Total domination critical graphs with respect to relative complements

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Abstract

A set S of vertices of a graph G is a total dominating set if every vertex of $V(G)$ is adjacent to some vertex in S. The total domination number $\gamma_t(G)$ is the minimum cardinality of a total dominating set of G. Let G be a spanning subgraph of $K_{s,s}$ and let H be the complement of G relative to $K_{s,s}$; that is, $K_{s,s} = G \oplus H$ is a factorization of $K_{s,s}$. The graph G is *k_t-critical relative to $K_{s,s}$* if $\gamma_t(G) = k$ and $\gamma_t(G+e) < k$ for all $e \in E(H)$. We study k_t-critical graphs relative to $K_{s,s}$ for small values of k. In particular, we characterize the 3_t-critical and 4_t-critical graphs.

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1 Introduction

Let $G = (V, E)$ be a graph and let $S, X \subseteq V$. We say that S *dominates* X, written $S \succ X$, if every vertex in $X - S$ is adjacent to a vertex in S and we say that S *totally dominates* X, written $S \succ_t X$, if every vertex in X is adjacent to a vertex in S (other than itself). In particular, if $X = V - S$, then we call the set S a *dominating set* of G and we write $S \succ G$, while if $X = V$, then we call S a *total dominating set* of G and we write $S \succ_t G$. If $S = \{s\}$, we write $\{s\} \succ X$ simply as $s \succ X$. The minimum cardinality of any dominating set (respectively, total dominating set) of G is the *domination number* $\gamma(G)$ (respectively, *total domination number* $\gamma_t(G)$). If S is a minimum dominating (respectively, minimum total dominating) set of G, we call S a $\gamma(G)$*-set* (respectively, $\gamma_t(G)$*-set*).

Total domination was introduced by Cockayne, Dawes, and Hedetniemi [4] and is studied, for example, in [5, 15]. For a more detailed treatment of domination related parameters and for terminology not defined here, the reader is referred to [2, 7]. In particular, for a vertex v in a graph $G = (V, E)$, the *open neighborhood* of v is $N(v) = \{u \in V \mid uv \in E\}$. We denote a path on n vertices by P_n. A nontrivial star is star of order at least two.

A graph G is said to be *domination critical* if $\gamma(G + e) = \gamma(G) - 1$ for every edge e in the complement $\overline{G}$ of G. This concept of domination critical graphs has been studied by, among others, Blitch [1], Sumner [16], Sumner and Blitch [17], and Wojcicka [19]; and a survey of this work is found in [18]. The domination critical graphs with domination number two were characterized in [17], but obtaining a characterization for domination critical graphs in general is a very difficult problem. In fact, it is still an open problem even restricted to graphs with domination number three. Haynes, Mynhardt, and van der Merwe [11]-[14] introduced and studied the *total domination edge critical graphs*, that is, graphs G such that $\gamma_t(G + e) < \gamma_t(G)$ for any edge $e \in E(\overline{G})$. Note that since $\gamma_t(G) \geq 2$ for any graph G, a total domination critical graph must have total domination number at least three. Whereas the addition of an edge to G from the complement $\overline{G}$ can change the domination number of G by at most one, it can change the total domination number by as much as two.

Proposition 1 [12] *If G is a graph with no isolated vertex, then for any edge $e \in E(\overline{G})$,*

$$\gamma_t(G) - 2 \leq \gamma_t(G + e) \leq \gamma_t(G).$$

The graphs G with the property $\gamma_t(G+e) = \gamma_t(G) - 2$ for any edge $e \in E(\overline{G})$ are called *supercritical*. It is shown in [11] that a graph G is supercritical if and only if G is the union of two or more nontrivial complete graphs.

As is the case with domination critical graphs, obtaining a characterization for total domination critical graphs is an open problem. Although families of total domination critical graphs with domination number three are characterized in [13, 14], not even all these graphs with the smallest possible total domination number have been characterized.

