Even $(m_1, m_2, \ldots, m_r)$-cycle systems of the complete graph

Shung-Liang Wu
National Lien-Ho Institute of Technology
Miaoli, Taiwan, R. O. C.

ABSTRACT. Let K_n be the complete graph on n vertices. In this paper, we find the necessary and sufficient conditions for the existence of $(m_1, m_2, \ldots, m_r)$-cycle system of K_n, where m_i $(1 \leq i \leq r)$ are positive even integers, and $\sum_{i=1}^{r} m_i = 2^k$ for $k \geq 2$. In particular, if $r = 1$ then there exists a cyclic 2^k-cycle system of K_n if and only if 2^k divides $|E(K_n)|$ and n is odd.

1 Introduction

The *m-cycle* $(v_0, v_1, \ldots, v_{m-1})$ is the graph induced by the edges $\{(v_i, v_{i+1}), (v_0, v_{m-1}) \mid i \in Z_{m-1}\}$. An *m-cycle system* of a graph $G = (V, E)$ is an ordered pair (V, C), where C is a set of m-cycles such that every edge of E belongs to exactly one m-cycle of C. Similarly, a $(m_1, m_2, \ldots, m_r)$*-cycle* is the union of edge-disjoint m_i-cycles for $1 \leq i \leq r$, and a $(m_1, m_2, \ldots, m_r)$*-cycle system* of a graph $G = (V, E)$ is also an ordered pair (V, C^*), where C^* is a set of $(m_1, m_2, \ldots, m_r)$-cycles such that every edge of E belongs to exactly one $(m_1, m_2, \ldots, m_r)$-cycle of C^*. A $(m_1, m_2, \ldots, m_r)$-cycle system is said to be *even* if each m_i $(1 \leq i \leq r)$ is even.

Let K_n denote the complete graph on n vertices and $K_n - F$ denote the complete graph on n vertices with a 1-factor F removed. The necessary conditions for the existence of a decomposition of K_n or $K_n - F$ into a $(m_1, m_2, \ldots, m_r)$-cycle are:

(1) $3 \leq m_i \leq n$, for $i = 1, 2, \ldots, r$;

(2) n is odd (even); and

(3) The number of edges in K_n $(K_n - F)$ is a multiple of $\sum_{i=1}^{r} m_i$.

The question of whether these necessary conditions are also sufficient was asked by Alspach [3]. Although the conditions have been shown to be sufficient in many cases, the question is still widely an open problem.

A great deal of work has been done on the m-cycle systems of K_n. For a survey of m-cycle systems refer to Linder and Rodger [8]. Notably, Alspach and Gavlas [4] and Sajna [14] have proved that Alspach's conjecture is true when all cycles are the same length.

In the case of $(m_1, m_2, \ldots, m_r)$-cycle systems of K_n, Alspach's conjecture is known to be true for all $n \le 10$ [9] and for all n when

(1) $m_i \in \{3,5\}$ for $i = 1,2,\ldots,r$ [1];

(2) $m_i \in \{4,5\}$ for $i = 1,2,\ldots,r$ [5];

(3) $m_i \in \{3,4,6\}$ (or $\{n-2, n-1, n\}$, $\{2^k, 2^{k+1}\}$ $(k \ge 2)$) for $i = 1,2,\ldots,r$ [6]; and when

(4) $m_i \in \{4,8\}$ (or $\{4,10\}$, $\{6,8\}$, $\{6,10\}$, $\{8,10\}$) for $i = 1,2,\ldots,r$ [2].

In this paper, it is shown that if $m_1, m_2, \ldots, m_r$ are positive even integers with $\sum_{i=1}^{r} m_i = 2^k$ for $k \ge 2$, then there exists an even $(m_1, m_2, \ldots, m_r)$-cycle system of K_n if and only if $\sum_{i=1}^{r} m_i$ divides $|E(K_n)|$ and n is odd.

2 The results

Let the vertex set of K_n be Z_n and let (a,b) be any edge of K_n. By $\|a-b\|$ we mean the *length* of the edge (a,b) that is defined as

$$\|a-b\| = \min\{|a-b|, n-|a-b|\}.$$

By observing the lengths of edges in K_n, we have that there are n edges of length q for each $1 \le q \le \lfloor (n-1)/2 \rfloor$ and if n is even, then there are $n/2$ edges of length $n/2$.

