Properties of closed meanders

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Abstract

In this paper various transformations of the set of closed meanders are introduced. Some of these are used in order to partition the above set and to find a representative of each class. Furthermore each closed meander is separated into shorter ones.

1 Introduction

This paper extends the study of meanders, which were presented as permutations in [4], by introducing various transformations in the case of closed meanders.

In section 2 of this paper we introduce a method to partition the set of closed meanders into equivalence classes and to find a representative of each class, in order to simplify the generation of closed meanders of greater size and to facilitate the methods used for their enumeration [1],[2], whereas in section 3 we describe a method to separate any closed meander into two shorter ones and we establish a relevant recursive formula.

A *closed meander of order* n is a closed self avoiding curve, crossing an infinite horizontal line $2n$ times. If we enumerate the crossing points along the horizontal line, the order in which they appear along the curve determines a permutation μ on $[2n] = \{1, 2, \ldots, 2n\}$. So we can describe a closed meander of order n through such a permutation μ, with the following properties:

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1) $\mu(1) = 1$;

2) The sets $U_\mu = \{\{\mu(i), \mu(i+1)\} : i = 1, 3, \ldots, 2n-1\}$ and $L_\mu = \{\{\mu(i), \mu(i+1)\} : i = 2, 4, \ldots, 2n\}$ are both nested.

Notice that, as opposed to the general case of meanders [4], here we can consider $\mu(1) = 1$, since closed meanders correspond to cyclic permutations, (called planar permutations [3],[5]). We take all numbers mod $2n$. It is clear that $\mu(i)$ is odd iff i is odd.

We recall that a set S of disjoint pairs of $[2n]$ such that $\bigcup_{\{a,b\}\in S} \{a, b\} = [2n]$ and for any $\{a, b\}, \{c, d\} \in S$ we never have $a < c < b < d$ is called *nested set* of pairs on $[2n]$.

We call *short pair* of the nested set S any pair of consecutive numbers that belongs to S and *outer pair* of S any pair $\{a, b\} \in S$ such that there is no pair $\{c, d\} \in S$ with $c < a < b < d$. Obviously, each nested set of pairs (and hence both U_μ and L_μ) contains at least one outer and one short pair.

For instance the permutation μ = 1 6 3 4 5 2 7 10 9 8 is a closed meander having:

$U_\mu = \{\{1,6\}, \{3,4\}, \{5,2\}, \{7,10\}, \{9,8\}\}$, with outer pairs $\{1,6\}$, $\{7,10\}$ and short pairs $\{3,4\}$, $\{9,8\}$,

$L_\mu = \{\{2,7\}, \{4,5\}, \{6,3\}, \{8,1\}, \{10,9\}\}$, with outer pairs $\{8,1\}$, $\{10,9\}$ and short pairs $\{4,5\}$, $\{10,9\}$.

In the corresponding geometrical representation, the nested edges correspond to nested pairs and the nature of short pairs and outer pairs is exposed (see Fig.1).

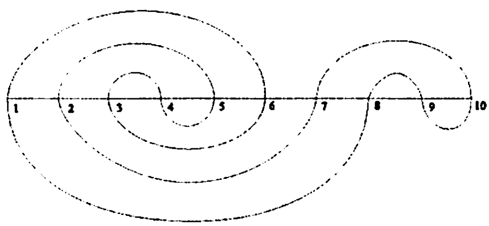

Figure 1: The closed meander μ = 1 6 3 4 5 2 7 10 9 8

Since in this paper we deal with closed meanders only, we will just call them meanders.

We denote with M_{2n} the set of all meanders of order n. In accordance with the cyclic property of the geometrical representation of each $\mu \in M_{2n}$, we define the following operations:

obverse μ^* is defined by $\mu^*(i) = \mu(2n+2-i), i \in [2n]$,
shift μ^+ is defined by $\mu^+(i) = \mu(h)-1$, where $h = \mu^{-1}(2)+i-1, i \in [2n]$,
conjugate μ^c is defined by $\mu^c(i) = 2n+2-\mu(i), i \in [2n]$ and
compatible $\bar{\mu}$ is defined by $\bar{\mu}(i) = 2n-\mu(c-i+1)$, where $c = \mu^{-1}(2n-1)$, $i \in [2n]$.

It is easy to prove that μ^*, μ^+, μ^c and $\bar{\mu}$ belong to M_{2n}.

