
*	*	*	*	*	*	*
6	*	*	*	*	*	4
*	3	5	*	*	*	*

7.39:9,8,0,14,15

1	*	3	*	*	*	*
*	*	*	*	6	7	*
3	*	1	*	*	*	6
*	5	*	7	*	*	*
*	*	*	*	*	*	1
6	*	*	*	3	*	2
*	*	2	*	*	5	*

7.40:15,14,0,14,15

1	*	3	*	*	*	7
*	*	*	*	6	*	5
3	*	1	*	*	2	*
*	5	*	7	*	*	*
*	*	2	*	3	4	*
*	*	*	*	1	*	*
*	6	*	*	*	*	*

7.41:20,12,0,14,15

1	*	*	*	*	6	7
*	*	4	3	*	*	*
*	*	*	*	7	*	6
*	*	*	*	2	*	1
5	*	*	*	*	1	*
*	3	*	2	*	*	*
7	*	5	*	*	*	*

7.42:18,20,0,15,15

*	2	3	*	*	*	*
*	*	*	*	*	*	5
*	4	*	5	*	*	6
*	*	*	*	3	*	*
*	*	7	*	2	3	*
6	*	*	*	*	*	*
*	*	*	6	*	5	1

7.43:16,10,0,14,14

1	*	*	4	*	*	*
*	*	4	*	6	*	*
*	*	*	*	7	2	6
*	5	*	*	3	*	2
5	*	*	1	*	*	*
*	7	*	*	*	*	3
*	3	*	*	*	*	*

7.43:16,10,0,14,15

1	*	3	*	*	*	*
2	*	*	3	6	*	*
*	4	*	*	*	2	*
*	5	*	*	*	*	*
*	*	*	2	1	*	*
6	*	*	1	*	*	3
*	*	5	*	*	4	*

7.44:19,14,0,14,15

*	*	*	4	*	6	*
2	1	*	*	*	*	*
*	*	1	*	7	*	6
*	*	*	*	*	*	*
5	*	7	*	1	*	*
*	*	*	*	*	4	3
7	3	2	*	*	*	*

7.45:22,18,0,15,15

1	2	3	*	*	*	*
2	1	*	*	*	7	*
*	*	*	5	*	*	6
4	*	*	*	3	1	*
*	*	*	6	*	*	4
*	3	*	*	*	*	*
*	*	5	*	*	*	*

7.46:19,12,0,14,15

1	*	*	4	*	*	*
*	1	*	*	6	*	5
*	*	*	5	*	2	*
*	*	*	*	*	*	*
*	*	2	*	*	4	*
6	3	7	*	*	*	*
7	6	*	*	*	*	1

7.47:14,10,0,14,15

1	*	3	*	*	*	*
*	*	*	*	*	7	5
*	4	*	5	*	*	6
*	*	*	*	3	1	*
*	*	*	6	*	*	*
*	*	7	*	4	*	*
*	6	*	*	2	*	4

7.48:15,12,0,14,15

1	*	3	*	*	*	*
*	*	*	*	6	7	*
*	4	*	*	7	2	*
*	5	*	*	*	*	*
*	7	2	*	4	*	*
6	*	*	1	*	*	*
*	*	*	*	*	4	3

7.49:23,10,0,14,15

1	*	3	*	5	*	*
*	*	*	3	6	*	*
*	4	*	*	*	2	*
*	*	7	6	*	*	2
*	*	*	*	*	1	4
6	*	5	*	3	*	*
*	*	*	*	*	*	*

7.50:19,12,0,14,15

1	*	3	4	*	*	*
*	*	*	*	*	7	5
3	*	1	5	*	*	*
4	*	*	*	1	*	*
*	*	6	*	*	*	*
*	7	*	*	*	5	*
*	6	*	*	4	*	*

7.51:15,10,1,14,15

*	*	3	*	*	*	7
2	*	*	*	6	*	*
*	*	1	*	*	*	*
4	*	*	6	*	*	*
*	3	*	7	*	*	1
*	7	5	*	*	1	*
*	*	*	*	4	*	3

7.52:27,18,1,15,15

1	2	3	*	*	*	*
*	*	4	*	6	*	*
*	*	*	5	7	*	*
*	5	*	6	*	*	*
*	*	2	*	*	1	4
*	*	*	*	*	4	3
*	*	*	*	*	*	1

