On the Geodetic Covers and Geodetic Bases of the Composition $G[K_m]$

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ABSTRACT

Given a connected graph G and two vertices u and v in G, $I_G[u,v]$ denotes the closed interval consisting of u, v and all vertices lying on some $u-v$ geodesic of G. A subset S of $V(G)$ is called a geodetic cover of G if $I_G[S] = V(G)$, where $I_G[S] = \cup_{u,v \in S} I_G[u,v]$. A geodetic cover of minimum cardinality is called a geodetic basis. In this paper, we give the geodetic covers and geodetic bases of the composition of a connected graph and a complete graph.

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1 Introduction

Let G be a connected simple graph and $u, v \in V(G)$, where $V(G)$ is the vertex set of G. The *distance* $d_G(u,v)$ between u and v in G is the length of a shortest path $P(u,v)$ in G. Any $u-v$ path of length $d_G(u,v)$ is called a $u-v$ *geodesic*. For any two vertices u and v of G, the set $I_G[u,v]$ is the closed interval consisting of u, v and all vertices lying on some $u-v$ geodesic of G. For convenience, we write $I_G(u,v) = I_G[u,v] \backslash \{u,v\}$.

A subset C of $V(G)$ is *convex* if for every two vertices $u, v \in C$, $I_G[u,v] \subseteq C$. For any subset S of $V(G)$, the *closure of* S is $I_G[S] = \bigcup_{u,v \in S} I_G[u,v]$. It can easily be verified that $S \subseteq I_G[S]$ and that $I_G[S] = S$ if and only if S is convex. A *geodetic cover* of G is a set $S \subseteq V(G)$ such that every vertex of G is contained in $I_G[S]$, that is, $I_G[S] = V(G)$. A *geodetic number* $g(G)$ of G is the minimum order of its geodetic covers, and any cover of order $g(G)$ is called a *geodetic basis*. These concepts were introduced in [9]. It was further investigated in [1], [2], [4], [5], [6] and [7].

A vertex in a graph G is an *extreme vertex* if the subgraph induced by its neighbors is complete. The set of extreme vertices in G is denoted by $Ex(G)$. For convention, we set $Ex(G) = V(K_1)$ if $G = K_1$. For other graph theoretic terms, which are assumed here, readers are advised to see [8].

A subset S of G is a *hull set* if there exists a non-negative integer p such that $I_G^p[S] = V(G)$, where $I_G^0[S] = S$, $I_G^1[S] = I_G[S]$, and $I_G^p[S] = I_G[I_G^{p-1}[S]]$. The smallest cardinality of a hull set in G is the *hull number* $h(G)$ of G. It is easily seen that every geodetic cover is a hull set; hence, $h(G) \leq g(G)$ for any connected graph G. In [3], the authors determined the hull number of the composition of any two connected graphs. Although geodetic covers are hull sets, the authors observed that it is quite difficult to determine the geodetic number of the composition of any two connected graphs. It is observed further that for non-extreme geodesic graphs (graphs where the geodetic bases and minimum hull sets are not the extreme vertices only), the sequence of iterates $\{I_G^p[S]\}_{p\geq 0}$, where S is any minimum hull set in G, cannot generally be used to obtain the geodetic number of the composition of graphs. Results would show that a much more complicated expression for the geodetic number of the composition

$G[K_m]$ is obtained in this paper than the one obtained by the authors for the hull number of the same graph. For some particular cases, one can easily see the nice relationships between the geodetic number and the hull number of the composition $G[K_m]$.

2 Results

The *composition* of two graphs G and H, denoted by $G[H]$, is the graph with $V(G[H]) = V(G) \times V(H)$ and $(u,v)(u',v') \in E(G[H])$ if and only if either $uu' \in E(G)$ or $u = u'$ and $vv' \in E(H)$.

In [3], the authors obtained the following results.

