Antimagic labelings of Mőbius grids

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Abstract

A d-antimagic labeling of a plane graph $G = (V, E, F)$ is a one-to-one mapping taking the vertices, edges and faces onto the integers $1, 2, \ldots, |V(G)| + |E(G)| + |F(G)|$ so that the s-sided face weights form an arithmetic progression of difference d. This paper describes d-antimagic labelings for Mőbius grids.

1 Introduction

Let $G = (V, E, F)$ be a finite connected plane graph without loops and multiple edges where $V(G)$, $E(G)$ and $F(G)$ are its vertex set, edge set and face set, respectively. General references for graph-theoretic notions are [14] and [15].

A labeling of *type* $(1, 1, 1)$ assigns labels from the set $\{1, 2, ..., |V(G)| + |E(G)| + |F(G)|\}$ to the vertices, edges and faces of plane graph G in such a way that each vertex, edge and face receives exactly one label and each number is used exactly once as a label.

A labeling of *type* $(1,1,0)$ is a bijection from the set $\{1,2,...,|V(G)|+|E(G)|\}$ to the vertices and edges of plane graph G.

If we label only vertices (respectively edges, faces) we call such a labeling a *vertex* (respectively *edge, face*) *labeling* and also the labeling is said to be of *type* $(1,0,0)$ (respectively *type*$(0,1,0)$, *type* $(0,0,1)$).

The *weight* of a face under a labeling is the sum of the labels (if present) carried by that face and the edges and vertices surrounding it.

A labeling of a plane graph G is called *d-antimagic* if for every number s the set of s-sided face weights is $W_s=\{a_s, a_s+d, a_s+2d, ..., a_s+(f_s-1)d\}$ for some integers a_s and d, $d \geq 0$, where f_s is the number of s-sided faces. We allow different sets W_s for different s.

Somewhat related types of antimagic labelings were defined by Hartsfield and Ringel in [9] and by Bodendiek and Walther in [6].

If $d=0$ then Ko-Wei Lih in [11] called such labelings *magic* (*face-magic*). Ko-Wei Lih [11] described 0-antimagic labelings of type $(1,1,0)$ for *wheels, friendship graphs* and *prisms.* 0-antimagic labelings of type $(1,1,1)$ for *grid* and *honeycomb* are given in [2] and [3].

If $d=1$ then d-antimagic labeling is called *consecutive.* Qu [13] and Kathiresan *et al.* [10] studied consecutive labelings for certain classes of plane graphs.

d-antimagic labelings of prisms and generalized Petersen graphs $P(n,2)$ can be found in [12] and [5]. Additional known results about face-antimagic labelings are given in [7] and [4].

Guy and Harary [8] defined the graph of Mőbius ladder M_k as follows: if $k=2n \geq 6$, then M_k consists of a circuit C_k of length k and the $\frac{k}{2}$ chords joining opposite pairs of vertices of C_k. They also showed that every Mőbius ladder M_k is minimally nonplanar, that is, its crossing number is one. In this paper we shall consider the Mőbius ladder more generally as a graph M_n^m.

Let $I=\{1,2,...,m\}$ and $J=\{1,2,...,n\}$ be index sets. For $n \geq 1$ and $m \geq 1$, let $P_{m+1} \times P_n$ be the Cartesian product of a path P_{m+1} on $m+1$ vertices with a path P_n on n vertices embedded in the plane. Let vertices $x_{i,j}$, $i \in I \cup \{m+1\}$ and $j \in J$ of $P_{m+1} \times P_n$, be labeled so that $x_{i,1}\ x_{i,2}\ x_{i,3}\ \dots\ x_{i,n-2}\ x_{i,n-1}\ x_{i,n}$ are vertices of the path $P_n(i)$, $i \in I \cup \{m+1\}$ and $x_{1,j}\ x_{2,j}\ x_{3,j}\ \dots\ x_{m-1,j}\ x_{m,j}\ x_{m+1,j}$ are vertices of the path $P_{m+1}(j)$, $j \in J$.