Recall that $V(K_n) = Z_n$. Assume the $2m$-cycle considered here to be a subgraph of K_n. A $2m$-cycle $(v_0, u_0, v_1, u_1, \ldots, v_{m-1}, u_{m-1})$ is *resoluble* if $\max\{v_0, v_1, \ldots, v_{m-1}\} < \min\{u_0, u_1, \ldots, u_{m-1}\}$. A resoluble $2m$-cycle is *exact* if the set of lengths of all edges is a sequence of distinct consecutive integers. For convenience, let S be the set of lengths of edges in a $2m$-cycle. An exact resoluble $(m_1, m_2, \ldots, m_r)$-cycle is defined similarly. Lemmas 2.1 to 2.6 can be obtained by easy computation, and so their proofs are omitted.

Lemma 2.1.

(1) $(0,4,1,2)$ *is an exact resoluble 4-cycle with* $S = \{1,2,3,4\}$.

(2) $(0, 4k, 1, 4k-1, \ldots, k-1, 3k+1, k, 3k-1, k+1, 3k-2, \ldots, 2k-1, 2k)$ *is an exact resoluble* $4k$*-cycle with* $S = \{1,2,\ldots,4k\}$ *for* $k \ge 2$.

Lemma 2.2.

(1) $(0,4,1,3,2,7)$ *is a resoluble 6-cycle with* $S=\{1,2,3,4,5,7\}$.

(2) $(2,11,1,12,0,8)$ *is a resoluble 6-cycle with* $S=\{6,8,9,10,11,12\}$.

(3) $(0,4k+1,1,4k,\ldots,k-2,3k+3,k-1,3k+1,k,3k,\ldots,2k-1,2k+1,2k,4k+3)$ *is a resoluble* $(4k+2)$*-cycle with* $S=\{1,2,\ldots,4k+1,4k+3\}$ *for* $k\geq 2$.

(4) $(2k,2k+3,2k-1,2k+4,\ldots,k+2,3k+1,k+1,3k+3,k,3k+4,\ldots,1,4k+3,0,2k+1)$ *is a resoluble* $(4k+2)$*-cycle with* $S=\{1,3,4,\ldots,4k+3\}$ *for* $k\geq 2$.

Suppose that $C=(v_0,u_0,v_1,u_1,\ldots,v_{m-1},u_{m-1})$ is an exact resoluble $2m$-cycle with $\max\{v_0,v_1,\ldots,v_{m-1}\}<\min\{u_0,u_1,\ldots,u_{m-1}\}$. For the sake of notational convenience, denote $(v_0,u_0+j,v_1,u_1+j,\ldots,v_{m-1},u_{m-1}+j)$ by $(v_0,u_0,v_1,u_1,\ldots,v_{m-1},u_{m-1})\oplus j$, or simply $C\oplus j$. Let C^* be an exact resoluble $(m_1,m_2,\ldots,m_r)$-cycle and the notation $C^*\oplus j$ has a similar meaning. Moreover, we will assume that $C+j=(v_0+j,u_0+j,v_1+j,u_1+j,\ldots,v_{m-1}+j,u_{m-1}+j)$ and C^*+j is also defined similarly.

Lemma 2.3. *If* $(v_0,u_0,v_1,u_1,\ldots,v_{2m-1},u_{2m-1})$ *is an exact resoluble* $4m$*-cycle with* $S=\{k,k+1,\ldots,k+4m-1\}$ *for* $k\geq 1$, *then* $(v_0,u_0,v_1,u_1,\ldots,v_{2m-1},u_{2m-1})\oplus j$ *is also an exact resoluble* $4m$*-cycle with* $S=\{k+j,k+j+1,\ldots,k+4m+j-1\}$. *In particular, if* C^* *is an exact resoluble* $(m_1,m_2,\ldots,m_r)$*-cycle with* $S=\{k,k+1,\ldots,k+4m-1\}$, *then* $C^*\oplus j$ *is still an exact resoluble* $(m_1,m_2,\ldots,m_r)$*-cycle with* $S=\{k+j,k+j+1,\ldots,k+4m+j-1\}$.