7.53:34,18,0,15,15

1	*	3	*	*	*	*
*	*	*	*	*	7	5
*	4	*	*	7	2	*
*	5	*	*	*	*	*
5	*	*	*	*	*	3
*	7	*	2	*	4	*
*	*	*	1	2	*	*

7.54:20,10,0,14,15

1	2	3	*	*	*	*
*	*	*	*	6	*	5
3	*	1	*	*	*	*
*	*	7	*	*	*	*
*	7	*	*	*	4	*
6	3	*	7	*	*	*
*	*	*	*	4	5	*

7.55:25,12,0,14,15

1	*	3	*	*	*	7
2	*	*	3	*	*	*
*	4	*	5	*	*	*
*	*	7	*	2	*	3
*	6	*	*	*	4	*
*	*	*	*	*	*	2
7	*	*	*	*	5	*

7.56:28,12,0,15,15

1	*	3	*	5	*	*
*	1	*	*	*	7	*
3	*	1	*	*	*	*
*	5	*	*	*	*	3
*	*	*	2	*	4	*
*	*	*	7	*	*	4
*	6	5	*	*	*	*

7.57:21,12,0,14,15

1	2	3	*	*	*	*
*	*	*	3	6	*	5
3	*	1	*	*	*	*
*	*	7	*	*	*	*
*	*	*	*	*	4	*
6	7	*	1	*	*	*
*	*	*	*	4	5	*

7.58:25,16,0,14,15

*	*	*	4	*	6	*
*	1	*	*	6	*	*
3	4	*	5	*	*	*
4	5	*	*	*	*	*
*	*	2	*	1	*	3
*	*	*	*	*	*	*
*	*	6	*	2	*	*

7.59:21,10,0,14,14

1	*	3	*	*	*	7
*	*	4	*	6	7	*
*	4	*	5	*	*	*
*	*	7	*	3	*	*
5	6	*	*	*	*	*
*	*	*	*	4	3	*
*	*	*	1	*	*	*

7.59:21,10,0,14,15

1	*	*	4	*	*	*
*	*	*	3	*	*	5
*	*	1	*	7	2	*
*	*	7	*	*	1	*
*	*	2	*	*	*	*
6	7	*	*	2	*	*
*	3	*	*	*	*	4

7.60:28,14,1,14,15

1	*	3	*	*	6	*
*	1	*	*	*	*	5
*	4	*	5	*	*	*
*	*	7	*	3	1	*
*	*	6	*	*	3	4
6	*	*	*	*	*	*
*	*	*	2	*	*	*

7.61:26,10,0,14,15

1	*	*	4	*	*	*
*	*	4	*	6	*	*
3	*	*	*	*	*	*
*	5	*	6	*	*	*
5	7	*	*	*	*	3
*	3	*	*	*	5	*
*	*	*	2	4	*	*

7.62:19,10,0,14,14

1	2	*	4	*	*	*
*	*	*	*	6	7	5
*	4	*	5	*	*	*
*	*	7	*	3	*	*
5	*	*	1	*	*	*
*	*	2	*	*	*	*
*	*	*	*	*	3	1

7.62:19,10,0,14,15

1	*	3	*	*	*	*
*	*	4	*	*	7	*
*	*	*	*	7	*	6
*	*	*	6	*	*	2
*	*	*	2	1	*	*
*	3	*	7	*	5	*
*	*	5	*	*	4	*

7.63:24,14,0,15,15

1	2	*	4	*	*	*
2	1	*	*	6	*	*
*	4	*	*	*	*	*
4	*	*	*	*	1	2
*	*	6	*	*	3	*
*	*	5	*	*	*	*
*	*	*	*	*	5	3

7.64:19,10,0,14,15

1	*	*	*	*	6	*
*	*	4	3	*	*	5
*	*	*	*	7	*	*
*	6	*	*	*	1	*
*	*	2	6	*	*	4
*	*	*	2	4	*	*
7	*	*	*	*	*	2

7.65:17,14,0,14,15

1	*	3	4	*	*	*
*	*	*	*	6	*	5
3	*	1	*	*	2	*
4	*	*	*	*	1	*
*	7	*	*	*	4	*
*	5	*	*	2	*	*
*	*	6	*	*	*	*

7.66:25,18,0,14,15

1	*	*	*	*	*	7
*	*	4	*	6	7	*
*	*	*	5	*	*	*
*	6	5	*	3	*	*
*	*	*	*	*	4	*
*	*	*	2	*	*	1
*	3	6	*	2	*	*