Theorem 2.1 *Let G be a connected graph and K_m the complete graph of order m. Then $h(G[K_m]) = h(G) + (m-1)|Ex(G)|$.*

Example 2.2 Consider $G = C_5$ and $C_5[K_2]$ as shown in Figure 1. Suppose $A = \{a, c, e\}$. Observe that A is a minimum hull set and a geodetic basis of C_5. By Theorem 2.1, $h(C_5[K_2]) = 3$. Now, it can be shown that $g(C_5[K_2]) = 5$ (see Figure 3).

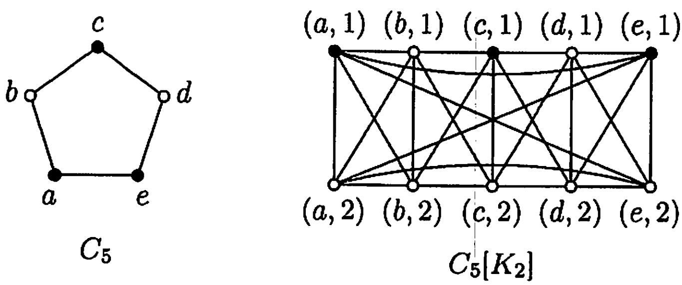

Figure 1: The graphs C_5 and $C_5[K_2]$ with $h(C_5) = g(C_5) = 3$ and $h(C_5[K_2]) = 3$.

The following observation follows from the above example.

Observation 2.3 *There exist a graph G and a positive integer m such that $g(G[K_m]) \neq g(G) + (m-1)|Ex(G)|$.*

We shall determine the geodetic number of $G[K_m]$. We start with a simple result characterizing the composition $G[K_m]$ with geodetic number equal to 2.

Theorem 2.4 *Let G be a connected graph and K_m the complete graph of order m. Then $g(G[K_m]) = 2$ if and only if either $g(G) = 2$ and $m = 1$ or $G = K_1$ and $m = 2$.*

Proof. Suppose $g(G[K_m]) = 2$, say $T = \{(v_1, a), (v_2, b)\}$ is a geodetic basis of $G[K_m]$. Consider the following cases:

Case 1. If $v_1 = v_2$, then $a \neq b$. This implies that $m \neq 1$. Since $I_{G[K_m]}[T] = T = V(G[K_m])$, $G[K_m] = K_2$. Thus, $G = K_1$ and $m = 2$.

Case 2. Let $v_1 \neq v_2$. Further, assume that $m \neq 1$. Then there exist $c \in V(K_m)$ such that $c \neq a$. Since $d_{G[K_m]}((v_1, a), (v_2, b)) = d_{G[K_m]}((v_1, a), (v_2, c))$, it follows that $(v_2, c) \notin I_{G[K_m]}[(v_1, a), (v_2, b)]$. This contradicts the assumption that $I_{G[K_m]}[T] = V(G[K_m])$. Thus, $m = 1$. Hence, $G[K_m] \cong G$. Since $g(G[K_m]) = 2$, $g(G) = 2$.

Conversely, suppose either $g(G) = 2$ and $m = 1$ or $G = K_1$ and $m = 2$. Then $g(G[K_m]) = 2$. ■

Let G be a connected graph and A a geodetic cover of G. A subset B of A with $Ex(G) \subseteq B$ satisfies the *geodetic interiority condition* (GIC) *with respect to* A if for every $a \in B \backslash Ex(G)$, there exist $x, y \in A$ such that $a \in I_G(x, y) = I_G[x, y] \backslash \{x, y\}$. A subset B of A in G satisfying GIC with respect to A is termed as a *GIC set with respect to A*. A GIC set with respect to A of maximum cardinality is called a *maximum GIC set with respect to A*. A geodetic cover of G that is a GIC set with respect to itself is called a *GIC set in G*.

Example 2.5 Consider the following graph G shown in Figure 2. Let $A = \{a, b, c, d, f\}$. Clearly, A is a geodetic cover of G. Moreover, A is a GIC set in G.

A GIC set B with respect to A is not necessarily a geodetic cover of G. A geodetic cover of G is not necessarily a GIC set in G. Clearly, $V(G)$ is a GIC set in G. Further, if $Ex(G)$ is a geodetic cover of G, then $Ex(G)$ is a GIC set in G.

