The Basis Number of The Composition of Theta Graphs with Some Graphs

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Abstract

The basis number of a graph G is defined to be the least integer k such that G has a k-fold basis for its cycle space. We investigate the basis number of the composition of theta graphs, a theta graph and a path, a theta graph and a cycle, a path and a theta graph, and a cycle and a theta graph.

Keywords: fold; basis number; cycle space.

1 Introduction.

The first important use of the basis number occurred in 1937 by MacLane [8] who proved that a graph is planar if and only if its basis number does not exceed 2. In 1981 Schmeichel [9] investigated the basis number of certain important classes of non-planar graphs, specifically, complete graphs and complete bipartite graphs. Then Banks and Schmeichel [4] proved that for $n \geq 7$, the basis number of Q_n is 4 , where Q_n is the n-cube. Thereafter many papers investigated the basis number of special types of graphs, in particular most of these papers focused on studying the basis number of graphs which obtained by special types of products on graphs such as cartesian product, direct product, strong product, semi-strong product, and the lexicographic product(or the composition of graphs), see [1], [2], [3], [4],[6].

In 1994 Hailat and Alzoubi [6] investigated the basis number of the composition of two paths, two cycles, a path and a cycle, a path and a star, a path and a wheel, a cycle and a star, a cycle and a wheel, a star and a path, a star and a cycle, a wheel and a path, a wheel and a cycle, a star and a wheel, and a star and a star.

The purpose of this paper is to investigate the basis number of the composition of two theta graphs, a theta graph and a path, a theta graph and a cycle, a path and a theta graph, and a cycle and a theta graph.

2 Notations and Preliminaries

In this paper, we consider only finite connected undirected graphs without loops and multiple edges. The terminology and notations will be standard except as indicated. For undefined terms we refer the reader to [5].

Let G=(V,E) be a graph where V and E are the vertex and the edge sets of G, respectively. If $e_1, e_2, \ldots, e_q$ is an ordering of the edges in G, then any subset S of edges corresponding to a $(0,1)$-vector $(a_1, \ldots, a_q)$ in the usual way, with $a_i = 1$ ($a_i = 0$) if and only if $e_i \in S$ ($e_i \notin S$). These vectors form a q-dimensional vector space over the field Z_2. Those vectors corresponding to the cycles in G generate a subspace of $(Z_2)^q$ called the *cycle space* of G, and denoted by $\mathcal{C}(G)$. In the sequel, we say the cycles themselves generate the $\mathcal{C}(G)$, rather than saying that the vectors corresponding to the cycles generate the $\mathcal{C}(G)$. It is well known that the dimension of $\mathcal{C}(G)$, denoted by dim $\mathcal{C}(G)$, is $q - p + 1$, where p and q denote, respectively, the number of vertices and edges in G.

A basis of $\mathcal{C}(G)$ is called a k-fold basis if each edge of G occurs in at most k of the cycles in the basis. The *basis number* of G, denoted by $b(G)$, is the smallest integer k such that $\mathcal{C}(G)$ has a k-fold basis.

The following result of MacLane [8] will be used frequently in the sequel:

Theorem 2.1. Let G be graph. Then G is palnar if and only if $b(G) \leq 2$.

The following lemma due to Hailat and Alzoubi [6] will be of great use in our results:

Lemma 2.1. *Let G be a graph with p vertices and q edges. If $|C|$ denotes the length of the cycle C, and $\mathcal{B} = \{C_1, C_2, \ldots, C_d\}$ be a k-fold basis of $\mathcal{C}(G)$, then $rd \leq \sum_{i=1}^{d} |C_i| \leq kq$ where $d = dim\ \mathcal{C}(G)$ and r is the girth of G.*

Definition 2.1. *The composition of two graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$, denoted by $G_1[G_2]$, is the graph with vertex-set $V(G_1[G_2]) = V_1 \times V_2$ and edge-set $E(G_1[G_2]) = \{(u_1, v_1)(u_2, v_2) : \text{either } u_1u_2 \in E_1 \text{ or } u_1 = u_2 \text{ and } v_1v_2 \in E_2\}$.*

It is worth mentioning that $G_1[G_2]$ and $G_2[G_1]$ are not isomorphic graphs for being $|E(G_1[G_2])| = p_1q_2 + p_2^2q_1$ and $|E(G_2[G_1])| = p_2q_1 + p_1^2q_2$.