7.67:16,12,0,14,15

1	*	*	4	*	*	*
*	*	4	*	*	*	5
*	*	*	*	*	2	6
*	6	5	*	3	*	*
*	7	*	*	*	*	*
*	*	*	1	2	*	*
*	5	6	*	*	*	3

7.68:41,14,0,14,15

1	*	3	*	*	*	*
*	1	*	*	*	7	*
*	*	*	5	7	*	*
4	*	*	*	3	*	2
*	*	*	*	2	4	3
*	*	*	*	*	*	4
*	5	*	6	*	*	*

7.69:27,14,0,14,15

1	*	3	*	*	*	*
*	*	4	*	6	*	*
*	*	*	5	*	2	*
*	6	*	*	3	*	*
5	*	*	2	*	*	*
*	3	7	*	*	*	4
7	*	*	*	*	*	1

7.70:24,14,0,14,15

1	2	3	*	*	*	*
*	*	4	*	*	7	*
3	*	1	*	7	*	*
*	*	*	6	*	*	2
*	6	*	*	*	*	*
*	*	7	*	1	5	*
*	5	*	*	*	*	*

7.71:33,18,0,15,15

1	2	3	*	*	*	*
*	*	*	*	*	7	5
3	*	1	5	*	*	*
*	*	*	6	*	1	*
*	*	2	*	4	*	*
*	3	7	*	*	*	4
*	*	*	*	2	*	*

7.71:33,18,0,15,16

1	*	3	*	*	*	*
*	*	*	*	6	*	5
3	4	*	*	7	*	*
*	*	*	*	1	*	*
*	*	*	6	*	*	2
*	*	*	2	*	4	*
7	*	1	*	*	3	*

7.72:47,10,1,14,15

1	*	3	*	*	*	*
*	*	*	3	*	7	*
*	*	2	*	7	5	*
*	*	*	7	*	2	*
*	6	*	*	*	*	1
6	*	*	*	2	*	4
*	*	5	*	*	*	*

7.73:24,9,1,14,15

1	*	3	*	*	*	*
2	*	*	*	*	7	*
*	4	*	*	7	5	*
*	5	*	7	*	*	*
*	*	1	*	*	*	2
*	3	*	5	*	4	*
*	*	*	*	*	*	4

7.74:37,18,1,14,15

1	2	*	*	*	*	*
*	*	4	3	6	*	*
*	*	*	*	*	5	*
*	5	*	*	*	1	*
*	*	*	6	3	*	2
6	*	*	*	4	*	*
*	*	*	2	*	*	4

7.75:24,15,1,15,15

1	*	3	*	*	6	*
*	*	*	*	*	*	*
3	*	*	*	*	5	6
*	*	7	*	*	3	*
*	*	*	7	2	*	*
6	*	1	*	*	*	4
*	*	*	2	4	*	*

7.76:31,6,3,14,15

1	*	3	*	5	*	*
*	*	*	3	6	*	5
*	4	2	*	*	*	*
*	*	*	7	1	*	*
5	*	*	1	*	*	*
*	7	*	*	*	4	*
*	*	*	*	*	2	*

7.77:21,10,0,15,15

1	*	3	4	*	6	*
*	*	*	*	6	7	*
3	*	2	5	*	*	*
*	*	*	*	*	*	*
5	*	*	1	*	*	3
*	7	*	*	4	*	*
*	*	1	*	*	*	*

7.78:23,14,0,14,15

1	*	3	*	*	*	*
*	*	*	*	6	7	5
3	*	2	*	*	*	*
*	*	*	*	1	*	*
*	6	*	*	*	2	4
*	7	*	*	*	*	*
*	5	*	6	*	4	*

7.79:23,10,0,15,15

1	*	3	4	*	*	*
*	*	4	3	6	*	*
*	4	*	*	7	*	*
*	*	*	*	*	5	2
*	*	7	*	3	*	*
6	*	*	*	*	2	*
*	5	*	*	*	*	*

7.80:20,10,0,14,15

1	*	3	4	*	*	*
*	*	*	*	*	7	5
3	*	*	5	*	*	*
*	*	6	*	*	5	2
*	*	*	*	*	*	*
*	7	*	*	2	*	*
*	*	1	6	3	*	*

7.81:25,10,0,14,15

1	*	3	*	*	6	*
*	*	*	*	*	*	5
3	*	2	*	*	1	*
*	*	6	*	1	*	*
*	*	*	*	4	3	*
*	7	*	*	*	*	4
7	5	*	*	*	*	*