There does not exist an exact resoluble $(4m+2)$-cycle for $m\geq 1$. Thus, we will amalgamate a $(4m_1+2)$-cycle and a $(4m_2+2)$-cycle such that they become exact and resoluble. We use the symbol $C_1\cup C_2$ to denote the union of two edge-disjoint graphs C_1, C_2. In Lemmas 2.4 and 2.5, assume that $(v_0,u_0,v_1,u_1,\ldots,v_{2m},u_{2m})$ and $(x_0,y_0,x_1,y_1,\ldots,x_{2k},y_{2k})$ are respectively $(4m+2)$ and $(4k+2)$-cycles with $\max\{v_0,v_1,\ldots,v_{2m}\}<\min\{u_0,u_1,\ldots,u_{2m}\}$ and $\max\{x_0,x_1,\ldots,x_{2k}\}<\min\{y_0,y_1,\ldots,y_{2k}\}$.

Lemma 2.4. *If* $(v_0,u_0,v_1,u_1,\ldots,v_{2m},u_{2m})\cup(x_0,y_0,x_1,y_1,\ldots,x_{2k},y_{2k})$ *is an exact resoluble* $(4m+2,4k+2)$*-cycle with* $S=\{p,p+1,\ldots,p+4m+4k+3\}$, *then* $(v_0,u_0,v_1,u_1,\ldots,v_{2m},u_{2m})\oplus j\cup(x_0,y_0,x_1,y_1,\ldots,x_{2k},y_{2k})\oplus j$ *is also an exact resoluble* $(4m+2,4k+2)$*-cycle with* $S=\{p+j,p+j+1,\ldots,p+4m+4k+j+3\}$.

Lemma 2.5. *If* $(v_0,u_0,v_1,u_1,\ldots,v_{2m},u_{2m})$ *is a* $(4m+2)$*-cycle with* $S=\{1,2,\ldots,4m+1,4m+3\}$, *and* $(x_0,y_0,x_1,y_1,\ldots,x_{2k},y_{2k})$ *is a* $(4k+2)$*-cycle with* $S=\{1,3,4,\ldots,4k+3\}$, *then* $(v_0,u_0,v_1,u_1,\ldots,v_{2m},u_{2m})\cup$

$(x_0, y_0, x_1, y_1, \ldots, x_{2k}, y_{2k}) \oplus (4m+1)$ *is an exact resoluble* $(4m+2, 4k+2)$*-cycle with* $S = \{1, 2, \ldots, 4m+4k+4\}$.

Lemma 2.6. *If* (a_i, b_i) *is an edge of* K_n *satisfying that* $\|a_i - b_i\| = i$, $1 \le i \le \lfloor n/2 \rfloor$, *then* $(a_i + c, b_i + c) \ne (a_j + d, b_j + d)$, *where all addition is taken mod* n *and* $1 \le i < j \le \lfloor n/2 \rfloor$.

Theorem 2.7. *Suppose that* $m_1, m_2, \ldots, m_r$ *are positive even integers with* $\sum_{i=1}^{r} m_i = 2^k$ *for* $k \ge 2$. *Then there exists an even* $(m_1, m_2, \ldots, m_r)$*-cycle system of* K_n *if and only if* $\sum_{i=1}^{r} m_i$ *divides* $|E(K_n)|$ *and* n *is odd.*

Proof: The necessity follows since each $(m_1, m_2, \ldots, m_r)$-cycle contains $\sum_{i=1}^{r} m_i$ edges, and each vertex in m_i-cycle $(1 \le i \le r)$ has even degree.

We begin with proving the following sufficiency: Let n be odd and let $m_1, m_2, \ldots, m_r$ be positive even integers with $\sum_{i=1}^{r} m_i = 2^k$ for $k \ge 2$. Obviously, $|E(K_n)| = n(n-1)/2$. Since n is odd and 2^k divides $n(n-1)/2$, we then have that $n = s \cdot 2^{k+1} + 1$ with $s \ge 1$ and $k \ge 2$. Unless specified otherwise, let $m_i \le m_{i+1}$, $1 \le i \le r-1$. We will split the proof into three cases, depending on whether $m_i = 4k_i$ for $1 \le i \le r$, $m_i = 4k_i + 2$ for $1 \le i \le r$, or there exists an integer t $(1 \le t \le r)$ with $m_i = 4k_i$ for $1 \le i \le t$ and $m_i = 4k_i + 2$ for $t+1 \le i \le r$. For convenience, let $C(i)$ denote the m_i-cycle for $1 \le i \le r$.