In this paper, we use the additive group Z_n of positive integers residue modulo n to denote the vertex-set of the graphs P_n, C_n, and θ_n , and thier edge sets are defined as follows:

$E(P_n) = \{i(i+1) : 0 \leq i \leq n-2\}$, $E(C_n) = \{i(i+1) : 0 \leq i \leq n-1\}$, $E(\theta_n) = \{i(i+1) : 0 \leq i \leq n-1\} \cup \{st\}$ where $s, t \in Z_n$ are non-consecative number. Note that the degree of every vertix of θ_n is of degree 2 except s and t which have degree 3.

We denote by P_n the path $012\cdots(n-1)$, C_n the cycle $012\cdots(n-1)0$ and θ_n to be C_n union an edge that connects a non adjacent vertices .

3 Main Results.

In this section, we investigate the basis number of the graphs $\theta_m[\theta_n]$, $\theta_m[C_n]$, $\theta_m[P_n]$, $P_m[\theta_n]$,
$C_m[\theta_n]$ where $m \geq 4$ and $n \geq 5$. Throughout the paper we consider l and t $(l < t)$ to be the vertices of θ_m of degree 3, and s and k are the vertices of θ_n of degree 3. One may see that $\theta_m[\theta_n]$ consists of $(m+1)$ edge-disjoint copies of the complete bipartite graph $K_{n,n}$. Note that m of these copies occur in the form $K_{(r,n),(r+1,n)}$ for $r \in Z_m$, taking into account that the last of the m copies has the form $K_{(m-1,n),(0,n)}$,and the $(m+1)th$ copy has the form $K_{(l,n),(t,n)}$where l and t are nonconsecutive numbers from Z_m. Also, $\theta_m[\theta_n]$ contains the following sets of edges:

$E_1 = \{(i,j)(i,j+1) : i \in Z_m, j \in Z_{n-1}\}, |E_1| = (n-1)m,$
$E_2 = \{(i,n-1)(i,0) : i \in Z_m\}, |E_2| = m,$
$E_3 = \{(i,k)(i,s) : i \in Z_m\}, |E_3| = m.$

Note that $\theta_m[\theta_n]$ has mn vertices and $n^2(m+1) + m(n+1)$ edges. Therefore, $\dim C(\theta_m[\theta_n]) = (m+1)(n^2+1)$.

The purpose of describing $\theta_m[\theta_n]$ as above is to simplify the way we choose the elements of the required basis.

Theorem 3.1. *For every $m \geq 4$ and $n \geq 5$, we have $3 \leq b(\theta_m[\theta_n]) \leq 4$. Moreover, $b(\theta_m[\theta_n]) = 4$ for each $m \geq 4$ and $n \geq 23$.*

Proof. For each $m \geq 4$ and $n \geq 5$, the graph $\theta_m[\theta_n]$ is non-planar since it contains many copies of the non-planar $K_{n,n}$. Then by Theorem 2.1 of Maclane's we have $b(\theta_m[\theta_n]) \geq 3$.
To prove that $b(\theta_m[\theta_n]) \leq 4$, we exhibit a 4-fold basis for $C(\theta_m[\theta_n])$. For each $r \in Z_m$, we set $B_r = H_r \cup K_r$; $|B_r| = n^2 - 1$, where H_r and K_r are sets of cycles defined by

$H_r = \{(r,i)(r+1,j)(r,i+1)(r+1,j+1)(r,i) : 0 \leq i,j \leq n-2\}$,
$|H_r| = (n-1)^2$,
$K_r = \{(r,0)(r+1,i)(r+1,i+1)(r,0),\ (r+1,n-1)(r,i)(r,i+1)$
$(r+1,n-1) : 0 \leq i \leq n-2\}$

also we consider the following sets of cycles:

$T_1 = \{(l,i)\,(t,j)\,(l,i+1)\,(t,j+1)\,(l,i) : 0 \leq i,j \leq n-2\}$,

$T_2 = \{(l,0)\,(t,i)\,(t,i+1)\,(l,0),\ (t,n-1)\,(l,i)\,(l,i+1)\,(t,n-1) :$
$0 \leq i \leq n-2,\}$

$L = \{(i,k)\,(i+1,0)\,(i,s)\,(i,k) : i \in Z_m\}$,

$M = \{(i,0)\,(i+1,n-1)\,(i,n-1)\,(i,0) : i \in Z_m\}$.

Let $Q_1 = (0,0)\,(1,0)...(l,0)\,(t,0)\,(t+1,0)...(m-1,0)\,(0,0)$ and

$Q_2 = (l,0)\,(l+1,0)...(t,0)\,(l,0)$ be two cycles, then we define the following set:

$$B(\theta_m[\theta_n]) = (\bigcup_{r=0}^{m-1} B_r) \cup T_1 \cup T_2 \cup L \cup M \cup \{Q_1, Q_2\}$$

which we will prove that it is the required 4-fold basis for the cycle space $C(\theta_m[\theta_n])$. Since $|B(\theta_m[\theta_n])| = \left|\left(\bigcup_{r=0}^{m-1} B_r\right)\right| + |T_1| + |T_2| + |L| + |M| + 2 = m(n^2-1) + (n-1)^2 + 2(n-1) + m + m + 2 = (m+1)(n^2+1)$ =dim $C(\theta_m[\theta_n])$, it is enough to prove that $B(\theta_m[\theta_n])$ is linearly independent. By Theorem 2.4 of Schmeichel [9] , B_r is a basis for the cycle subspace $C\left(K_{(r,n),(r+1,n)}\right)$ for every $r \in Z_m$, and $T_1 \cup T_2$ is a basis for the cycle subspace $C\left(K_{(l,n),(t,n)}\right)$. Then each of the sets $T_1 \cup T_2$ and B_r for $r \in Z_m$ is linearly independent. Moreover, all of these sets are edge-disjoint, this implies that $\left(\bigcup_{r=0}^{m-1} B_r\right) \cup T_1 \cup T_2$ is linearly independent. All the cycles in the set $L \cup M$ are edge-disjoint, then any linear combination of cycles from it is a union of edge-disjoint cycles which ensures that $L \cup M$ is linearly independent. Moreover, every cycle in $L \cup M$ contains an edge that does not occur in any cycle of the set $\left(\bigcup_{r=0}^{m-1} B_r\right) \cup T_1 \cup T_2$. Hence any linear combination of cycles from $\left(\bigcup_{r=0}^{m-1} B_r\right) \cup T_1 \cup T_2 \cup L \cup M$ gives either a cycle or a union of edge-disjoint cycles which guarantees that $\left(\bigcup_{r=0}^{m-1} B_r\right) \cup T_1 \cup T_2 \cup L \cup M$ is a linearly independent set of cycles. It is easy to see that Q_1, Q_2 can not be written as a linear combination of $\left(\bigcup_{r=0}^{m-1} B_r\right) \cup T_1 \cup T_2 \cup L \cup M$. Thus $B(\theta_m[\theta_n])$ is linearly independent. Therefore, $B(\theta_m[\theta_n])$ is a basis of the cycle space $C(\theta_m[\theta_n])$. Following the way we constructed $B(\theta_m[\theta_n])$ one can see easily that the fold of any edge of $\theta_m[\theta_n]$ in $B(\theta_m[\theta_n])$ does not exceeds 4. Therefore $B(\theta_m[\theta_n])$ is a 4-fold basis of the cycle space $C(\theta_m[\theta_n])$.