7.82:19,8,0,14,15

1	*	3	*	*	*	7
2	*	*	*	*	7	*
3	*	2	*	7	*	*
*	*	*	*	*	5	2
*	*	*	6	*	4	*
*	5	*	1	*	*	*
*	6	*	*	*	*	*

7.83:19,8,0,14,15

*	2	*	*	*	6	*
2	*	*	3	*	7	*
*	4	*	*	*	*	6
*	3	*	*	1	*	*
5	*	*	*	*	*	*
*	*	*	*	4	*	1
7	*	5	*	*	2	*

7.84:22,14,0,15,15

1	*	3	*	*	*	*
*	*	*	*	6	7	5
3	*	2	*	*	*	*
*	*	6	*	*	5	*
*	6	*	*	*	*	4
*	7	*	1	*	*	*
*	5	*	*	*	4	*

7.85:12,11,0,14,15

1	*	3	*	*	*	*
*	*	4	*	*	7	5
*	4	*	*	*	*	*
*	5	*	*	*	*	3
*	*	*	1	2	*	*
6	*	*	2	*	*	*
*	3	*	*	4	5	*

7.86:19,8,0,14,15

1	*	3	*	5	*	*
2	*	4	*	*	7	*
3	*	*	*	*	*	*
*	*	*	7	*	2	*
5	*	*	*	3	*	*
*	*	5	*	4	*	1
*	*	*	6	*	*	*

7.87:19,8,0,14,15

1	*	3	*	5	*	*
*	1	*	*	6	*	5
*	*	*	*	*	*	6
*	*	*	7	*	2	*
5	6	*	*	*	*	*
*	*	*	2	3	*	4
*	*	*	*	*	4	*

7.88:18,7,0,14,15

1	*	3	*	*	*	*
2	*	*	*	6	*	5
*	4	*	5	*	*	*
*	*	6	*	1	*	*
*	*	*	*	*	4	*
6	*	*	1	2	*	*
*	3	*	*	*	5	*

7.89:17,7,0,14,15

1	*	3	*	*	*	7
*	*	*	*	*	*	*
3	*	*	*	7	*	6
*	*	6	7	*	*	3
*	*	*	2	4	*	*
6	*	*	*	*	4	*
*	*	*	*	2	5	*

7.90:23,6,0,14,15

1	*	3	4	*	*	*
*	*	4	3	*	*	5
3	*	*	*	7	*	*
*	5	*	*	*	2	*
*	7	*	*	2	*	*
*	*	*	1	*	*	*
*	*	5	*	*	*	1

7.91:22,12,0,14,15

1	*	*	*	*	*	*
*	1	*	3	*	7	*
*	4	2	*	*	*	6
*	*	*	7	*	*	*
*	*	*	*	2	*	4
*	*	*	*	4	*	*
7	*	5	1	*	*	*

7.92:22,6,0,14,14

1	*	3	*	*	*	7
*	*	*	*	6	*	*
*	*	2	5	*	*	*
4	5	*	*	*	*	*
*	*	*	6	2	*	*
*	3	7	*	*	*	1
7	*	*	*	*	*	2

7.92:22,6,0,14,15

1	*	3	*	5	*	*
*	*	4	*	*	7	5
*	4	*	*	*	*	*
*	*	*	*	*	*	3
*	7	*	6	*	*	*
6	*	*	2	*	*	*
*	*	5	*	*	3	4

7.93:14,9,0,14,15

*	2	*	*	5	*	7
*	*	4	*	*	*	*
*	*	*	*	7	*	*
4	*	6	*	*	3	*
5	*	1	6	*	*	*
6	*	*	*	*	*	*
*	*	*	*	2	*	3

7.94:22,9,0,14,14

1	*	*	4	*	6	*
*	*	4	*	6	7	*
*	*	*	*	*	1	*
*	5	*	*	*	*	2
5	*	1	6	*	*	*
*	3	*	*	*	*	*
*	*	*	*	2	*	3

7.94:22,9,0,14,15

1	*	*	*	5	*	*
*	*	4	*	*	7	*
*	4	*	5	*	*	*
*	*	*	*	*	*	1
5	*	*	1	2	*	*
6	7	*	*	*	*	*
*	3	*	*	*	4	*