Case 1: Suppose that $m_i = 4k_i$ for $1 \le i \le r$.

We proceed depending on whether $m_i = 4$ or $m_i > 4$.

Subcase 1.1: Suppose that $m_1 = m_2 = \cdots = m_j = 4$ for some j with $1 \le j \le r$.

For $1 \le i \le j$, let $C(i) = (0, 4, 1, 2) \oplus 4(i-1)$. If $j < r$, then let $C(i) = (0, 4k_i, 1, 4k_i - 1, \ldots, k_i - 1, 3k_i + 1, k_i, 3k_i - 1, k_i + 1, 3k_i - 2, \ldots, 2k_i - 1, 2k_i) \oplus \sum_{a=1}^{i-1} m_a$ for $j+1 \le i \le r$.

Subcase 1.2: Suppose that $m_i \ge 8$ for $1 \le i \le r$.

For $1 \le i \le r$, let $C(i) = (0, 4k_i, 1, 4k_i - 1, \ldots, k_i - 1, 3k_i + 1, k_i, 3k_i - 1, k_i + 1, 3k_i - 2, \ldots, 2k_i - 1, 2k_i) \oplus \sum_{a=1}^{i-1} m_a$.

Case 2: Suppose that $m_i = 4k_i + 2$ for $1 \le i \le r$.

Note that r is even since $\sum_{i=1}^{r} m_i = 2^k$. We proceed depending on whether $m_i = 6$ or $m_i > 6$.

Subcase 2.1: Suppose that $m_1 = m_2 = \cdots = m_j = 6$ for some j with $1 \le j \le r$.

Suppose first that j is even, say $j = 2p$. Let $r - j = 2q$. For each i with $1 \le i \le p$, let $C(2i-1) = (0, 4, 1, 3, 2, 7) \oplus 12(i-1)$ and $C(2i) = (2, 11, 1, 12, 0, 8) \oplus 12(i-1)$. If $j < r$, then for each i with $1 \le i \le q$, let $C(j+2i-1) = (0, 4k_{j+2i-1} + 1, 1, 4k_{j+2i-1}, \ldots, k_{j+2i-1} - 2, 3k_{j+2i-1} + 3, k_{j+2i-1} - 1, 3k_{j+2i-1} + 1, k_{j+2i-1}, 3k_{j+2i-1}, \ldots, 2k_{j+2i-1} - 1, 2k_{j+2i-1} + 1, 2k_{j+2i-1}, 4k_{j+2i-1} + 3) \oplus \sum_{a=1}^{j+2i-2} m_a$ and $C(j+2i) = (2k_{j+2i}, 2k_{j+2i} +$

$3, 2k_{j+2i}-1, 2k_{j+2i}+4, \dots, k_{j+2i}+2, 3k_{j+2i}+1, k_{j+2i}+1, 3k_{j+2i}+3, k_{j+2i}, 3k_{j+2i}+4, \dots, 1, 4k_{j+2i}+3, 0, 2k_{j+2i}+1) \oplus ((\sum_{a=1}^{j+2i-1} m_a)-1)$.

The case when j is odd is similar and is omitted.

Subcase 2.2: Suppose that $m_j > 6$ for $1 \le j \le r$.

Let $r = 2p$. For each i with $1 \le i \le p$, let $C(2i-1) = (0, 4k_{2i-1}+1, 1, 4k_{2i-1}, \dots, k_{2i-1}-2, 3k_{2i-1}+3, k_{2i-1}-1, 3k_{2i-1}+1, k_{2i-1}, 3k_{2i-1}, \dots, 2k_{2i-1}-1, 2k_{2i-1}+1, 2k_{2i-1}, 4k_{2i-1}+3) \oplus \sum_{a=1}^{2i-2} m_a$ and $C(2i) = (2k_{2i}, 2k_{2i}+3, 2k_{2i}-1, 2k_{2i}+4, \dots, k_{2i}+2, 3k_{2i}+1, k_{2i}+1, 3k_{2i}+3, k_{2i}, 3k_{2i}+4, \dots, 1, 4k_{2i}+3, 0, 2k_{2i}+1) \oplus ((\sum_{a=1}^{2i-1} m_a)-1)$.