Now, for $n \geq 1$, $m \geq 1$, we denote by M_n^m (Mőbius grid) the graph with

$V(M_n^m) = V(P_{m+1} \times P_n) = \{x_{i,j} : i \in I \cup \{m+1\},\ j \in J\}$ and $E(M_n^m) = \{x_{i,j}x_{i,j+1} : i \in I \cup \{m+1\},\ j \in J - \{n\}\} \cup \{x_{i,j}x_{i+1,j} : i \in I, j \in J\} \cup \{x_{i,n}x_{m+2-i,1} : i \in I \cup \{m+1\}\}$.

If we consider the Mőbius grid M_n^m drawn in Euclidean space and not on the Euclidean plane then the face set $F(M_n^m)$ is unambiguous and contains mn 4-sided faces.

In this paper we consider the case when n is odd, $n \geq 3$, $m \geq 1$, and we describe d-antimagic labelings of type $(1,1,1)$ for M_n^m, $d \in \{1,2,4\}$.

2 Upper bounds for d

In this section we provide upper bounds for the parameter d for the vertex labeling and the edge labeling of the Mőbius grid M_n^m.

Theorem 1 *For every Mőbius grid M_n^m, $n \geq 2$, $m \geq 4$, there does not exist a d-antimagic vertex labeling with $d \geq 5$.*

Proof. Suppose that g_1 is a d-antimagic vertex labeling of M_n^m. If the vertices $x_{1,j}$ and $x_{m+1,j}$, $j \in J$, receive labels $1, 2, ..., 2n$ or, at the other extreme, labels $mn-n+1, mn-n+2, ..., mn+n$, or anything in between, then

$$\begin{aligned} 2mn(mn+1) + 2n^2 &\leq 2\sum_{j=1}^{n} g_1(x_{1,j}) + 2\sum_{j=1}^{n} g_1(x_{m+1,j}) + 4\sum_{i=2}^{m}\sum_{j=1}^{n} g_1(x_{i,j}) \\ &\leq 2mn(mn+2n+1) - 2n^2 \ . \end{aligned} \qquad (1)$$

The sum of all the weights of the 4-sided faces in M_n^m is

$$a + (a+d) + (a+2d) + ... + a + (mn-1)d = mna + d\binom{mn}{2}. \qquad (2)$$

The minimum possible weight of a 4-sided face is at least 10.

From (1) and (2) we get the following Diophantine equation

$$2\sum_{j=1}^{n} g_1(x_{1,j}) + 2\sum_{j=1}^{n} g_1(x_{m+1,j}) + 4\sum_{i=2}^{m}\sum_{j=1}^{n} g_1(x_{i,j}) = mna + d\binom{mn}{2}. \tag{3}$$

From (3) it follows that if $m \geq 8$, $n \geq 2$ then $d < 5$.

On the other hand, the maximum weight of a 4-sided face under a d-antimagic vertex labeling is no more than

$$\sum_{i=1}^{4}(|V(M_n^m)| + 1 - i) = 4mn + 4n - 6$$

and $a + (|F(M_n^m)| - 1)d \leq 4mn + 4n - 6$.

If $m \geq 4$ and $n \geq 2$, then $d < 5$. □

Theorem 2 *For every graph M_n^m, $n \geq 2$, $m \geq 4$, there is no d-antimagic edge labeling with $d \geq 9$.*

Proof. Suppose that g_2 is a d-antimagic edge labeling of M_n^m. Under the edge labeling g_2, the edges $x_{1,j}x_{1,j+1}$, $x_{m+1,j}x_{m+1,j+1}$, $j \in J - \{n\}$ and $x_{1,n}x_{m+1,1}$, $x_{m+1,n}x_{1,1}$ can receive the smallest labels $1, 2, ..., 2n$ or, at the other extreme, the largest labels $2mn - n + 1, 2mn - n + 2, ..., 2mn + n$, or anything in between.