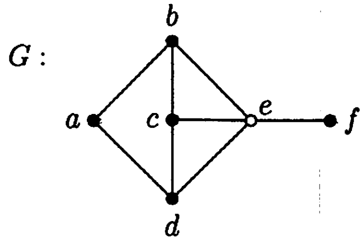

Figure 2: A graph G illustrating a GIC set in G.

Theorem 2.6 *Let G be a connected graph, A a geodetic cover of G and B a GIC set with respect to A. Then*

$$T = Ex(G[K_m]) \cup [(B \backslash Ex(G)) \times \{v_0\}] \cup [(A \backslash B) \times V(K_m)] \quad (1)$$

is a geodetic cover of $G[K_m]$ for every $v_0 \in V(K_m)$.

Proof. Let A be a geodetic cover of G, B a GIC set with respect to A and $v_0 \in V(K_m)$. Suppose $(x, v) \in V(G[K_m])$ and (1) holds. If $x \in Ex(G)$, then $(x, v) \in Ex(G[K_m]) \subseteq T \subseteq I_{G[K_m]}[T]$. If $x \in B \backslash Ex(G)$, then by the interiority condition, there exist $w, z \in A$ such that $x \in I_G(w, z)$. Let

$$P(w, z) = [w, w_1, w_2, \ldots, w_k, z]$$

be a $w - z$ geodesic, where $x = w_r$ $(1 \leq r \leq k)$. This implies that

$$P^* = [(w, v_0), (w_1, v_0), \ldots, (w_{r-1}, v_0), (x, v), (w_{r+1}, v_0), \ldots, (z, v_0)]$$

is a $(w, v_0) - (z, v_0)$ geodesic. Hence,

$$(x, v) \in I_{G[K_m]}[(w, v_0), (z, v_0)] \backslash \{(w, v_0), (z, v_0)\}.$$

Since $(w, v_0), (z, v_0) \in T$, $(x, v) \in I_{G[K_m]}[T]$. If $x \in A \backslash B$, then by the definition of T, $(x, v) \in T \subseteq I_{G[K_m]}[T]$. Now, let $x \in I_G[A] \backslash A$. Since A is a geodetic cover of G, there exist $p, q \in A$ such that $x \in I_G[p, q]$. Let $P(p, q) = [p, p_1, p_2, \ldots, p_k, q]$, where $x = p_r$ for some $1 \leq r \leq k$, be a $p - q$ geodesic. Then

$$P'((p, v), (q, v)) = [(p, v), (p_1, v), \ldots, (p_k, v), (q, v)]$$

is a $(p,v)-(q,v)$ geodesic containing vertex (x,v). Observe that $(p,v),(q,v) \in T$ because $p,q \in A$. Thus,

$$(x,v) \in I_{G[K_m]}[(p,v),(q,v)] \subseteq I_{G[K_m]}[T]$$

In any case, we have $V(G[K_m]) \subseteq I_{G[K_m]}[T]$. This shows that $I_{G[K_m]}[T] = V(G[K_m])$. ■

Corollary 2.7 *Let G be a connected graph and A a geodetic cover of G. Then $A \times V(K_m)$ is a geodetic cover of $G[K_m]$.*

Proof. Let A be a geodetic cover of G. Since $B = Ex(G)$ is a *GIC* set with respect to A, the result now follows from Theorem 2.6. ■

Corollary 2.8 *Let G be a connected graph and A a geodetic cover of G. If A is a GIC set in G, then $Ex(G[K_m]) \cup ((A \backslash Ex(G)) \times \{v_0\})$ is a geodetic cover of $G[K_m]$ for every $v_0 \in V(K_m)$.*

Proof. Use Theorem 2.6, where $B = A$. ■

Lemma 2.9 [6] *Every geodetic cover of a graph contains its extreme vertices.*

In what follows, if $T \subseteq V(G[K_m])$, then

$$T_f = \{x \in V(G) : (x,a) \in T \text{ for some } a \in V(K_m)\}.$$

Lemma 2.10 *Let G be a connected graph and T a geodetic cover of $G[K_m]$. Then $T = Ex(G[K_m]) \cup D$, where $Ex(G[K_m]) \cap D = \emptyset$, and T_f is a geodetic cover of G.*

Proof. Let T be a geodetic cover of $G[K_m]$. Then by Lemma 2.9, $T = Ex(G[K_m]) \cup D$, where $Ex(G[K_m]) \cap D = \emptyset$.