7.95:12,10,0,14,14

1	2	*	4	*	*	*
2	1	*	*	6	*	*
*	*	2	*	7	*	*
*	*	*	*	*	*	*
*	*	*	*	*	3	4
*	*	*	*	*	5	3
7	*	5	*	1	*	*

7.95:12,10,0,14,15

1	*	3	*	*	*	*
*	*	*	*	*	7	5
3	4	*	*	*	*	*
*	5	*	7	*	2	*
*	*	1	6	*	*	*
*	*	*	2	*	5	4
*	*	*	*	*	*	2

7.96:13,5,0,14,15

1	*	*	4	*	*	*
*	*	*	*	6	*	*
*	*	*	5	*	1	*
4	*	*	*	*	*	3
*	3	6	*	2	*	*
*	7	*	*	3	*	*
*	6	*	1	*	*	*

7.97:7,6,0,14,14

1	*	3	*	*	*	7
*	*	*	*	*	*	*
3	4	*	5	*	*	*
*	*	7	*	1	*	3
*	*	*	*	2	4	*
*	*	*	2	*	5	*
7	*	5	*	*	*	*

7.97:7,6,0,14,15

1	*	3	*	*	6	*
2	*	4	*	*	*	*
*	*	*	5	7	*	*
*	5	*	6	*	*	*
*	*	1	*	*	4	2
*	7	*	*	*	*	4
*	*	*	*	*	*	1

7.98:21,9,0,14,15

1	*	3	*	*	*	*
2	*	*	*	6	*	*
*	4	*	5	*	*	*
*	*	*	6	1	*	*
*	*	*	*	*	4	*
*	7	5	*	*	3	*
*	3	*	*	*	5	4

7.99:25,10,0,14,15

1	*	3	*	*	6	*
2	*	4	*	*	*	*
*	*	*	5	7	*	*
*	5	*	*	1	*	*
*	*	*	7	*	4	*
6	*	*	*	*	2	4
*	*	6	*	*	*	*

7.100:16,8,0,14,15

1	*	*	4	*	*	*
2	*	*	*	6	7	*
*	4	*	5	*	*	*
*	5	*	*	*	3	*
*	*	*	*	3	2	*
6	*	*	*	2	*	*
*	*	6	*	*	*	1

7.101:25,12,0,14,15

1	*	3	*	5	*	*
*	1	*	*	*	*	*
*	*	*	5	7	*	*
*	5	7	*	1	*	*
*	*	*	*	*	2	3
*	*	*	2	*	4	*
7	*	*	*	*	*	4

7.102:26,11,0,14,15

*	*	*	4	*	*	7
2	*	*	*	6	*	*
*	*	*	*	7	*	6
4	5	*	*	*	2	*
5	*	*	*	*	4	*
*	*	*	*	*	3	*
*	*	6	*	1	*	*

7.103:22,7,0,14,14

1	*	3	*	*	*	7
*	*	*	*	6	7	*
3	*	2	*	*	*	*
*	5	*	6	*	*	*
*	*	1	*	2	*	3
*	7	*	*	*	*	2
*	*	*	*	*	5	*

7.103:22,7,0,14,15

1	*	3	*	*	6	*
2	*	*	*	6	7	*
*	4	*	5	*	*	*
*	5	*	*	3	*	*
*	*	1	*	*	3	*
6	*	*	*	*	*	*
*	*	*	2	*	*	4

7.104:21,10,0,14,15

1	*	*	*	*	*	7
*	1	4	*	*	*	*
*	4	*	5	*	*	*
*	5	*	6	*	2	*
*	*	*	*	2	*	3
*	*	*	*	*	*	2
7	*	*	*	1	3	*

7.105:30,10,0,14,15

1	*	3	*	*	*	*
*	*	4	*	6	*	*
*	4	*	*	7	*	*
*	5	7	*	3	*	*
5	*	*	2	*	*	*
*	3	*	*	*	*	4
*	*	*	1	*	*	2

7.106:26,9,0,14,15

1	*	3	*	*	*	*
*	*	*	*	6	*	5
3	*	2	*	*	1	*
*	*	1	*	2	*	*
*	7	*	*	*	*	4
*	5	*	*	*	4	*
7	*	*	*	*	2	*

7.107:16,10,0,14,15

1	*	*	*	*	*	*
*	1	*	3	*	*	5
*	*	*	5	*	*	6
*	*	*	*	*	5	3
*	*	*	2	*	*	*
6	*	7	*	4	*	*
7	*	*	1	*	*	*