To complete the proof of our theorem, we proceed by eliminating any possibility for the cycle space $C(\theta_m[\theta_n])$ to have a 3-fold basis. Now, suppose that $\mathcal{B}$ is a 3-fold basis of the cycle space $C(\theta_m[\theta_n])$. We consider the following three cases:

Case 1. $\mathcal{B}$ contains only 3-cycles. Since any 3-cycle in $\theta_m[\theta_n]$ must contain an edge from the sets E_1,E_2,E_3, or from the set $\{(l,i)(t,i),(l,i)(l+1,i),(l+1,i)(t,i) : i \in Z_n\}$ (if θ_m contains a 3-cycle say $l(l+1)tl$), the number of 3-cycles in $\theta_m[\theta_n]$ that can be in $\mathcal{B}$ is at most $3mn+3m+9n = 3m(n+1)+9n$, because the fold of any edge is at most 3. Thus $|\mathcal{B}| = (m+1)(n^2+1) \leq 3m(n+1)+9n$. But this inequality does not hold for any $m \geq 4$ and $n \geq 5$, a contradiction.

Case 2. $\mathcal{B}$ consists only of cycles of length greater than or equal to 4. Lemma 2.1 implies that $4(m+1)(n^2+1) \leq 3(m+1)n^2 + m(n+1)$. Then $(m+1)(n^2+1) \leq 3m(n+1)$. But this inequality cannot hold for any $m \geq 4$ and $n \geq 4$, a contradiction.

Case 3. $\mathcal{B}$ contains s cycles of length 3 and t cycles of length greater than or equal to 4. Since the fold of every edge in $\theta_m[\theta_n]$ is at most 3 in $\mathcal{B}$, at most $3s$ edges are used to form the s 3-cycles. Since $|E(\theta_m[\theta_n])| = n^2(m+1)+m(n+1)$,we have $t \leq \left[\frac{3(n^2(m+1)+m(n+1))-3s}{4}\right]$,where $[x]$ is the greatest integer less than or equal to x. Then $(n^2+1)(m+1) = \dim C(\theta_m[\theta_n]) = s+t \leq 3mn+3m+9n+\left[\frac{3(n^2(m+1)+m(n+1))-3s}{4}\right]$, thus $(n^2+1)(m+1) \leq 3mn+3m+9n+\left[\frac{3(n^2(m+1)+m(n+1))-3}{4}\right]$ since $s \geq 1$. But $\left[\frac{3(n^2(m+1)+m(n+1))-3}{4}\right] \leq \frac{3(n^2(m+1)+m(n+1))-3}{4}$, and so $(n^2+1)(m+1) \leq 3mn+3m+9n+\frac{3(n^2(m+1)+m(n+1))-3}{4}$. This implies that $4(n^2+1)(m+1) \leq 12mn+12m+36n+3n^2(m+1)+3m(n+1)-3$. Therefore, $n^2(m+1)+7 \leq 15nm+15m+36n$, thus, $n^2+\frac{7}{m+1} \leq 15n(\frac{m}{m+1}) + 11\frac{m}{m+1}+\frac{36n}{m+1}$. Since $m \geq 4$, $n^2-22n-11 < 0$. Solving the inequality $n^2-22n < 11$ implies that $n \leq 22$, a contradiction.

Now we consider $\theta_m[C_n]$. Note that $\theta_m[C_n]$ is a subgraph of $\theta_m[\theta_n]$ obtained by deleting the set E_3. It is easy to see that $\theta_m[C_n]$ has mn vertices and $n^2(m+1)+mn$ edges. Thus, $\dim C(\theta_m[C_n]) = n^2(m+1)+1$.

Theorem 3.2. *For every $m \geq 4$ and $n \geq 5$, we have $3 \leq b(\theta_m[C_n]) \leq 4$. Moreover, $b(\theta_m[C_n]) = 4$ for each $m \geq 4$ and $n \geq 15$.*

Proof. It is clear that $\theta_m[C_n]$ is non-planar for each $m \geq 4$ and $n \geq 5$, so by Theorem 2.1 of Maclane's we have $b(\theta_m[C_n]) \geq 3$.