Pick $u \in V(G)$ and $v \in V(K_m)$. If $u \in T_f$, then $u \in I_G[T_f]$. If $u \notin T_f$, then $t = (u,v) \notin T$ and there exist $t_1 = (w,a), t_2 = (x,b) \in T$ such that $t \in I_{G[K_m]}[t_1,t_2]$. Since $u \notin T_f$, $u \neq w$ and $u \neq x$. This implies that $d_G(w,x) = d_{G[K_m]}(t_1,t_2) \neq 1$. Hence, $u \in I_G[w,x]$. Since $w,x \in T_f$, it follows that $u \in I_G[T_f]$. Therefore, T_f is a geodetic cover of G. ■

Theorem 2.11 *Let G be a connected graph and T a geodetic cover of $G[K_m]$. Then T_f is a geodetic cover of G and there exists a GIC set B with respect to T_f such that*

$$\begin{aligned} T &= Ex(G[K_m]) \cup [(T_f \backslash B) \times V(K_m)] \cup \\ &\quad \{(u,v) \in T : u \in B \backslash Ex(G), v \in V(K_m)\}. \end{aligned}$$

Proof. Suppose T is a geodetic cover of $G[K_m]$. Then T_f is a geodetic cover of G by Lemma 2.10. Note that $T_f \times V(K_m)$ is a geodetic cover of $G[K_m]$ by Corollary 2.7.

If $T = T_f \times V(K_m)$, then choose $B = Ex(G)$ and B is a *GIC* set with respect to T_f. If $T \neq T_f \times V(K_m)$, then $T_f \times V(K_m)$ is a proper subset of T, that is, $T_f \times V(K_m) \subsetneq T$. Observe that $T_f \neq Ex(G)$. Thus, there exists $s \in T_f \backslash Ex(G)$ and there exists $t \in V(K_m)$ such that $(s,t) \notin T$. Also, since $s \in T_f \backslash Ex(G) \subseteq T_f$, there exists $t^* \in V(K_m)$ such that $(s,t^*) \in T$. Let $S = \{(s',t') \in [(T_f \backslash Ex(G)) \times V(K_m)] \backslash T\}$. Then $S \neq \emptyset$. Define $B = Ex(G) \cup S_f$, where $S_f = \{x : (x,v) \in S$ for some $v \in V(K_m)\}$. Note that $S_f \neq \emptyset$. Then $Ex(G) \subsetneq B \subseteq T_f$. Suppose there exists $x \in B \backslash Ex(G) = S_f$ such that $x \notin I_G(y,z)$ for all $y,z \in T_f$. Then $(x,v) \in T$ for all $v \in V(K_m)$. This is contrary to the definition of S_f. Hence, B is a *GIC* set with respect to T_f.

By the definition of B, we get the desired expression for T. ■

The following is a consequence of Theorem 2.6 and Theorem 2.11.

Corollary 2.12 *Let G be a connected graph and K_m the complete graph of order $m \geq 2$. Then*

$$g(G[K_m]) = (m-1)|Ex(G)| + min\ L,$$

where $L = \{m|A| - (m-1)|B| : A$ is a geodetic cover in G and B is a maximum GIC set with respect to $A\}$.

The next results follow from Corollary 2.12.

Corollary 2.13 *Let G be a connected graph and K_m the complete graph of order $m \geq 2$. If G has no extreme vertices, then*

$$g(G[K_m]) = \min L,$$

where $L = \{m|A|-(m-1)|B| : A$ is a geodetic cover of G and B is a maximum GIC set with respect to $A\}$.