7.108:14,8,0,14,14

1	*	3	*	*	*	7
*	*	*	*	*	*	5
*	4	*	5	*	*	*
*	*	1	*	2	*	3
*	*	*	2	*	4	*
*	5	7	*	*	*	*
7	*	*	*	1	*	*

7.108:14,8,0,14,15

1	*	3	*	*	*	7
*	*	4	*	*	*	*
*	4	*	5	*	*	*
*	6	*	*	*	5	*
*	*	*	*	*	*	1
*	*	7	*	2	*	3
7	*	*	6	1	*	*

7.109:25,16,0,14,15

*	2	*	*	*	*	7
*	1	*	3	6	*	*
3	*	*	5	*	*	*
*	*	*	*	*	2	*
5	*	*	6	*	*	*
*	*	7	*	*	4	2
*	*	6	*	*	*	*

7.110:25,10,0,14,14

1	*	*	4	*	*	7
2	*	*	*	6	*	*
*	*	*	*	*	*	*
4	6	*	7	*	*	*
*	*	*	*	2	3	*
*	*	7	1	*	*	2
*	3	*	*	*	5	*

7.110:25,10,0,14,15

1	*	3	*	*	*	*
*	*	*	*	*	7	5
3	4	*	*	*	*	*
4	3	6	*	*	*	*
*	*	*	2	*	*	6
*	*	*	*	4	2	*
*	6	*	5	*	*	*

7.111:14,8,0,15,15

*	2	*	4	*	*	*
*	*	4	*	*	*	5
3	*	5	*	*	1	*
*	*	6	*	*	5	1
*	7	*	*	*	*	6
*	*	*	2	*	*	*
7	*	*	*	4	*	*

7.112:18,9,0,15,15

1	*	3	*	*	6	*
*	*	4	*	6	*	5
3	*	*	*	*	1	*
*	5	*	7	*	*	*
*	*	*	*	*	2	*
6	*	*	*	3	*	*
*	*	*	2	*	*	4

7.113:28,9,0,14,15

1	*	3	*	*	*	*
*	1	*	*	*	7	5
3	*	*	6	*	*	*
*	5	*	*	*	*	*
*	*	*	2	*	4	*
*	7	*	*	4	5	*
*	*	6	*	*	*	4

7.114:19,8,0,14,15

*	2	3	*	*	*	7
*	1	*	3	6	*	*
*	*	*	*	7	*	*
4	*	*	*	*	*	*
5	*	*	*	*	2	*
*	3	7	*	1	*	*
*	*	*	5	*	4	*

7.115:19,10,0,15,15

1	*	3	*	*	*	*
*	*	*	*	6	7	*
3	*	*	*	*	*	2
4	*	1	*	2	*	*
*	*	*	*	3	*	1
*	*	*	5	*	*	4
*	6	*	*	*	5	*

7.116:23,6,0,14,15

*	*	*	*	*	6	7
*	1	4	3	*	*	*
*	4	*	*	*	*	2
*	*	*	*	*	2	6
*	*	*	1	*	*	*
6	*	*	5	*	*	*
*	3	*	*	1	*	*

7.117:21,10,0,14,14

1	*	3	4	*	*	*
*	*	*	*	6	7	*
3	*	5	*	*	*	*
4	*	1	*	*	*	*
*	6	*	*	2	*	3
*	7	*	*	*	*	1
*	*	*	*	*	5	*

7.117:21,10,0,14,15

1	*	3	4	*	*	*
*	1	*	*	6	7	*
*	*	*	*	7	1	*
*	5	*	*	*	*	*
*	6	*	2	*	*	*
*	*	2	*	*	*	3
7	*	*	*	*	5	*

7.118:17,5,0,14,15

*	2	*	*	*	*	7
*	*	4	3	6	*	*
*	*	*	*	*	*	2
*	5	*	7	*	2	*
*	*	6	*	*	4	*
6	*	*	*	1	*	*
*	*	*	*	4	*	*

7.119:17,7,0,14,14

1	*	*	4	*	*	*
*	*	*	*	6	7	*
*	*	5	*	7	*	*
4	*	1	*	*	*	*
*	7	*	1	*	*	3
*	*	*	2	*	*	4
*	6	*	*	*	3	*

7.119:17,7,0,14,15

*	2	*	4	*	*	*
*	*	*	*	*	*	5
*	4	*	*	7	*	*
*	*	1	*	*	*	6
5	*	6	*	*	*	3
*	*	*	2	4	5	*
*	6	*	*	*	*	*