The sum of all the edge labels used to calculate the weights of the 4-sided faces is equal to

$$\begin{aligned} 2mn(2mn+1) + n^2 &\leq \sum_{j=1}^{n-1} g_2(x_{1,j}x_{1,j+1}) + \sum_{j=1}^{n-1} g_2(x_{m+1,j}x_{m+1,j+1}) \\ &+ g_2(x_{1,n}x_{m+1,1}) + g_2(x_{m+1,n}x_{1,1}) + 2\sum_{i=2}^{m}\sum_{j=1}^{n-1} g_2(x_{i,j}x_{i,j+1}) \\ &+ 2\sum_{i=1}^{m}\sum_{j=1}^{n} g_2(x_{i,j}x_{i+1,j}) + 2\sum_{i=2}^{m} g_2(x_{i,n}x_{m+2-i,1}) \\ &\leq 2mn(2mn + 2n + 1) - n^2 . \end{aligned} \tag{4}$$

Combining (2) and (4), we get an inequality and by direct computation we deduce that if $m \geq 8$, $n \geq 2$ then $d < 9$.

Under a d-antimagic edge labeling g_2, the maximum 4-sided face weight is no more than

$$\sum_{i=1}^{4}(|E(M_n^m)| + 1 - i) = 8mn + 4n - 6$$

and from the inequality $a + (|F(M_n^m)| - 1)d \leq 8mn + 4n - 6$ we get that if $m \geq 4$, $n \geq 2$, then $d < 9$, which completes the proof.

□

Applying Theorem 1, Theorem 2 and the fact that under d-antimagic face labeling $F(M_n^m) \to \{1, 2, ..., |F(M_n^m)|\}$ the parameter d is no more than 1, we obtain the following theorem.

Theorem 3 *Let M_n^m, $n \geq 2$, $m \geq 4$, be a Mőbius grid which admits d_1-antimagic vertex labeling g_1, d_2-antimagic edge labeling g_2 and 1-antimagic face labeling g_3, $d_1 \geq 0$, $d_2 \geq 0$. If the labelings g_1, $|V(M_n^m)| + g_2$ and $|V(M_n^m)| + |E(M_n^m)| + g_3$ combine to a d-antimagic labeling of type $(1,1,1)$ then the parameter $d \leq 13$.*

3 The results

We have proved [1] that if n is odd, $n \geq 3$ and $m \geq 1$, then the Mőbius grid M_n^m has a magic (0-antimagic) labeling of type $(1,1,1)$.

In this paper we shall prove that for n odd, $n \geq 3$ and $m \geq 1$, the graph M_n^m is d-antimagic of type $(1,1,1)$ for $d = 1, 2, 4$.

If n is odd, $n \geq 3$ and $m \geq 1$, we construct a vertex labeling g_4 and an edge labeling g_5 of M_n^m in the following way.

$g_4(x_{i,j}) =$

$$\begin{cases} (m+1)\frac{j-1}{2}+\frac{i+1}{2} & \text{if } i \text{ and } j \text{ are odd, } m \geq 1 \\ (m+1)\frac{j}{2}+\frac{2-i}{2} & \text{if } i \text{ and } j \text{ are even, } m \geq 1 \\ \frac{m+1}{2}(2n-j+2)-\frac{m}{2}+\frac{1-i}{2} & \text{if } i \text{ is odd, } j \text{ is even and } m \text{ is even} \\ \frac{m+1}{2}(2n-j+1)+\frac{1-i}{2} & \text{if } i \text{ is odd, } j \text{ is even and } m \text{ is odd} \\ (m+1)(n+\frac{1-j}{2})+\frac{i-m}{2} & \text{if } i \text{ is even, } j \text{ is odd and } m \text{ is even} \\ \frac{m+1}{2}(2n-j)+\frac{i}{2} & \text{if } i \text{ is even, } j \text{ is odd and } m \text{ is odd.} \end{cases}$$

$$g_5(x_{i,n}x_{m+2-i,1}) = \begin{cases} \frac{i+1}{2} & \text{if } i \text{ is odd, } m \geq 1 \\ (m+1)n+\frac{i-m}{2} & \text{if } i \text{ is even and } m \text{ is even} \\ (m+1)n+\frac{i-m-1}{2} & \text{if } i \text{ is even and } m \text{ is odd.} \end{cases}$$