Corollary 2.14 *Let G be a connected graph and A a geodetic cover of G. Then $g(G[K_m]) \leq (m-1)|Ex(G)| + m|A|$. Further, if A is a GIC set in G, then $g(G[K_m]) \leq (m-1)|Ex(G)| + |A|$.*

Proof. Let B be a maximum GIC set with respect to A (B may be $\emptyset$). Then $m|A| - (m-1)|B| \in L$. Hence, by Corollary 2.12, $g(G[K_m]) \leq (m-1)|Ex(G)| + m|A| - (m-1)|B| \leq (m-1)|Ex(G)| + m|A|$.

Now, suppose A is a GIC set in G. Then $m|A| - (m-1)|A| = |A| \in L$. By Corollary 2.12, $g(G[K_m]) \leq (m-1)|Ex(G)| + |A|$. ■

The following examples show that strict inequality in Corollary 2.14 can happen.

Example 2.15 As in Figure 1, consider $G = C_5$ and $m = 2$. Clearly, $S = \{a, c, e\}$ is a minimum hull set in C_5, where $I^1_{C_5}[S] = I_{C_5}[S] = V(C_5)$. Hence, $I^0_{C_5}[S] = S$ is a geodetic cover of C_5. Further, S is not a GIC set in G. Also, it can be verified that (see Figure 3) $T = \{(a,2), (b,1), (c,2), (d,1), (e,2)\}$ is a geodetic basis of $C_5[K_2]$. Hence, by Corollary 2.14, $g(C_5[K_2]) = |T| = 5 < 2|S| = 6$.

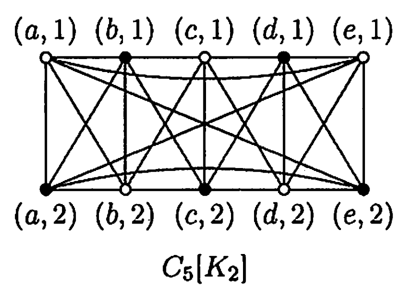

Figure 3: The graph $C_5[K_2]$ with $g(C_5[K_2]) = 5$.

Observation 2.16 *There exist a graph G and a positive integer m such that if S is a minimum hull set in G and p is the smallest non-negative integer with $I_G^p[S] = V(G)$, the iterate $I_G^{p-1}[S]$ is not a GIC set in G and $g(G[K_m])$ is not equal to $(m-1)|Ex(G)| + m|I_G^{p-1}[S]|$.*

Example 2.17 Consider the graph H in Figure 4. Observe that $S' = \{t, u\}$ is a minimum hull set in H. Now, $I_H^2[S'] = \{t, u, v, w, x, z\}$ and $I_H^3[S'] = V(H)$. Thus, $I_H^2[S'] = T'$ is a geodetic cover (but is not a geodetic basis) of H. Moreover, T' is a GIC set in H. The other minimum hull set in H is $S^* = \{u, z\}$, where, $I_H[S^*] = \{u, v, w, x, y, z\}$ and $I_H^2[S^*] = V(H)$. Note that $I_H[S^*] = T^*$ is a geodetic cover (not basis) of H and is a GIC set in H. Since $|T'| = |T^*| = |T|$, we have $(m-1)|Ex(H)| + |T| = 7$ for $m = 2$. In Figure 5, it can be verified that $\{(u,1), (u,2), (v,1), (x,1), (z,1)\}$ is a geodetic basis of $H[K_2]$. Thus, $g(H[K_2]) = 5$. Therefore, $g(H[K_2]) < (2-1)|Ex(H)| + |T| < (2-1)|Ex(H)| + 2|T|$.

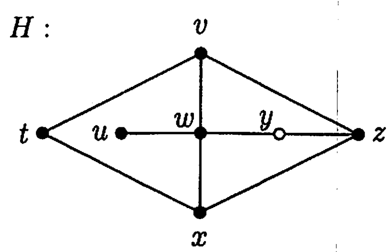

Figure 4: A graph with a geodetic cover shown which is not a basis.