We define $B(\theta_m[C_n]) = B(\theta_m[\theta_n]) \backslash L$, where $B(\theta_m[\theta_n])$ and L are the same sets as in Theorem 3.1. So $|B(\theta_m[C_n])| = (n^2+1)(m+1) - m = n^2(m+1)+1$ =dim $C(\theta_m[C_n])$. Since $B(\theta_m[\theta_n])$ is a basis of the cycle space $C(\theta_m[\theta_n])$, then $B(\theta_m[C_n])$ is a basis for the cycle subspace $C(\theta_m[C_n])$. Also, since $B(\theta_m[\theta_n])$ is a 4-fold basis, then so is

$B(\theta_m[\theta_n])$.Hence $3 \le b(\theta_m[C_n]) \le 4$.

On the other hand. Suppose that $\mathcal{B}$ is a 3-fold basis of the cycle space $C(\theta_m[C_n])$. We consider the following three cases:

Case1. $\mathcal{B}$ consists only of 3-cycles. Then $|\mathcal{B}| \le 3mn + 9n$, because any 3-cycle of $\theta_m[C_n]$ must contain an edge either from E_1, E_2, or from the set $(l,i)(t,i)(l,i)(l+1,i)(l+1,i)(t,i) : i \in Z_n$ (if θ_m contains a 3-cycle say $l(l+1)tl$) and the fold of each of these edges is at most 3. Thus $|\mathcal{B}| \le 3mn + 9n < n^2(m+1) + 1 =$dim $C(\theta_m[C_n])$ for each $m \ge 4$ and $n \ge 5$, a contradiction.

Case2. $\mathcal{B}$ consists only of cycles of length greater than or equal to 4. Then by Lemma 2.1 we have $4\left(n^2(m+1)+1\right) \le 3\left(n^2(m+1)+mn\right)$. But this inequality does not hold for any $n,m \ge 1$, a contradiction.

Case3. $\mathcal{B}$ contains s 3-cycles and t cycle of length at least 4. Since $|E(\theta_m[C_n])| = n^2(m+1) + mn$ and $\mathcal{B}$ is a 3-fold basis, the most number of edges that can be used to form the s 3-cycles is $3s$. So, the number of edges left to form the other cycles counting folds is $3\left(n^2(m+1)+mn\right)$ $-3s$, which implies that $t \le \left\lceil \frac{3n^2(m+1)+3mn-3s}{4} \right\rceil$. Then $n^2(m+1)+1 =$dim $C(\theta_m[C_n]) = |\mathcal{B}| = s + t \le 3mn + 9n + \left\lceil \frac{3n^2(m+1)+3mn-3}{4} \right\rceil$ since $s \ge 1$. But $\left\lceil \frac{3n^2(m+1)+3mn-3}{4} \right\rceil \le \frac{3n^2(m+1)+3mn-3}{4}$, then $n^2(m+1)+1 \le 3mn + 9n + \frac{3n^2(m+1)+3mn-3}{4}$. Multiplying this inequality by 4 and rearranging the terms leads to the inequality $n^2(m+1) \le 15mn + 36n - 7$. Thus, $n^2 \le 15n(\frac{m}{m+1}) + \frac{36n}{m+1} - \frac{7}{m+1}$. Since $m \ge 4$, $n^2 \le 22n$. It implies that $n < 22$, a contradiction.

We consider the graph $\theta_m[P_n]$ as a subgraph of $\theta_m[\theta_n]$ that can be obtained by deleting the sets of edges E_2 and E_3. Note that $\theta_m[P_n]$ has mn vertices and $n^2(m+1)+m(n-1)$ edges, so dim $C(\theta_m[P_n]) = n^2(m+1) - m + 1$.