$g_5(x_{i,j}x_{i+1,j}) =$

$$\begin{cases} (m+1)n+\frac{n+1}{2}(m-i)+\frac{j+1}{2} & \text{if } j \text{ is odd, } i \in I, \, m \geq 1 \\ 2mn+n-\frac{n-1}{2}i+\frac{j}{2} & \text{if } j \text{ is even, } i \in I, \, m \geq 1. \end{cases}$$

If m is even then

$$g_5(x_{i,j}x_{i,j+1}) = \begin{cases} (m+1)\frac{n-j}{2}+\frac{i+1}{2} & \text{if } i \text{ and } j \text{ are odd} \\ (m+1)\frac{n-j+1}{2}+\frac{i-m}{2} & \text{if } i \text{ and } j \text{ are even} \\ (m+1)\frac{n+j}{2}+\frac{i-m}{2} & \text{if } i \text{ is even and } j \text{ is odd} \\ (m+1)\frac{n+j-1}{2}+\frac{i+1}{2} & \text{if } i \text{ is odd and } j \text{ is even.} \end{cases}$$

If m is odd then

$$g_5(x_{i,j}x_{i,j+1}) = \begin{cases} \frac{m+1}{2}(n-j)+\frac{i+1}{2} & \text{if } i \text{ is odd and } j \in J-\{n\} \\ \frac{m+1}{2}(n+j-1)+\frac{i}{2} & \text{if } i \text{ is even and } j \in J-\{n\}. \end{cases}$$

Theorem 4 *For n odd, $n \geq 3$ and $m \geq 1$, the M_n^m has a 0-antimagic vertex labeling.*

Proof. Label the vertices of M_n^m by the labeling g_4. It is easy to verify that the vertex labeling g_4 uses each integer $1, 2, \ldots, |V(M_n^m)|$ exactly once. By direct computation we obtain that the common weight for all the 4-sided faces is $2n(m+1)+3$ if m is even and $2n(m+1)+2$ if m is odd. □

Theorem 5 *For n odd, $n \geq 3$ and $m \geq 1$, the M_n^m has a 1-antimagic edge labeling.*

Proof.

The labeling g_5 is a bijection from $E(M_n^m)$ onto the set $\{1, 2, \ldots, |E(M_n^m)|\}$. If m is even then the set of weights of the 4-sided faces of M_n^m consists of the consecutive integers $\frac{7mn}{2} + 3n + 3, \frac{7mn}{2} + 3n + 4, \ldots, \frac{7mn}{2} + 3n + m + 2, \frac{7mn}{2} + 3n + m + 3, \ldots, \frac{9mn}{2} + 3n + 1, \frac{9mn}{2} + 3n + 2$.

If m is odd then the set of weights of the 4-sided faces of M_n^m receive the consecutive integers $\frac{7mn+5}{2} + 3n, \frac{7mn+5}{2} + 3n + 1, \ldots, \frac{7mn+5}{2} + 3n + m - 1, \frac{7mn+5}{2} + 3n + m, \ldots, \frac{9mn+5}{2} + 3n - 2, \frac{9mn+5}{2} + 3n - 1$.

□

Theorem 6 *If n is odd, $n \geq 3$ and $m \geq 1$, then the Mőbius grid M_n^m has a 2-antimagic labeling of type* $(1, 1, 1)$.

Proof.

Label the vertices and the edges of M_n^m by g_4 and $|V(M_n^m)| + g_5$, respectively. From Theorem 4 and Theorem 5, it follows that the obtained labeling successively assumes values $1, 2, \ldots, |V(M_n^m)| + |E(M_n^m)|$ and the weights of the 4-sided faces constitute an arithmetic sequence of difference 1.

If we complete the face labeling with values in the set $\{|V(M_n^m)| + |E(M_n^m)| + 1, |V(M_n^m)| + |E(M_n^m)| + 2, ..., |V(M_n^m)| + |E(M_n^m)| + |F(M_n^m)|\}$ then the resulting labeling of type $(1, 1, 1)$ can be

(i) 0-antimagic with the common weight for all the 4-sided faces equal to $\frac{27mn}{2} + 11n + 6$ (if m is even) or $\frac{27mn+9}{2} + 11n$ (if m is odd) or

(ii) 2-antimagic with the weights of the 4-sided faces in the set $\{w : w = \frac{25mn}{2} + 11n + 6 + k, 1 \leq k \leq mn, m \text{ even}\}$ or in the set $\{w : w = \frac{25mn+5}{2} + 11n + 2 + k, 1 \leq k \leq mn, m \text{ odd}\}$.