If G is a spanning subgraph of F, then the graph $F - E(G)$ is the *complement of G relative to F* with respect to a fixed embedding of G into F. The idea of a relative complement of a graph was suggested by Cockayne [3] and is studied in [6]. We shall assume that the complete bipartite graph $K_{s,s}$ has partite sets $\mathcal{L}$ and $\mathcal{R}$ (representing "left" and "right"), and that $G \oplus H = K_{s,s}$ is a factorization of $K_{s,s}$. (If G and H are graphs on the same vertex set but with disjoint edge sets, then $G \oplus H$ denotes the graph whose edge set is the union of their edge sets.) Notice that if G is uniquely embeddable in $K_{s,s}$, then H is unique. We henceforth consider only spanning subgraphs G of $K_{s,s}$ such that G is uniquely embeddable in $K_{s,s}$. We denote the relative complement H of G by $\overline{G}$. (The rest of this paper deals only with relative complements, so confusion with complements in the ordinary sense is unlikely.)

Haynes and Henning [8] studied domination critical graphs with respect to the relative complement, that is, the graphs G such that $\gamma(G+e) = \gamma(G) - 1$ for all $e \in E(\overline{G})$. In this paper, we study the same concept for total domination. We say that a graph G is *total domination edge critical relative to $K_{s,s}$*, or just *k_t-critical*, if $\gamma_t(G+e) < \gamma_t(G) = k$ for any edge $e \in E(\overline{G})$. For an example of a k_t-critical graph, let G be obtained from a star $K_{1,k}$, $k \geq 3$, by subdividing $k-1$ edges once and then adding an edge between two leaves at distance 3 apart. The graph G is illustrated in Figure 1.

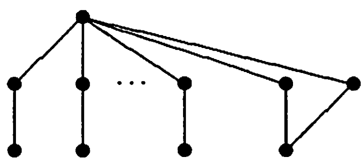

Figure 1: A k_t-critical graph of order $2k$ for $k \geq 3$.

Obviously, since Proposition 1 considers adding an arbitrary edge from the ordinary complement, it also applies to adding an edge from the relative complement. We note that adding an edge to a bipartite graph from its relative complement can change the total domination number by 0, 1, or 2. For example, the path $P_6: u_1, u_2, u_3, u_4, u_5, u_6$ is a subgraph of $K_{3,3}$ where all three possibilities occur. In particular, $\gamma_t(P_6 + u_1u_6) = \gamma_t(P_6) = 4$, $\gamma_t(P_6 + u_3u_6) = \gamma_t(P_6) - 1 = 3$, and $\gamma_t(P_6 + u_2u_5) = \gamma_t(P_6) - 2 = 2$.

If $\gamma_t(G) = k$ and $\gamma_t(G + e) = k - 2$ for an edge $e \in E(\overline{G})$, then e is called a *two-edge*. As before, if every edge in $E(\overline{G})$ is a two-edge, then we say that G is *k_t-supercritical relative to $K_{s,s}$*. We note that since $\gamma_t(G) \geq 2$, if G is k_t-supercritical, then $\gamma_t(G) \geq 4$. In [10], we characterized the disconnected k_t-supercritical graphs relative to $K_{s,s}$ and those for small values of k. In particular, the following result is given.

Theorem 2 [10] *A connected graph G is 4_t-supercritical relative to $K_{s,s}$ if and only if G is obtained from $K_{s,s}$ by removing the edges of a perfect matching.*

It is shown in [9] that no tree is total domination critical, and the k_t-critical trees with respect to $K_{s,s}$ are characterized as follows.

Theorem 3 [9] *A tree T is k_t-critical with respect to $K_{s,s}$ if and only if T is a subdivided star $K^*_{1,k-2}$, for $k \geq 5$, with exactly one edge subdivided twice as shown in Figure 2.*

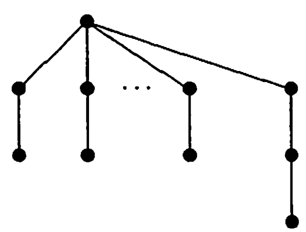

Figure 2: A subdivided star with exactly one edge subdivided twice.

Considering graphs that are total domination critical with respect to the relative complement as opposed to the ordinary complement, the problem of characterizing them becomes slightly less difficult. In particular, we are able to characterize the total domination critical graphs relative to $K_{s,s}$ with small total domination number; that is, we characterize the 3_t-critical graphs in Section 2 and the 4_t-critical graphs in Section 3. (Since $\gamma_t(G) \geq 2$ for any graph G, no graph is 2_t-critical.) Since the remainder of this paper deals only with relative complements, we will often omit the phrase "relative to $K_{s,s}$".