Theorem 3.3. *For each $m \ge 4$ and $n \ge 5$, we have* $3 \le b(\theta_m[P_n]) \le 4$. *Moreover,* $b(\theta_m[P_n]) = 4$ *for each $m \ge 4$ and $n \ge 24$.*

Proof. It is clear that $\theta_m[P_n]$ is non-planar and that the set $B(\theta_m[P_n]) = B(\theta_m[\theta_n]) \setminus (L \cup M)$ is a 4-fold subbasis of $B(\theta_m[\theta_n])$ for the cycle subspace $C(\theta_m[P_n])$ of the cycle space $C(\theta_m[\theta_n])$. Hence $3 \le b(\theta_m[P_n]) \le 4$. To prove that $b(\theta_m[P_n]) = 4$, we use the same arguments of Theorem 3.2 taking into account that any 3-cycle in $\theta_m[P_n]$ must have an edge from the set E_1, or the set $\{(l,i)(t,i) : i \in Z_n\}$.

Now we consider the graph $C_m[\theta_n]$. Note that $C_m[\theta_n]$ is a subgraph of $\theta_m[\theta_n]$ obtained by deleting the $K_{(l,n),(t,n)}$ copy of $K_{n,n}$. Also, this graph has mn vertices, $n^2m + m(n+1)$ edges, and $\dim C(C_m[\theta_n]) = m(n^2+1)+1$.

Theorem 3.4. *For each $m \geq 4$ and $n \geq 5$, we have $3 \leq b(C_m[\theta_n]) \leq 4$. Moreover, $b(C_m[\theta_n]) = 4$ for each $m \geq 4$ and $n \geq 16$.*

Proof. It is easy to see that $C_m[\theta_n]$ is non-planar, and that $B(C_m[\theta_n]) = (B(\theta_m[\theta_n]) \cup \{Q\}) \setminus (T_1 \cup T_2 \cup \{Q_1, Q_2\})$ is a 4-fold subbasis of $B(\theta_m[\theta_n])$ for the cycle subspace $C(C_m[\theta_n])$ of the cycle space $C(\theta_m[\theta_n])$, where $Q = (0,0)(1,0)\ldots(m-1,0)(0,0)$ and Q_1, Q_2, T_1, and T_2 are defined in Theorem 3.1. Then $3 \leq b(C_m[\theta_n]) \leq 4$. On the other hand, to prove that $b(\theta_m[P_n]) = 4$, we use the same arguments of Theorem 3.2 with the fact that any 3-cycle in $C_m[\theta_n]$ must contain an edge from the set $E_1 \cup E_2 \cup E_3$.

Now we consider the graph $P_m[\theta_n]$. Note that $P_m[\theta_n]$ is a subgraph of $C_m[\theta_n]$ obtained by deleting the mth copy of $K_{n,n}$, which has the form $K_{(m-1,n),(0,n)}$. Also, $P_m[\theta_n]$ has mn vertices and $n^2(m-1) + m(n+1)$ edges, and so $\dim C(P_m[\theta_n]) = n^2(m-1) + m + 1$.

Theorem 3.5. *For each $m \geq 4$ and $n \geq 5$, we have $3 \leq b(P_m[\theta_n]) \leq 4$. Moreover, $b(P_m[\theta_n]) = 4$ for each $m \geq 4$ and $n \geq 30$.*

Proof. It is obvious that the set $B(P_m[\theta_n]) = (B(C_m[\theta_n]) \cup M') \setminus (B_{m-1} \cup M \cup \{Q\})$ is a 4-fold subbasis of $B(C_m[\theta_n])$ for the cycle subspace $C(P_m[\theta_n])$ of the cycle space $C(C_m[\theta_n])$, where M and B_{m-1} are defined in Theorem 3.1 and $M' = \{(i,0)(i,1)\ldots(i,n-1)(0,0) : i \cup Z_m\}$. Also, $P_m[\theta_n]$ is non-planar, so $3 \leq b(P_m[\theta_n]) \leq 4$ for each $m \geq 4$ and $n \geq 5$. To prove that $b(P_m[\theta_n]) = 4$, we follow the same arguments of Theorem 3.2, just considering that any 3-cycle in $P_m[\theta_n]$ must have an edge from the set $E_1 \cup E_3$.

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