□

Theorem 7 *If n is odd, $n \geq 3$ and $m \geq 1$ then the Mőbius grid M_n^m has an 1-antimagic labeling of type* $(1, 1, 1)$.

Proof.

Let us distinguish two cases.

Case 1. If m is odd.

Label the vertices of M_n^m by the labeling g_4 and the edges by the labeling $|V(M_n^m)|+g_5$. We obtain a labeling of type $(1,1,0)$ and from Theorem 4 and Theorem 5, it follows that the set of the weights of the 4-sided faces is equal to $W = \{\frac{19mn+7}{2}+9n+1, \frac{19mn+7}{2}+9n+2, \frac{19mn+7}{2}+9n+3, \dots, \frac{19mn+7}{2}+9n+mn-1, \frac{19mn+7}{2}+9n+mn\}$. Denote by $w_k = \frac{19mn+7}{2}+9n+k$, $k=1,2,\dots,mn$, the elements of the set W.

Define the face labeling $g_6 : F(M_n^m) \to \{|V(M_n^m)|+|E(M_n^m)|+1, |V(M_n^m)|+|E(M_n^m)|+2, \dots, |V(M_n^m)|+|E(M_n^m)|+|F(M_n^m)|\}$ such that for $1 \le k \le mn$

$$g_6(f_k) = \begin{cases} |V(M_n^m)|+|E(M_n^m)|+\frac{mn+2-k}{2} & \text{if } k \text{ is odd} \\ |V(M_n^m)|+|E(M_n^m)|+mn+1-\frac{k}{2} & \text{if } k \text{ is even}. \end{cases}$$

Now, we label the faces of M_n^m by the labeling g_6 in such a way that the face label $g_6(f_k)$ is given to the 4-sided face which has weight w_k, for all $1 \le k \le mn$. It is easy to verify that under the resulting labeling of type $(1,1,1)$, the set of weights of the 4-sided faces is equal to

$\{w_k + g_6(f_k) : 1 \le k \le mn\} = \{|V(M_n^m)| + |E(M_n^m)| + 10mn + 9n + 5, |V(M_n^m)|+|E(M_n^m)|+10mn+9n+6, \dots, |V(M_n^m)|+|E(M_n^m)|+11mn+9n+4\}$.

Case 2. If m is even.

Label the vertices and edges of M_n^m as follows.

$g_7(x_{i,j}) = 2g_4(x_{i,j})$ for $i \in I \cup \{m+1\}$ and $j \in J$.

$g_8(x_{i,n}x_{m+2-i,1}) = 2g_5(x_{i,n}x_{m+2-i,1}) - 1$ for $i \in I \cup \{m+1\}$.

$g_8(x_{i,j}x_{i,j+1}) = 2g_5(x_{i,j}x_{i,j+1}) - 1$ for $i \in I \cup \{m+1\}$ and $j \in J - \{n\}$.

$g_8(x_{i,j}x_{i+1,j}) = 3mn + n(2-i) + j$ for $i \in I$ and $j \in J$.

From Theorem 4, we know that the vertex labeling g_4 is 0-antimagic with common weight $2n(m+1)+3$. Under the labeling g_7, the common weight for all the 4-sided faces is $4n(m+1)+6$. It can be seen that the labelings g_7 and g_8 combine to give a labeling of type $(1,1,0)$ and weights of the 4-sided faces form two sets

$W = \{w_i : w_i = 7mn + 6n - m + 1 + 2i, 1 \le i \le m\}$ and

$U = \{u_k : u_k = 6mn + 6n + m + 1 + 2k, 1 \le k \le m(n-1)\}$

i.e., the weights of the 4-sided faces constitute two arithmetic progressions with difference 2.