2 The set $\mathbf{M_{2n}}$

For the set M_{2n} we use the following notations:
$M_{2n}(j) = \{\mu \in M_{2n} : \mu(2) = j\}$,
$\bar{M}_{2n}(k) = \{\mu \in M_{2n} : \mu(2n) = k\}$.
It is easy to prove that:
$|M_{2n}(j)| = |\bar{M}_{2n}(j)| = |M_{2n}(2n+2-j)|$.

Proposition 2.1 *The set $M_{2n}(2)$ can be generated from the set M_{2n-2}.*

Proof: For each $\mu \in M_{2n-2}$, let $\{\mu(q), \mu(q+1)\}$ be an outer pair of L_μ. Define $\hat{\mu} \in M_{2n}(2)$ with $\hat{\mu}(i+2) = 2+\mu(q+i), i \in [2n-2]$.

Conversely, for each $\hat{\mu} \in M_{2n}(2)$ let $\hat{\mu}(r) = 3$. Define $\mu \in M_{2n-2}$ with:

$$\mu(i) = \begin{cases} \hat{\mu}(i+r-1)-2, & i \in \{1,2,\ldots,2n-r+1\} \\ \hat{\mu}(i+r+1-2n)-2, & i \in \{2n-r+2,\ldots,2n-2\}. \end{cases}$$

It is clear from the above that from each outer pair of L_μ of each $\mu \in M_{2n-2}$, we create a different $\hat{\mu} \in M_{2n}(2)$ and that in this manner we create all the elements of $M_{2n}(2)$. □

Notice that the set U_μ of $\mu \in M_{2n-2}$ equals to the set L_{μ^*}. So, we can create $M_{2n}(2)$ by using only half of the elements of M_{2n-2} (e.g. the ones that have $\mu(2) < \mu(2n)$), provided that for each such meander μ we use not only each outer pair of L_μ, but each outer pair of U_μ too, in a similar way.

It is easy to see that for every nested set S on $[2n]$, its set of outer pairs is the set:

$$\{\{a_i+1, a_{i+1}\} \in S : i \in \{0,1,\ldots,r\}, a_0 = 0, a_{r+1} = 2n\}.$$

Corollary 2.2 *The cardinality of $M_{2n}(2)$ equals to the number of outer pairs that belong to the nested sets L_μ of all the elements μ of M_{2n-2}.*

Let $\{\mu(p), \mu(p+1)\}$ be a short pair of the set U_μ of a meander $\mu \in M_{2n}$. Then, the set that we obtain if we delete $\{\mu(p), \mu(p+1)\}$ from U_μ, as well as the set that we obtain if we delete the pairs $\{\mu(p-1),\mu(p)\}$, $\{\mu(p+1),\mu(p+2)\}$ from $L_\mu \cup \{\mu(p-1),\mu(p+2)\}$ are obviously nested. We can get the corresponding result for any short pair of L_μ, too.

We thus have the following result.

Proposition 2.3 *Let $\mu \in M_{2n}$ and $\{\mu(p),\mu(p+1)\} \neq \{1,2\}$ be a short pair of either U_μ or L_μ. Then the permutation t on $[2n-2]$ with :*

$$t(i) = \begin{cases} \mu(i), & if\ i<p\ and\ \mu(i)<m \\ \mu(i)-2, & if\ i<p\ and\ \mu(i)>m+1 \\ \mu(i+2), & if\ i\geq p\ and\ \mu(i+2)<m \\ \mu(i+2)-2, & if\ i\geq p\ and\ \mu(i+2)>m+1 \end{cases}$$

where $m = min\{\mu(p),\mu(p+1)\}$, is a meander of M_{2n-2}.

Notice that if $\{1,2\} \in U_\mu$ (resp. L_μ) then we can define a meander $t \in M_{2n-2}$, in a way similar to that of proposition 2.3, as follows:

$$t(i) = \begin{cases} \mu(i+i_1-1)-2, & i=1,2,\ldots,2n-i_1+1 \\ & (resp.\ i=1,2,\ldots,2n-i_1) \\ \mu(i+i_1+1)-2, & i=2n-i_1+2,\ldots,2n-2 \\ & (resp.\ i=2n-i_1+1,\ldots,2n-2) \end{cases}$$

where $i_1 = \mu^{-1}(3)$.