2 3_t-critical graphs

The following result characterizes 3_t-critical graphs.

Theorem 4 *Let $K_{s,s}$ have partite sets $\mathcal{L}$ and $\mathcal{R}$. For $s \geq 3$, a graph G is 3_t-critical relative to $K_{s,s}$ if and only if*

(1) *there exists a vertex v of $\mathcal{L}$ such that* $\deg(v) = s$, *and*

(2) *each vertex of $\mathcal{R}$ has degree $s - 1$.*

Proof. Let G be a graph with the two properties listed in the theorem. Clearly, no two adjacent vertices dominate G, and so $\gamma_t(G) \geq 3$. However, the vertex v together with any two vertices in $\mathcal{R}$ totally dominate G. Therefore, $\gamma_t(G) = 3$. Since for any edge $xy \in E(\overline{G})$ where $x \in \mathcal{L}$ and $y \in \mathcal{R}$, $y \succ \mathcal{L} - \{x\}$, it follows that $\{v, y\} \succ_t G + xy$. Hence, the graphs G are 3_t-critical.

Conversely, assume that G is 3_t-critical. We show first that G has a vertex of degree s. Suppose that G has no vertex of degree s. Let $S = \{x, y, x\}$ be a $\gamma_t(G)$-set. Since S induces a P_3, we may assume that $x \in \mathcal{L}$ and $\{y, z\} \subset \mathcal{R}$. But then $\deg(x) = s$, a contradiction. Hence, G has a vertex of degree s.

Let v be a vertex of degree s in G. We may assume that $v \in \mathcal{L}$, that is, $v \succ \mathcal{R}$. Since $\gamma_t(G) = 3$, $s \geq 3$ and no vertex in $\mathcal{R}$ dominates $\mathcal{L}$. Hence, $\deg(u) \leq s - 1$ for each $u \in \mathcal{R}$. For each $u \in \mathcal{R}$, let $\overline{u}$ denote a vertex in $\mathcal{L}$ that is not adjacent to u in G. Let S be a $\gamma_t(G + u\overline{u})$-set. Since G is 3_t-critical, $|S| = 2$ and at least one of u and $\overline{u}$ is in S. If $u \notin S$, then $S = \{\overline{u}, x\}$ where $x \in \mathcal{R} - \{u\}$. But then $x \succ \mathcal{L}$, and so $\deg(x) = s$, a contradiction. Hence, $u \in S$ and $\overline{u}$ is the only vertex in $\mathcal{L}$ that is not adjacent to u in G. Thus, $\deg(u) = s - 1$ for all $u \in \mathcal{R}$. □

3 4_t-critical graphs

Our aim in this section is to characterize the 4_t-critical graphs relative to $K_{s,s}$. Note that if G is a 4_t-critical graph, then for any edge $e \in E(\overline{G})$, $2 \le \gamma_t(G+e) \le 3$. Theorem 3 implies that no tree is 4_t-critical. If G is 4_t-supercritical, then, by Theorem 2, G is obtained from $K_{s,s}$ by removing the edges of a perfect matching. Our next result characterizes the disconnected 4_t-critical graphs.

Theorem 5 *If G is a disconnected 4_t-critical graph relative to $K_{s,s}$, then $G = K_2 \cup K_{s-1,s-1}$.*

Proof. Since $\gamma_t(G) = 4$, G has exactly two components. If $u \in \mathcal{L}$ and $v \in \mathcal{R}$ are non-adjacent vertices in the same component of G, then $G + uv$ has two components. Thus any total dominating set of $G + uv$ contains at least two vertices from each component, and so $\gamma(G+uv) \ge 4$, contradicting the fact that G is 4_t-critical. Hence each component is a complete bipartite graph. If $G = 2K_2$, then $s = 2$ and G is 4_t-supercritical relative to $K_{s,s}$ by Theorem 2. Hence, we may assume that $s \ge 3$.