7.120:21,5,0,14,14

1	*	3	*	5	*	*
*	1	*	*	*	*	*
3	*	5	*	7	*	*
*	*	*	7	*	2	6
*	*	*	*	*	4	*
*	*	7	*	*	5	*
*	6	*	*	*	*	4

7.120:21,5,0,14,15

1	*	3	*	*	*	7
2	*	*	*	*	*	*
3	*	*	*	7	*	2
*	5	*	*	*	3	*
*	*	*	*	2	4	*
*	*	7	*	*	*	1
*	6	*	5	*	*	*

7.121:18,10,0,14,15

1	2	3	*	*	*	*
*	*	*	*	*	7	5
3	*	*	*	7	*	*
*	*	2	*	*	*	*
*	*	6	*	*	*	4
6	3	*	2	*	*	*
*	*	*	5	*	4	*

7.122:18,8,0,14,15

1	*	3	*	*	*	*
*	*	*	*	6	*	5
*	*	*	6	*	*	2
4	*	*	*	1	3	*
*	*	*	*	*	2	*
*	*	7	5	*	*	*
7	*	1	*	4	*	*

7.123:17,8,0,14,15

1	*	*	4	*	*	*
*	1	4	*	*	*	5
*	*	*	*	7	*	*
*	*	2	*	*	3	6
5	*	*	*	*	*	1
*	*	7	*	2	*	*
*	*	*	5	*	2	*

7.124:23,12,0,14,15

1	*	*	4	*	6	*
*	*	*	*	*	*	5
*	*	*	6	*	1	*
*	5	6	*	*	*	3
*	*	*	2	*	*	*
*	7	*	*	3	*	*
*	3	*	*	*	4	*

7.125:13,2,0,14,14

1	*	3	*	5	*	*
*	*	*	*	*	7	*
3	*	5	6	*	*	*
4	*	*	*	1	*	3
*	6	*	2	*	*	*
*	7	*	*	*	*	*
*	*	1	*	*	4	*

7.125:13,2,0,14,15

1	*	3	4	*	*	*
*	*	4	3	*	*	5
3	*	5	*	*	*	*
4	*	*	*	2	*	*
*	7	*	*	*	*	6
*	*	*	*	*	5	*
*	6	*	*	*	2	*

7.126:25,12,0,15,15

1	*	*	4	*	*	*
*	*	4	*	*	*	5
*	*	*	*	*	*	2
*	*	*	*	3	2	*
5	*	7	1	*	*	*
*	7	*	*	*	3	*
7	6	1	*	*	*	*

7.127:21,10,0,15,15

1	*	3	*	5	*	*
*	*	*	3	6	*	5
*	4	*	*	*	1	*
4	*	*	7	*	*	*
*	7	*	*	*	*	6
*	*	*	*	*	*	*
*	*	*	5	2	*	3

7.128:11,6,0,14,15

1	*	*	*	*	*	*
*	1	*	3	*	*	5
*	*	*	*	7	*	*
*	*	7	*	3	2	*
*	*	*	*	2	4	*
6	*	*	5	*	*	1
*	*	*	*	*	*	4

7.129:21,7,0,14,14

1	*	3	4	*	*	*
*	*	*	*	*	7	5
*	*	*	*	7	*	2
*	*	*	*	*	2	*
5	6	1	*	*	*	*
6	*	*	*	*	*	*
*	3	6	*	1	*	*

7.129:21,7,0,14,15

1	*	3	*	*	6	*
*	*	*	3	*	*	5
*	4	*	*	7	*	*
4	7	1	*	*	*	*
*	*	*	*	2	4	*
*	*	*	*	*	*	*
*	*	*	1	*	5	3

7.130:23,4,0,14,15

1	*	*	4	*	*	7
2	*	*	3	*	*	*
*	*	5	*	*	*	2
4	*	*	*	*	*	*
*	6	*	*	*	3	*
*	*	*	1	2	*	4
*	5	6	*	*	*	*

7.131:32,8,0,14,15

1	*	3	4	*	*	*
*	*	4	3	*	7	*
3	*	*	*	*	*	*
*	5	*	*	*	*	6
*	6	*	*	*	*	2
*	*	1	5	2	*	*
*	*	*	*	*	5	*