Practically, in order to construct $t \in M_{2n-2}$ it is enough to delete $\mu(p)$, $\mu(p+1)$ and reduce by 2 each value of μ that is greater than them, starting with 1 and keeping the order unchanged.

For example, for $\mu = 1\ 4\ 7\ 6\ 5\ 8\ 3\ 2 \in M_8$ and the short pair $\{3,2\} \in U_\mu$ we get $t = 1\ 2\ 5\ 4\ 3\ 6 \in M_6$, whereas for the short pair $\{6,5\} \in L_\mu$ we get $t = 1\ 4\ 5\ 6\ 3\ 2 \in M_6$. Finally, for $\mu = 1\ 2\ 5\ 4\ 3\ 8\ 7\ 6 \in M_8$ and the short pair $\{1,2\} \in U_\mu$ we get $t = 1\ 6\ 5\ 4\ 3\ 2 \in M_6$.

Let now $\{\mu(q),\mu(q+1)\}$ be an outer pair of the set U_μ or L_μ of a meander μ and let $\bar{m} = max\{\mu(q),\mu(q+1)\}$; notice that $\bar{m}$ is always even. Since the repetitive application of proposition 2.3 for every short pair $\{\mu(p),\mu(p+1)\}$ of U_μ or L_μ, with $min\{\mu(p),\mu(p+1)\} \geq \bar{m}$ gives a meander, we have the following result.

Proposition 2.4 *Let $\mu \in M_{2n}$ and $\{\mu(q),\mu(q+1)\}$ be an outer pair of U_μ or L_μ. Then the permutation τ on $[\bar{m}]$ having $\tau(1)=1$ and $\tau(i+1)=\mu(v)$ where v is the minimum element of $\{i+1,i+2,\ldots,2n\}$ with $\mu(v) \leq \bar{m}$, is a meander of $M_{\bar{m}}$.*

Practically, in order to construct $\tau \in M_{\bar{m}}$ it is enough to retain in the existing order, only the values $\mu(i)$ with $\mu(i) \leq \bar{m}$.

For example, for $\mu = 1\ 8\ 7\ 6\ 11\ 10\ 9\ 12\ 5\ 2\ 3\ 4\ 13\ 16\ 15\ 14 \in M_{16}$ and the outer pair $\{1,8\} \in U_\mu$ we get $\tau = 1\ 8\ 7\ 6\ 5\ 2\ 3\ 4 \in M_8$, whereas for the outer pair $\{9,12\} \in U_\mu$ we get $\tau = 1\ 8\ 7\ 6\ 11\ 10\ 9\ 12\ 5\ 2\ 3\ 4 \in M_{12}$ and for the outer pair $\{14,1\} \in L_\mu$ we get $\tau = 1\ 8\ 7\ 6\ 11\ 10\ 9\ 12\ 5\ 2\ 3\ 4\ 13\ 14 \in M_{14}$.

If τ is obtained from μ according to proposition 2.4 for some outer pair of U_μ, we will write $\tau \prec \mu$. Furthermore we denote:

$$M_{2n}/\tau = \{\mu \in M_{2n} : \tau \prec \mu\}.$$

For each $\mu \in M_{2n}$ we define $\mu^r, r \in N$ as follows:
$\mu^0 = \mu, \mu^1 = \mu^+, \mu^r = (\mu^{r-1})^+$.

Obviously, $\mu^r \in M_{2n}$ and $\mu^{2n} = \mu$. In some cases though, we get $\mu^p = \mu$, with $p < 2n$, p being a divisor of $2n$.

Thus, using the above operations we can partition M_{2n} into equivalence classes
$\Sigma(\mu) = P(\mu) \cup P(\mu^*)$ where $P(\mu) = \{\mu^0, \mu^1, \ldots, \mu^{p-1}\}$.

Since it is obvious that $(\mu^*)^+ = (\mu^+)^*$, $P(\mu^*)$ consists of the obverse meanders of $P(\mu)$.

Proposition 2.5 *For each class $\Sigma(\mu)$, there exists $\rho \in \Sigma(\mu) \cap M_{2n}(2)$.*

Proof: Since there exists at least one short pair in U_μ, i.e. one $i \in [2n]$ such that $\mu(i) = u$ and either $\mu(i+1) = u+1$ or $\mu(i-1) = u+1$, then either $\rho = \mu^{u-1}$ or $\rho = (\mu^{u-1})^*$ respectively. □

If we order lexicographically the elements of each equivalence class $\Sigma(\mu)$ of M_{2n}, we call *representative* of each class its smallest element; obviously, according to proposition 2.5, if α is a representative of such a class, then $\alpha \in M_{2n}(2)$.