Note that in each subcase $C(i) \cup C(i+1)$ constitutes an exact resoluble $(4k_i+2, 4k_{i+1}+2)$-cycle for $i = 1, 3, \dots, r-1$.

Case 3: Suppose that for some t with $1 \le t < r$, we have that $m_i = 4k_i$ for $1 \le i \le t$ and $m_i = 4k_i + 2$ for $t+1 \le i \le r$.

Assume that $m_i \le m_{i+1}$ for $1 \le i \le t-1$ and that $m_j \le m_{j+1}$ for $t+1 \le j \le r-1$. Of course, $r-t$ is even since $\sum_{i=1}^{r} m_i = 2^k$. We proceed depending on whether $m_1 = 4$, $m_1 > 4$, $m_{t+1} = 6$ or $m_{t+1} > 6$.

Subcase 3.1: Suppose that $m_1 = m_2 = \cdots = m_p = 4$ for some p with $1 \le p \le t$ and that $m_{t+1} = m_{t+2} = \cdots = m_q = 6$ for some q with $t+1 \le q \le r$.

Suppose first that $q-t$ is even, say $q-t = 2w$. Since $r-t$ and $q-t$ are both even, we have that $r-q$ is even as well, say $r-q = 2z$. We now define the cycles $C(i)$ for $1 \le i \le r$. For $1 \le i \le p$, let $C(i) = (0, 4, 1, 2) \oplus 4(i-1)$. Next for $p+1 \le i \le t$, let $C(i) = (0, 4k_i, 1, 4k_i-1, \dots, k_i-1, 3k_i+1, k_i, 3k_i-1, k_i+1, 3k_i-2, \dots, 2k_i-1, 2k_i) \oplus \sum_{a=1}^{i-1} m_a$. To define the cycles $C(i)$ for $t+1 \le i \le q$, let $C(t+2j-1) = (0, 4, 1, 3, 2, 7) \oplus \sum_{a=1}^{t+2j-2} m_a$ and $C(t+2j) = (2, 11, 1, 12, 0, 8) \oplus \sum_{a=1}^{t+2j-1} m_a$ for $1 \le j \le w$. Finally, to define the cycles $C(i)$ for $q+1 \le i \le r$, let $C(q+2j-1) = (0, 4k_{q+2j-1}+1, 1, 4k_{q+2j-1}, \dots, k_{q+2j-1}-2, 3k_{q+2j-1}+3, k_{q+2j-1}-1, 3k_{q+2j-1}+1, k_{q+2j-1}, 3k_{q+2j-1}, \dots, 2k_{q+2j-1}-1, 2k_{q+2j-1}+1, 2k_{q+2j-1}, 4k_{q+2j-1}+3) \oplus \sum_{a=1}^{q+2j-2} m_a$ and $C(q+2j) = (2k_{q+2j}, 2k_{q+2j}+3, 2k_{q+2j}-1, 2k_{q+2j}+4, \dots, k_{q+2j}+2, 3k_{q+2j}+1, k_{q+2j}+1, 3k_{q+2j}+3, k_{q+2j}, 3k_{q+2j}+4, \dots, 1, 4k_{q+2j}+3, 0, 2k_{q+2j}+1) \oplus ((\sum_{a=1}^{q+2j-1} m_a) -1)$ for $1 \le j \le z$.

Analogously, $C(i) \cup C(i+1)$ constitutes an exact resoluble $(4k_i+2, 4k_{i+1}+2)$-cycle for $i = t+1, t+3, \dots, r-1$.

When $q-t$ is odd, the proof is similar and is omitted.

Subcase 3.2: Suppose that $m_i > 4$ for $1 \le i \le t$ and that $m_j > 6$ for $t+1 \le j \le r$ $(1 \le t < r)$.