Define two sets of labels on the faces

$g_9 : F_1(M_n^m) \to \{|V(M_n^m)| + |E(M_n^m)| + 1, |V(M_n^m)| + |E(M_n^m)| + 2, \ldots, |V(M_n^m)| + |E(M_n^m)| + m\}$ and

$g_{10} : F_2(M_n^m) \to \{|V(M_n^m)| + |E(M_n^m)| + m + 1, |V(M_n^m)| + |E(M_n^m)| + m + 2, \ldots, |V(M_n^m)| + |E(M_n^m)| + |F(M_n^m)|\}$

as follows:

$F_1(M_n^m) \cup F_2(M_n^m) = F(M_n^m)$

$g_9(f_i) = |V(M_n^m)| + |E(M_n^m)| + m + 1 - i$ for $f_i \in F_1(M_n^m)$ and $1 \le i \le m$

$g_{10}(f_k) = |V(M_n^m)| + |E(M_n^m)| + |F(M_n^m)| + 1 - k$ for $f_k \in F_2(M_n^m)$ and $1 \le k \le m(n-1)$.

Label the faces of M_n^m by the labelings g_9 and g_{10} so that the face label $g_9(f_i)$ ($g_{10}(f_k)$) is assigned to the 4-sided face with w_i (u_k) for all $1 \le i \le m$ ($1 \le k \le m(n-1)$).

The labelings g_7, g_8 and g_9, g_{10} combine to a labeling of type $(1,1,1)$ and, under this labeling, the set of weights of the 4-sided faces is equal to

$\{w_i + g_9(f_i) : 1 \le i \le m\} \cup \{u_k + g_{10}(f_k) : 1 \le k \le m(n-1)\} =$

$\{|V(M_n^m)| + |E(M_n^m)| + 7mn + 6n + 3, |V(M_n^m)| + |E(M_n^m)| + 7mn + 6n + 4, \ldots, |V(M_n^m)| + |E(M_n^m)| + 7mn + 6n + m + 2\} \cup$

$\{|V(M_n^m)| + |E(M_n^m)| + 7mn + 6n + m + 3, |V(M_n^m)| + |E(M_n^m)| + 7mn + 6n + m + 4, \ldots, |V(M_n^m)| + |E(M_n^m)| + |F(M_n^m)| + 7mn + 6n + 2\}$.

Thus M_n^m has a 1-antimagic labeling of type $(1,1,1)$.

□

Theorem 8 *If n is odd, $n \ge 3$ and $m \ge 1$, then the Mőbius grid M_n^m has a 4-antimagic labeling of type* $(1,1,1)$.

Proof.

Define the new labelings g_{11} and g_{12} in such a way that

$g_{11}(u) = 2g_4(u) - 1$ for every vertex $u \in V(M_n^m)$ and

$g_{12}(e) = 2g_5(e)$ for every edge $e \in E(M_n^m)$.

The labeling g_{11} uses the values $1, 3, 5, \dots, 2mn + 2n - 1$ and the labeling g_{12} uses the values $2, 4, 6, \dots, 4mn + 2n$.

From Theorem 4, it follows that under the labeling g_{11} all the 4-sided faces have the same weight. In light of Theorem 5, we can see that under the labeling g_{12} the weights of the 4-sided faces constitute an arithmetic progression of difference 2.

Now, we label the vertices and the edges of M_n^m by g_{11} and g_{12}, respectively. We are able to arrange the face values $2mn + 2n + 1, 2mn + 2n + 3, 2mn + 2n + 5, \dots, 4mn + 2n - 3, 4mn + 2n - 1$ to the faces of M_n^m in such a way that the resulting labeling is a labeling of type $(1, 1, 1)$ and

(i) the weights of all the 4-sided faces are the same or

(ii) the weights of all the 4-sided faces constitute an arithmetic sequence of difference 4.

Thus we arrive at the desired result.

□

4 Conclusion

In this paper, we have studied d-antimagic labelings for Mőbius grid M_n^m. We have shown that for $n \geq 3$ (n odd), $m \geq 1$ and $d \in \{0, 1, 2, 4\}$ there exist d-antimagic labelings of type $(1, 1, 1)$.

We conclude with the following open problem.

Open Problem 1 *Find other possible values of the parameter d and corresponding d-antimagic labelings of type* $(1, 1, 1)$ *for* M_n^m.

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