For example the class of $\mu = 1\ 6\ 7\ 8\ 3\ 4\ 5\ 2$ has $\mu^5 = 1\ 2\ 3\ 6\ 7\ 8\ 5\ 4$ as representative, whereas the class of $\mu = 1\ 8\ 7\ 2\ 5\ 4\ 3\ 6$ has $(\mu^2)^* = 1\ 2\ 3\ 8\ 5\ 6\ 7\ 4$ as representative.

Let $A_{2n} \subset M_{2n}$ be the set of representatives of the classes of M_{2n}. Notice that if $\alpha = \alpha(1)\alpha(2)\ldots\alpha(2n) \in A_{2n}$, then $\alpha(2n) \neq 2n$, except for the trivial meander with $\alpha(i) = i, i \in [2n]$.

Clearly we have $M_{2n} = \bigcup_{\alpha \in A_{2n}} \Sigma(\alpha)$.

Proposition 2.6 *Every $\alpha \in A_{2n}$ can be constructed from $\mu \in \Sigma(\beta)$ where $\beta \in A_{2n-2}$, for some outer pair of L_μ not containing the number $2n-2$.*

Proof: For each $\mu \in \Sigma(\beta)$ and for each outer pair of L_μ, apply the procedure described in the proof of proposition 2.1, thus creating a set of meanders belonging to $M_{2n}(2)$. The set A_{2n} is a subset of this set. Notice that there is no need to use the outer pair of L_μ containing the number $2n-2$, since this will create a meander of $M_{2n}(2)$ with last element $2n$, which does not belong to A_{2n}. □

3 The set $\mathrm{M_{2n}(j,k)}$

We define $M_{2n}(j,k) = M_{2n}(j) \cap \bar{M}_{2n}(k)$; notice that j is even. It is easy to prove that:

$|M_{2n}(j,k)| = |M_{2n}(k,j)| = |M_{2n}(2n+2-j, 2n+2-k)|$,

$|M_{2n}| = \sum_{j,k \in \mathrm{I}} |M_{2n}(j,k)|$, where $\mathrm{I} = \{2,4,\ldots,2n\}$.

Let $\mu \in M_{2n}(j,k)$ and $J = \{j+1, j+2, \ldots, 2n\}$. Define the 1-1 mapping $s_1 : [2n-j] \to J$ as follows:
$s_1(1) = \mu(v_1)$, where v_1 is the minimum element of $[2n]$ with $\mu(v_1) > j$ and $s_1(i+1) = \mu(v)$, where v is the minimum element of $\{\mu^{-1}(s_1(i)) + 1,$ $\mu^{-1}(s_1(i)) + 2, \ldots, 2n\}$ with $\mu(v) > j$.

Practically, in order to get s_1 it is enough to retain, in the existing order, only the values $\mu(i)$ with $\mu(i) > j$.

We have the following lemma.

Lemma 3.1 *Let μ_1 be the permutation on $[2n-j]$ with $\mu_1(i) = s_1(i_1 - i + 1) - j$, where $i_1 = s_1^{-1}(j+1)$. Then $\mu_1 \in M_{2n-j}$.*

Proof: Apply repetitively proposition 2.3 for each short pair $\{\mu(p),\mu(p+1)\}$ of μ, where $\mu(p) \le j$, and get the obverse of the remaining permutation. □

For example, for $\mu = 1\ 6\ 5\ 4\ 9\ 8\ 7\ 10\ 3\ 2\ 11\ 14\ 13\ 12 \in M_{14}(6,12)$ we have $v_1 = 5$ and therefore $s_1 = 9\ 8\ 7\ 10\ 11\ 14\ 13\ 12$, $i_1 = 3$ and finally, $\mu_1 = 1\ 2\ 3\ 6\ 7\ 8\ 5\ 4 \in M_8$.