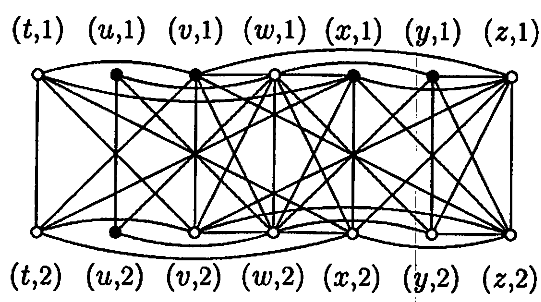

Figure 5: A graph $H[K_2]$ with $g(H[K_2]) = 5$.

Observation 2.18 *There exist a graph G and a positive integer m such that if S is a minimum hull set in G, and p is the smallest nonnegative integer with $I_G^p[S] = V(G)$, the iterate $I_G^{p-1}[S]$ is a GIC set in G but $g(G[K_m])$ is neither equal to $(m-1)|Ex(G)| + m|I_G^{p-1}[S]|$ nor equal to $(m-1)|Ex(G)| + |I_G^{p-1}[S]|$.*

Based on Observations 2.16 and 2.18, the sequence of iterates cannot always be used to determine the geodetic number of the composition $G[K_m]$.

Corollary 2.19 *Let G be a connected graph. Then*

$$g(G[K_m]) \leq (m-1)|Ex(G)| + |V(G)|.$$

In particular, if G has no extreme vertices, then $g(G[K_m]) \leq |V(G)|$.

Proof. Let $A' = V(G)$. Then A' is a geodetic cover. Since A' is a maximum GIC set in G, $m|A'| - (m-1)|A'| = |A'| = |V(G)| \in S$, where $S = \{m|A| - (m-1)|B| : A$ is a geodetic cover of G and B is a maximum GIC set with respect to A in $G\}$. Therefore, by Corollary 2.12, $g(G[K_m]) \leq (m-1)|Ex(G)| + |V(G)|$. ■

To illustrate Corollary 2.19, consider the following examples.

Example 2.20 In Figure 3, $g(C_5[K_2]) = 5 = |V(C_5)|$.

Example 2.21 Given a graph G in Figure 6, the composition $G[K_2]$ is shown in Figure 7. Observe that $A = \{(v,1),(v,2),(y,1),(y,2)\}$ is a geodetic basis of $G[K_2]$. This implies that $g(G[K_2]) = 4 < 6 = |Ex(G)| + |V(G)|$. Hence, $g(G[K_2]) < (2-1)|Ex(G)| + |V(G)|$.

A graph G is an *extreme geodesic graph* if every vertex in G lies on a $u - v$ geodesic for some pair u, v of extreme vertices.

Corollary 2.22 *Let G be a connected graph. Then $m|Ex(G)| \leq g(G[K_m])$. Moreover, if G is an extreme geodesic graph, then*

$$g(G[K_m]) = m|Ex(G)|.$$

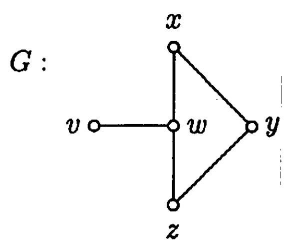

Figure 6: A graph G, where $|V(G)| = 5$ and $|Ex(G)| = 1$.

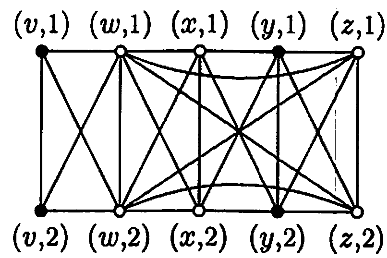

Figure 7: A graph $G[K_2]$ with $g(G[K_2]) = 4$.

Proof. Suppose $S = \{m|A| - (m-1)|B| : A$ is a geodetic cover of G and B is a maximum GIC set with respect to $A\}$. Since $Ex(G) \subseteq B \subseteq A$, $S = \{m|A \backslash B| + |B \backslash Ex(G)| + |Ex(G)| : A$ is a geodetic cover of G and B is a maximum GIC set with respect to $A\}$. So, we have $|Ex(G)| \leq \min S$. Thus, $g(G[K_m]) \geq m|Ex(G)|$ by Corollary 2.12.