Let G_1 and G_2 be the two components of G. For $i = 1, 2$, let $\mathcal{L}_i$ and $\mathcal{R}_i$ be the partite sets of G_i where $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2$ (and $\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2$). Let $u \in \mathcal{L}_1$ and $v \in \mathcal{R}_2$, and let S be a $\gamma_t(G+uv)$-set. If one of u or v is not in S, then S must contain at least two vertices from each component of G. But then $|S| \ge 4$, a contradiction. Hence, S contains both u and v. If $|S| = 2$, then $G = 2K_2$ and $s = 2$, a contradiction. Hence, $|S| \ge 3$. Let w be the vertex of S, different from u and v. We may assume that $w \in \mathcal{R}_1$. Then $\mathcal{R}_2 = \{v\}$ and v dominates $\mathcal{L}_2$. Since $s \ge 3$, $|\mathcal{R}_1| \ge 2$. If $|\mathcal{L}_2| \ge 2$, then $\gamma_t(G + wz) = 4$ for any vertex $z \in \mathcal{L}_2$, a contradiction. Hence, $|\mathcal{L}_2| = 1$. Thus, $G_2 = K_2$ and $G_1 = K_{s-1,s-1}$. □

Before proceeding further, we introduce some more notation. Let G be a k_t-critical graph relative to $K_{s,s}$ of diameter m. Let $u_0, u_1, \ldots, u_m$ be a diametrical path of G. For $i = 0, 1, \ldots, m$, let $V_i = \{x \mid d(u_0, x) = i\}$. If all edges are present between the two independent sets V_i and V_{i+1}, we say that $[V_i, V_{i+1}]$ is full. Necessarily, $V_0 = \{u_0\}$, $[V_0, V_1]$ is full, and $u_i \in V_i$ for $i = 1, 2, \ldots, m$. For $i = 0, 1, \ldots, m$, let v_i denote an arbitrary vertex of V_i (possibly, $v_i = u_i$).

We will use the following observations.

Observation 6 [10] *If $\gamma_t(G+uv) = \gamma_t(G) - 2$ for a graph G and an edge $uv \in E(\overline{G})$, then every $\gamma_t(G+uv)$-set contains both u and v.*

Observation 7 [12] *If $\gamma_t(G+uv) < \gamma_t(G)$ for a graph G and an edge $uv \in E(\overline{G})$, then every $\gamma_t(G+uv)$-set S contains at least one of u and v. Moreover, if without loss of generality, $u \in S$ and $v \notin S$, then u is the only neighbor of v in S.*

From Observation 7, we note that for a 4_t-critical graph, if $\gamma_t(G+uv) = 3$, then there exists a set W of cardinality 3 that totally dominates $G+uv$ where at least one of u and v is in W. Note that since G is bipartite, W induces a P_3. We use the fact that any $\gamma(G+uv)$-set induces a P_3 often in the following proofs without restating it to avoid excessive repetition. If exactly one of u and v belongs to W, without loss of generality, say u, then $S = W - \{u\}$ totally dominates $G-v$ and we write $[u, S] \mapsto v$. In particular, when we write $[u, S] \mapsto v$ it is understood that v is not dominated by S. To simplify the notation, if we write $[u, S] \mapsto v$, then we shall assume that $S = \{x, y\}$. First we give bounds on the diameter of a 4_t-critical graph.

Theorem 8 *If G is a connected 4_t-critical graph relative to $K_{s,s}$, then $3 \leq diam(G) \leq 4$.*

Proof. Let G be a connected 4_t-critical graph. If $\text{diam}(G) \leq 2$, then $G = K_{s,s}$ and $\gamma_t(G) < 4$, a contradiction. Hence, $\text{diam}(G) \geq 3$. Assume that $\text{diam}(G) \geq 5$. Let $u_0, u_1, \ldots, u_5$ be a diametrical path of G, and partition V into the sets V_i as described previously. Consider $G + u_0v_5$. Then Observation 7 states that at least one of u_0 and v_5 is in every $\gamma_t(G+u_0v_5)$-set W. If both u_0 and v_5 are in W, then at least one of V_2 and V_3 is not dominated. Thus, either $[u_0, S] \mapsto v_5$ or $[v_5, S] \mapsto u_0$. If $[u_0, S] \mapsto v_5$, then it is not possible to dominate V_4 since $S \cup \{u_0\}$ induces a P_3, while if $[v_5, S] \mapsto u_0$, then is is not possible to dominate V_1. Both cases produce a contradiction. Hence, $\text{diam}(G) \leq 4$. □