Let $\mu \in M_{2n}(j,k)$ and $K = \{2,3,\ldots,k-1\}$; notice that k is even. Define the 1-1 mapping $s_2 : [k-2] \to K$ as follows:
$s_2(1) = \mu(w_1)$, where w_1 is the minimum element of $[2n]$ with $1 < \mu(w_1) < k$ and
$s_2(i+1) = \mu(w)$, where w is the minimum element of $\{\mu^{-1}(s_2(i)) + 1,$ $\mu^{-1}(s_2(i)) + 2, \ldots, 2n\}$ with $\mu(w) < k$.

Practically, in order to get s_2 it is enough to retain, in the existing order, only the values $\mu(i)$ with $1 < \mu(i) < k$.

We have the following lemma.

Lemma 3.2 *Let μ_2 be the permutation on $[k-2]$ with $\mu_2(i) = k - s_2(i + i_2 - 1)$, where $i_2 = {s_2}^{-1}(k-1)$. Then $\mu_2 \in M_{k-2}$.*

Proof: Apply repetitively proposition 2.3 for each short pair $\{\mu(p), \mu(p+1)\}$ of either U_μ or L_μ, where $\mu(p) > k$ and then for the short pair $\{k, 1\}$; finally apply the operation $(\mu^c)^+$ to the remaining permutation. □

For example, for $\mu = 1\ 6\ 5\ 4\ 9\ 8\ 7\ 10\ 3\ 2\ 11\ 14\ 13\ 12 \in M_{14}(6, 12)$ we have $w_1 = 2$, $s_2 = 6\ 5\ 4\ 9\ 8\ 7\ 10\ 3\ 2\ 11$, $i_2 = 10$ and finally, $\mu_2 = 1\ 6\ 7\ 8\ 3\ 4\ 5\ 2\ 9\ 10 \in M_{10}$.

Given that $|M_{2n}(j,k)| = |M_{2n}(k,j)|$, we suppose for the rest of this section that $j < k$.

Let $\mu \in M_{2n}(j,k)$ and let μ_1, μ_2 be the meanders obtained according to lemmas 3.1 and 3.2 respectively. We then have the following results.

Lemma 3.3 *There exists an outer pair $\{\mu_1(q_1), \mu_1(q_1+1)\}$ of U_{μ_1} and an outer pair $\{\mu_2(q_2, \mu_2(q_2+1)\ \}$ of U_{μ_2} with $\mu_1(q_1+1) = \mu_2(q_2+1) = k - j$.*

Proof: It is easy to demonstrate that if it was not true, then the outer pairs $\{1, j\}$ of U_μ and $\{k, 1\}$ of L_μ would cross other pairs of U_μ and L_μ respectively. □

Proposition 3.4 *The meanders τ_1, τ_2 with $\tau_1 \prec \mu_1$ and $\tau_2 \prec \mu_2$, corresponding to the outer pairs of U_{μ_1}, U_{μ_2} which contain the number $k - j$, are compatible.*

Proof: Notice that by the definitions of s_1, s_2 their values form permutations of J, K respectively, since $k = j mod(k-j)$. These values occur in the same order as in μ; their places though change.

Suppose that $\mu(i) \in \{j+1, j+2, \ldots, k\}$ and that it occupies in s_1 the $({s_1}^{-1}(j+1) + p)$-th place (i.e. the p-th place after the value $j+1$), $p = 0, 1, \ldots, k-j-1$, among the (ordered) values $j+1, j+2, \ldots, k$ of s_1. In μ_1 (and hence in τ_1, too) this value turns into $\mu(i) - j$, occupying now the $(k-j-p+1)$-th place among the (ordered) values $1, 2, \ldots, k-j$ in μ_1 and τ_1.

So, $\tau_1(k-j-p+1) = \mu(i) - j$.

Similarly, from s_2 we get $\tau_2({\tau_2}^{-1}(k-j-1) + p) = k - \mu(i)$.

So, $\tau_1(k-j-p+1) + \tau_2({\tau_2}^{-1}(k-j-1)+p) = \mu(i) - j + k - \mu(i) = k - j$, i.e. τ_1, τ_2 satisfy the definition of compatibility, for meanders of the set M_{k-j} with $2n = k - j$, $i = k - j - p + 1$, $c = {\tau_2}^{-1}(k-j-1)$. □

For example, for $\mu = 1\ 6\ 5\ 4\ 9\ 8\ 7\ 10\ 3\ 2\ 11\ 14\ 13\ 12$ (with $j = 6$, $k = 12$ and $k - j = 6$), after using lemmas 3.1, 3.2 we have: $\mu_1 = 1\ 2\ 3\ 6\ 7\ 8\ 5\ 4$, $\mu_2 = 1\ 6\ 7\ 8\ 3\ 4\ 5\ 2\ 9\ 10$ and hence $\tau_1 = 1\ 2\ 3\ 6\ 5\ 4$, $\tau_2 = 1\ 6\ 3\ 4\ 5\ 2$ which are actually compatible.