Now, suppose G is an extreme geodesic graph. Then $Ex(G)$ is a geodetic cover of G. Since $Ex(G)$ is a maximum GIC set in G, $|Ex(G)| \in S$. This implies that $\min S \leq |Ex(G)|$. Hence, $g(G[K_m]) \leq m|Ex(G)|$ by Corollary 2.12. Accordingly, $g(G[K_m]) = m|Ex(G)|$. ■

The following example follows from Corollary 2.22.

Example 2.23 *Let n and m be positive integers. Then*

1. $g(K_n[K_m]) = mn$ *for* $n \geq 2$ *and*

2. $g(P_n[K_m]) = 2m$ *for* $n \geq 3$.

Theorem 2.24 *Let G be a connected graph and A a geodetic basis that is a GIC set in G. Then $Ex(G[K_m]) \cup [(A \backslash Ex(G)) \times \{v_0\}]$ is a geodetic basis of $G[K_m]$ for every $v_0 \in V(K_m)$.*

Proof. Let A be a geodetic basis that is a GIC set in G and $v_0 \in V(K_m)$. Then, by Corollary 2.8,

$$T = Ex(G[K_m]) \cup [(A \backslash Ex(G)) \times \{v_0\}]$$

is a geodetic cover of $G[K_m]$. Assume that T is not a geodetic basis of $G[K_m]$. Let T^* be a geodetic basis of $G[K_m]$. Then $|T^*| < |T|$, T_f^* is a geodetic cover of G and there exists a GIC set B^* with respect to T_f^* such that $T^* = Ex(G[K_m]) \cup [(T_f^* \backslash B^*) \times V(K_m)] \cup \{(u,v) \in T^* : u \in B^* \backslash Ex(G), v \in V(K_m)\}$ by Theorem 2.11. Thus,

$$T' = Ex(G[K_m]) \cup [(T_f^* \backslash B^*) \times V(K_m)] \cup [(B^* \backslash Ex(G)) \times \{v_0\}]$$

is a geodetic cover of $G[K_m]$ by Theorem 2.6. Hence,

$$|T'| = m|T_f^*| + (m-1)(|Ex(G)| - |B^*|) \leq |T^*|.$$

Since T^* is minimum,

$$|T^*| = m|T_f^*| + (m-1)(|Ex(G)| - |B^*|) < |T| = |A| + (m-1)|Ex(G)|.$$

This implies that $m|T_f^*| - (m-1)|B^*| < |A|$. Since $B^* \subseteq T_f^*$, it follows that $|T_f^*| < |A|$. This contradicts the assumption that A is a geodetic basis of G. Therefore, T is a geodetic basis of $G[K_m]$. ■

The following result is a direct consequence of Theorem 2.24.

Corollary 2.25 *Let G be a connected graph and K_m the complete graph of order m. If G has a geodetic basis that is a GIC set in G, then $g(G[K_m]) = g(G) + (m-1)|Ex(G)|$. In particular, if G has no extreme vertices, then $g(G[K_m]) = g(G)$.*

Example 2.26 In Figure 8, consider the graph G, where $g(G) = 5$. Observe that $A = \{a, b, c, d, e\}$ is a geodetic basis of G. Further, A is a GIC set in G. Thus, by Corollary 2.25, $g(G[K_m]) = m + 4$. In can also be shown that $g((G \backslash \{e\})[K_m]) = 4 = g(G \backslash \{e\})$.

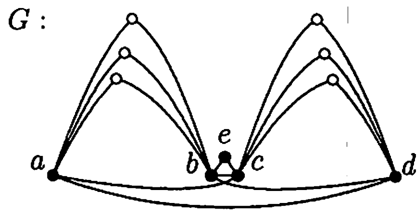

Figure 8: A graph G whose geodetic basis is a GIC set in G.

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