Note that $r-t$ is even, say $r-t = 2z$. For $1 \le i \le t$, let $C(i) = (0, 4k_i, 1, 4k_i-1, \dots, k_i-1, 3k_i+1, k_i, 3k_i-1, k_i+1, 3k_i-2, \dots, 2k_i-$

$1, 2k_i) \oplus \sum_{a=1}^{i-1} m_a$. For $t+1 \le i \le r$, let $C(t+2j-1) = (0, 4k_{t+2j-1}+1, 1, 4k_{t+2j-1}, \ldots, k_{t+2j-1}-2, 3k_{t+2j-1}+3, k_{t+2j-1}-1, 3k_{t+2j-1}+1, k_{t+2j-1}, 3k_{t+2j-1}, \ldots, 2k_{t+2j-1}-1, 2k_{t+2j-1}+1, 2k_{t+2j-1}, 4k_{t+2j-1}+3) \oplus \sum_{a=1}^{t+2j-2} m_a$ and $C(t+2j) = (2k_{t+2j}, 2k_{t+2j}+3, 2k_{t+2j}-1, 2k_{t+2j}+4, \ldots, k_{t+2j}+2, 3k_{t+2j}+1, k_{t+2j}+1, 3k_{t+2j}+3, k_{t+2j}, 3k_{t+2j}+4, \ldots, 1, 4k_{t+2j}+3, 0, 2k_{t+2j}+1) \oplus ((\sum_{a=1}^{t+2j-1} m_a) - 1)$ for $1 \le i \le z$.

As before, $C(i) \cup C(i+1)$ constitutes an exact resoluble $(4k_i+2, 4k_{i+1}+2)$-cycle for $i = t+1, t+3, \ldots, r-1$.

Subcase 3.3: Suppose that $m_1 = m_2 = \cdots = m_p = 4$ for some p with $1 \le p \le t$ and that $m_j > 6$ for $t+1 \le j \le r$.

The proof is similar to Subcases 3.1 and 3.2 and is omitted.

Subcase 3.4: Suppose that $m_i > 4$ for $1 \le i \le t$ and that $m_{t+1} = m_{t+2} = \cdots = m_q = 6$ for some q with $t+1 \le q \le r$.

The proof is similar to Subcases 3.1 and 3.2 and is omitted.

Let $C^*(0)$ be the edge-disjoint union of the m_i-cycles $C(i)$ for $i = 1, 2, \ldots, r$. By routine calculation and Lemmas 2.1 to 2.5, it follows that $C^*(0)$ is an exact resoluble $(m_1, m_2, \ldots, m_r)$-cycle with edge lengths $S^*(0) = \{1, 2, \ldots, 2^k\}$. Let $C^*(i) = C^*(0) \oplus i \cdot 2^k$ for $i = 1, 2, \ldots, s-1$. By Lemma 2.3, each $C^*(i)$ for $1 \le i \le r$ is an exact resoluble $(m_1, m_2, \ldots, m_r)$-cycles with edge lengths $S^*(i) = \{1+i \cdot 2^k, 2+i \cdot 2^k, \ldots, (i+1) \cdot 2^k\}$. Let C_0 be the union of the edge-disjoint $(m_1, m_2, \ldots, m_r)$-cycles $C^*(0), C^*(1), \ldots, C^*(s-1)$ and observe that the set $S_0 = \cup_{i=0}^{s-1} S^*(i)$ is the set of edge lengths for C_0. Let $C_j = C_0 + j$ for $j = 1, 2, \ldots, n-1$. By Lemma 2.6, the set $\{C_j \mid 0 \le j \le n-1\}$ of subgraphs of K_n are pairwise mutually edge-disjoint, and since each C_i contain s $(m_1, m_2, \ldots, m_r)$-cycles and has $s \cdot 2^k$ edges, it follows that $\{C_j \mid 0 \le j \le n-1\}$ is an even $(m_1, m_2, \ldots, m_r)$-cycle system of K_n. □

Let $V(K_n) = Z_n$. An m-cycle system of K_n $(K_n - F)$ is a set $\mathcal{C}$ of m-cycles such that every edge of K_n $(K_n - F)$ belongs to exactly one m-cycle of $\mathcal{C}$. An m-cycle system of K_n $(K_n - F)$ is *cyclic* if the m-cycle $C = (v_0, v_1, \ldots, v_{m-1}) \in \mathcal{C}$ implies that $C+1 = (v_0+1, v_1+1, \ldots, v_{m-1}+1)$ is also in $\mathcal{C}$. For results on cyclic m-cycle systems of the complete graph, the interested reader can refer to [7], [10], [11], [12] and [13]. If $r = 1$, as an immediate consequence of Theorem 2.7, we have the following.