7.132:28,7,0,14,15

1	*	3	*	*	*	7
2	*	*	*	6	*	*
*	4	*	6	*	*	*
*	5	*	1	*	*	*
*	*	2	*	*	1	3
*	*	*	5	*	4	*
7	*	*	*	*	*	*

7.133:26,8,0,14,15

1	*	*	4	*	*	*
*	1	*	*	*	7	*
*	*	*	6	*	*	*
*	*	*	*	2	*	6
5	*	*	*	*	1	*
*	*	*	*	*	5	2
*	*	6	2	*	*	3

7.134:16,5,0,14,14

1	*	3	4	*	*	*
*	*	*	*	*	7	5
3	*	*	6	*	*	*
*	7	1	*	*	*	*
*	*	*	*	3	*	4
6	*	*	1	*	*	*
*	5	*	*	*	4	*

7.134:16,5,0,14,15

1	*	3	*	*	*	*
*	*	*	*	6	*	*
*	*	*	6	7	*	4
*	*	5	*	*	1	*
*	7	*	*	3	*	2
*	*	*	*	*	5	*
7	4	*	*	*	*	6

7.135:31,11,0,14,15

1	*	3	*	5	*	*
2	*	*	3	*	*	*
*	*	1	*	*	*	4
*	6	*	*	2	*	*
5	*	*	2	3	*	*
*	*	*	*	*	5	*
*	4	*	*	*	*	6

7.136:22,6,0,14,15

1	*	3	*	*	*	*
*	1	*	*	6	7	*
*	*	*	6	*	*	4
*	*	5	*	*	3	*
*	4	*	2	*	*	6
*	*	*	5	*	*	*
*	*	*	*	4	*	2

7.137:23,10,0,14,15

1	*	3	*	*	*	7
*	*	*	3	6	*	*
*	5	*	6	*	*	*
*	*	7	*	*	1	2
5	4	*	*	*	*	*
*	*	*	*	*	*	*
7	*	*	*	4	*	1

7.138:34,11,0,14,15

*	*	*	4	*	6	7
2	*	4	*	*	*	*
*	*	1	6	*	*	4
*	*	*	5	*	3	*
5	*	*	*	2	*	*
*	*	*	*	3	*	*
*	*	6	*	*	*	1

7.139:36,13,0,13,15

1	*	3	*	*	6	*
2	*	*	*	6	7	*
*	5	*	*	*	*	2
*	*	*	*	*	1	*
*	4	7	*	*	*	*
*	*	*	5	*	*	4
7	*	6	*	*	*	*

7.140:31,8,0,14,15

1	2	3	*	*	*	*
*	*	*	*	*	7	5
*	*	*	*	*	4	*
*	6	*	*	3	*	*
5	*	*	*	*	*	4
*	*	7	*	2	*	*
*	3	2	5	*	*	*

7.141:45,16,0,15,15

1	*	*	4	*	*	7
*	1	4	*	*	*	5
*	*	1	*	*	*	*
*	*	*	*	2	5	*
*	*	*	*	3	*	*
*	*	*	2	*	3	*
7	*	6	*	*	*	4

7.142:31,10,0,14,15

1	*	3	*	*	*	*
*	*	*	*	*	7	*
3	5	*	*	*	*	*
*	*	*	7	2	5	*
*	4	*	*	*	2	*
*	7	*	2	4	*	*
*	*	6	*	1	*	*

7.143:20,4,0,14,15

1	*	3	*	*	*	*
*	*	*	5	*	7	*
3	*	*	*	*	5	*
4	*	1	*	2	*	*
*	*	*	*	4	*	*
*	7	*	2	*	*	*
*	*	6	*	1	*	4

7.144:15,4,0,14,15

1	*	3	*	5	*	*
2	*	*	*	*	7	3
*	4	*	1	*	*	*
*	*	1	*	2	*	5
*	7	*	*	*	4	*
*	*	*	*	*	*	*
*	*	5	*	*	*	2

7.145:11,2,0,14,15

1	*	*	*	*	*	*
*	*	4	*	6	*	*
*	*	*	*	*	2	*
*	*	*	3	2	5	*
*	3	*	7	*	*	2
6	*	1	*	*	*	*
*	5	*	*	*	3	4

7.146:7,3,0,14,15

1	*	3	*	*	*	*
*	1	4	*	6	*	*
*	*	*	7	*	2	6
4	*	*	*	3	*	*
*	*	*	*	*	*	1
*	7	*	2	*	*	5
*	*	*	*	*	5	*

7.147:15,1,0,14,15

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