Let $\mathcal{L}$ and $\mathcal{R}$ be partite sets of $K_{s,s}$, and let $\mathcal{G}$ be the family of graphs G such that G is a connected spanning subgraph of $K_{s,s}$ for $s \geq 4$ and the following conditions hold:

(1) There exists a vertex in $\mathcal{L}$ with degree s,

(2) no pair of vertices in $\mathcal{R}$ dominates $\mathcal{L}$, and

(3) for each nonadjacent pair $u \in \mathcal{L}$ and $v \in \mathcal{R}$, there exists a vertex $w \in \mathcal{R}$ such that $\{v, w\} \succ \mathcal{L} - \{u\}$.

Let $\mathcal{H}$ be the family of spanning subgraphs G of $K_{s,s}$ such that the relative complement of G is the disjoint union of at least three nontrivial stars. Note that if $s \geq 3$ and $\overline{G} = sK_2$, then $G \in \mathcal{H}$. Hence, by Theorem 2, $\mathcal{H}$ contains the 4_t-supercritical graphs.

We now characterize the 4_t-critical graphs.

Theorem 9 *A connected graph G is 4_t-critical relative to $K_{s,s}$ if and only if $G \in \mathcal{G} \cup \mathcal{H}$.*

Proof. Suppose $G \in \mathcal{G} \cup \mathcal{H}$. We first show that $\gamma_t(G) \geq 4$. Clearly, no two adjacent vertices dominate G, and so $\gamma_t(G) \geq 3$. Suppose that $S = \{x, y, z\}$ is a $\gamma_t(G)$-set. Since S induces a P_3, we may assume that $x \in \mathcal{L}$ and $\{y, z\} \subset \mathcal{R}$. Hence, $x \succ \mathcal{R}$, and so $\deg(x) = s$, while $\{y, z\} \succ \mathcal{L}$. But then $G \notin \mathcal{G} \cup \mathcal{H}$, a contradiction. Hence, $\gamma_t(G) \geq 4$.

FACT: If $G \in \mathcal{H}$, then G is 4_t-critical.

PROOF. Each vertex of G is either the center of a star or an endvertex of a star in $\overline{G}$. If $\overline{G} = sK_2$, then G is 4_t-supercritical by Theorem 2 and therefore 4_t-critical. Hence we may assume that there is a vertex $u \in \mathcal{L}$ that is the center of a star in $\overline{G}$ of order at least 3. Since $|\mathcal{L}| = |\mathcal{R}|$, there is therefore a vertex $v \in \mathcal{R}$ that is the center of a star in $\overline{G}$ of order at least 3. Let u_1 (v_1, respectively) be adjacent to u (v, respectively) in $\overline{G}$. Then, $\{u, v, u_1, v_1\}$ totally dominates G. Therefore, $\gamma_t(G) = 4$. To see that G is 4_t-critical, consider $G + uv$ where $u \in \mathcal{L}$ and $v \in \mathcal{R}$. We may assume that u is the center and v is an endvertex of the same star in $\overline{G}$. Then, $\{u, u', v\} \succ_t G + uv$ for any vertex $u' \in \mathcal{L} - \{u\}$, and so $\gamma_t(G + uv) \leq 3$. The result follows. □

FACT: If $G \in \mathcal{G}$, then G is 4_t-critical.

PROOF. Let $x \in \mathcal{L}$ be a vertex of G such that $x \succ \mathcal{R}$. Then there exists a pair of nonadjacent vertices $u \in \mathcal{L}$ and $v \in \mathcal{R}$. Moreover, there is a vertex $w \in \mathcal{R}$ where $\{v, w\} \succ \mathcal{L} - \{u\}$. Thus, $\{x, v, w, z\} \succ_t G$ where $z \in N(u)$ implying that $\gamma_t(G) = 4$. To see that G is 4_t-critical, consider $G + uv$ where $u \in \mathcal{L}$ and $v \in \mathcal{R}$. By condition (3), we know that there exists a vertex w such that $\{v, w\} \succ \mathcal{L}$ in $G + uv$. By condition (1), there exists a vertex z that dominates $\mathcal{R}$. Thus, $\{v, w, z\} \succ_t G + uv$, and so $\gamma_t(G + uv) \leq 3$. The result follows. □

The sufficiency now follows from the above two facts.