We also have the following result.

Proposition 3.5 *To each pair of meanders $\mu_1 \in M_{2n-j}, \mu_2 \in M_{k-2}$ for which the meanders τ_1, τ_2 of M_{k-j} with $\tau_1 \prec \mu_1, \tau_2 \prec \mu_2$ are compatible, corresponds a unique element of $M_{2n}(j,k)$.*

Proof: From lemmas 3.1, 3.2 and 3.3 we obtain that $s_1(i) = j + \mu_1(q_1 + 1 - i), i \in [2n - j]$ and $s_2(i) = k - \mu_2(q_2 + i), i \in [k-2]$. Let $\{a,b\} \in U_{\tau_1}$ (resp. L_{τ_1}); since τ_1 and τ_2 are compatible, $\{k-j-b, k-j-a\} \in L_{\tau_2}$ (resp. U_{τ_2}).

Both a and $k-j-a$ turn into $a+j$ in s_1 and s_2 respectively; similarly b and $k-j-b$ turn into $b+j$; (in the special case that a or b is equal to $k-j$, it turns into k in s_1 and into j in s_2). Notice that if a, $b \neq k-j$, then since $\tau_1, \tau_2 \in M_{k-j}$ we have that $a, b \in [k-j-1]$ and hence $a+j, b+j \in J \cap K = \{j+1, j+2, \ldots, k-1\}$. So $a+j$, $b+j$ occur in both s_1 and s_2, having possibly between them some elements of the set $\{k+1, k+2, \ldots, 2n\}$ in s_1, or of $\{2, 3, \ldots, j-1\}$ in s_2; (the pair containing k in s_1 corresponds to the pair containing j in s_2).

This means that the (ordered) values of s_1 and s_2 can be partitioned into $\frac{k-j}{2}$ consecutive subsequences, each of them having length at least 2 and ending with an element of $J \cap K$, with the last subsequence of s_1 ending with k.

We obtain μ by writing after $\mu(1) = 1$ alternatively these subsequences of s_2 and s_1, keeping of course each element of $J \cap K$ only once. □

For example, for $\mu_1 = 1\ 2\ 3\ 6\ 7\ 8\ 5\ 4 \in M_8$, $\tau_1 = 1\ 2\ 3\ 6\ 5\ 4 \in M_6$, $\mu_2 = 1\ 6\ 7\ 8\ 3\ 4\ 5\ 2\ 9\ 10 \in M_{10}$, $\tau_2 = 1\ 6\ 3\ 4\ 5\ 2 \in M_6$ we get $2n = 16$, $j = 6$, $k = 12$.
Since $k - j = 6$ we have $q_1 = 3$, $q_2 = 1$, $s_1 = 9\ 8\ 7\ 10\ 11\ 14\ 13\ 12$ and $s_2 = 6\ 5\ 4\ 9\ 8\ 7\ 10\ 3\ 2\ 11$.
$J \cap K = \{7, 8, 9, 10, 11\}$, so s_1, s_2 are partitioned as follows:
$s_1 = 9\ 8\ /\ 7\ 10\ /\ 11\ 14\ 13\ 12$, $s_2 = 6\ 5\ 4\ 9\ /\ 8\ 7\ /\ 10\ 3\ 2\ 11$, finally giving $\mu = 1\ 6\ 5\ 4\ 9\ 8\ 7\ 10\ 3\ 2\ 11\ 14\ 13\ 12 \in M_{14}(6, 12)$.

From propositions 3.4 and 3.5 we get the following result.

Corollary 3.6 *The following relation holds:*

$$|M_{2n}(j,k)| = \sum_{\tau \in M_{k-j}} |M_{2n-j}/\tau||M_{k-2}/\bar{\tau}|.$$

The above formula enables us to determine the cardinality of M_{2n}, given that

$$|M_{2n}| = \sum_{j,k \in I} |M_{2n}(j,k)|, I = \{2,4,\ldots,2n\}.$$

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