Corollary 2.8. *Let k be a positive integer (≥ 2). Then there exists a cyclic 2^k-cycle system of K_n if and only if 2^k divides $|E(K_n)|$ and n is odd.*

Corollary 2.9. *Let $m_1, m_2, \ldots, m_r$ and n be positive even integers with $4 \le m_i \le n$ for $1 \le i \le r$, $\sum_{i=1}^{r} m_i = 2^k$, and $2^{k+1} \mid (n-2)$. Then there exists an even $(m_1, m_2, \ldots, m_r)$-cycle system of $K_n - F$, where F is a 1-facter of K_n.*

Proof: Since $2^{k+1} \mid (n-2)$, it follows that $n = s \cdot 2^{k+1} + 2$ for some positive integer s. Letting $F = \{(0, s \cdot 2^k + 1), (1, s \cdot 2^k + 2), \ldots, (s \cdot 2^k, s \cdot 2^{k+1} + 1)\}$ and using the construction given in Theorem 2.7, we obtain the desired conclusion. □

By Corollary 2.9, we also have the following:

Corollary 2.10. *Let k be a positive integer (≥ 2) and $2^{k+1} \mid (n-2)$. Then there exists a cyclic 2^k-cycle system of $K_n - F$, where F is a 1-factor of K_n.*

Acknowledgements. The author is grateful to the referee for his considerable effort to help him rewrite this paper into a more readable form.

References

[1] P. Adams, D.E. Bryant, A. Khodkar, 3,5-cycle decompositions, *J. Combin. Des.* **6** (1998), 91–110.

[2] P. Adams, D.E. Bryant, A. Khodkar, On Alspach's conjecture with two even cycle lengths, *Discrete Math.* **223** (2000), 1-12.

[3] B. Alspach, Research problems, Problem 3, *Discrete Math.* **36** (1981), 333.

[4] B. Alspach and H. Gavlas, Cycle decompositions of K_n and $K_n - I$, *J. Combin. Theory Ser. B* **81** (2001), 77–99.

[5] D.E. Bryant, H.L. Fu, A. Khodkar, (m, n)-cycle systems, *J. Statist. Plann. Inference*, to appear.

[6] K. Heinrich, P. Horák and A. Rosa, On Alspach's conjecture, *Discrete Math.* **77** (1989), 97–121.

[7] A. Kotzig, Decompositions of a complete graph into $4k$-gons. (Russian) *Mat.-Fyz. Casopis Sloven. Akad. Vied* **15** (1965), 229–233.

[8] C.C. Lindner and C.A. Rodger, Decompositions into cycles II, Cycles systems, in: J.H. Dinitz, D.R. Stinson (Eds.) *Contemporary Design Theory: a Collection of Surveys*, Wiley, New York, pp. 325–369.

[9] A. Rosa, Alspach's conjecture is true for $n \leq 10$, *Math. Reports*, McMaster University, to appear.

[10] A. Rosa, A note on Steiner triple systems (Slovak), *Mat. Fyz. Casopis* **16** (1966), 285–290.

[11] A. Rosa, On cyclic decompositions of the complete graph into $(4m+2)$-gons, *Mat. Fyz. Casopis Sloven. Akad. Vied* **16** (1966), 349–352.

[12] A. Rosa, On cyclic decompositions of the complete graph into polygons with odd number of edges (Sloven), *Casopis Pest Mat.* **91** (1966), 53–56.

[13] A. Rosa, On decompositions of the complete graph into $4k$-gons. (Russian) *Mat. Casopis Sloven. Akad. Vied* **17** (1967), 242–246.

[14] M. Sajna, Cycle decompositions of K_n and $K_n - I$, Ph.D. Thesis, Simon Fraser University, July 1999.