Next we consider the necessity. Suppose that G is 4_t-critical. Let $u \in \mathcal{L}$ and $v \in \mathcal{R}$ be nonadjacent vertices in G. Observation 7 implies that one of $\{u, v\} \succ_t G + uv$, $\{u, v, x\} \succ_t G + uv$, $[u, S] \mapsto v$, and $[v, S] \mapsto u$ holds. We consider two cases.

Claim 1 *If G has no vertex of degree s, then $G \in \mathcal{H}$.*

Proof. We first prove the following fact.

FACT: At least one of u and v has degree $s - 1$ in G.

PROOF. Suppose $\deg(u) \le s-2$. Hence, $\{u, v\} \not\succ_t G + uv$ and there exists a vertex $w \neq v$ such that $uw \in E(\overline{G})$. We show that $\deg(v) = s - 1$. If $[u, S] \mapsto v$, then we may assume that $x \in \mathcal{L}$ (to dominate w) and $y \in \mathcal{R}$ (to dominate $\mathcal{L}$). But then $\deg(y) = s$, a contradiction. Hence, $\{u, v, x\} \succ_t G + uv$ or $[v, S] \mapsto u$. If $\{u, v, x\} \succ_t G + uv$, then $x \in \mathcal{L}$ (to dominate w), and so $\deg(v) = s - 1$, as desired. If $[v, S] \mapsto u$, then, since no vertex has degree s, $\{x, y\} \subset \mathcal{L}$ and so $\deg(v) = s - 1$, as desired. Hence, if $\deg(u) \le s - 2$, then $\deg(v) = s - 1$. Similarly, if $\deg(v) \le s - 2$, then $\deg(u) = s - 1$. □

It follows from the above fact that at least one of u and v is a leaf in $\overline{G}$. This is true for every pair of nonadjacent vertices with one vertex in $\mathcal{L}$ and the other in $\mathcal{R}$. Hence, since each vertex of $\overline{G}$ has degree at least 1, $\overline{G}$ is the disjoint union of nontrivial stars. Moreover, since G is a connected subgraph of $K_{s,s}$, $\overline{G}$ is the disjoint union of at least three nontrivial stars. Thus, $G \in \mathcal{H}$. This proves Claim 1. □

Claim 2 *If G has a vertex of degree s, then $G \in \mathcal{G}$.*

Proof. Without loss of generality, we may assume that $z \in \mathcal{L}$ has degree s. Since $\gamma_t(G) = 4$, we know then that $s \ge 4$, no vertex in $\mathcal{R}$ has degree s, and no pair of vertices in $\mathcal{R}$ dominates $\mathcal{L}$. Hence conditions (1) and (2) hold. Since G is connected, every vertex in $\mathcal{L}$ has a neighbor in $\mathcal{R}$ implying that no vertex in $\mathcal{R}$ can have degree $s - 1$. Hence, $\deg(v) \le s-2$ for each $v \in \mathcal{R}$. In particular, $\{u, v\} \not\succ_t G + uv$. If $\{u, v, w\} \succ_t G + uv$, then $w \in \mathcal{R}$. But then $\{v, w, z\} \succ_t G$, a contradiction. If $[u, S] \mapsto v$, then, since each vertex in $\mathcal{R}$ has degree at most $s - 2$, both x and y must belong to $\mathcal{R}$. But then $\{x, y, z\} \succ_t G$, a contradiction. Hence, $[v, S] \mapsto u$. Then, we may assume that $x \in \mathcal{R}$ and $y \in \mathcal{L}$. Thus, $\{v, x\} \succ \mathcal{L} - \{u\}$, and condition (3) holds. Hence, $G \in \mathcal{G}$. This proves Claim 2. □

The necessity now follows from Claims 1 and 2. □

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