Erratum to "Inducing regularization of graphs, multigraphs and pseudographs" [Ars Combin. 65 (2002) 129–133]

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For $X \in \{\mathcal{M}, \mathcal{P}\}$, let X-graph stand for $\mathcal{P}$-graph (pseudograph) if $X = \mathcal{P}$, otherwise for $\mathcal{M}$-graph (multigraph) if $X = \mathcal{M}$. Define X_r^p to be the class of X-graphs F where r is an upper bound on maximum degree and p is the upper bound on the edge multiplicity in F. Given an X-graph G and large enough integers r and p, an r-regular X_r^p-graph F is called *X_r^p-regularization* (or *inducing regularization within X_r^p*) of G if F contains G as an induced sub-X-graph.

Given an X-graph G, let F be a smallest inducing X_r^p-regularization of G. Then $V(F) = V(G) \cup U$ where $U := \{u_1, u_2, \ldots, u_t\}$ is a set of t new vertices u_j where t (≥ 0) is uniquely determined, see Theorem in [1]. Moreover, F is the edge-disjoint union of G, a bipartite $V(G)$–U multigraph B, and an X-graph H with $V(H) = U$. Our aim here is to rectify the construction of H presented in [1] in case $t \geq 2$ and $tr > \sigma := \sum_{v \in V(G)}(r - \deg_G(v))$, σ being the sum of vertex r-deficiencies in G.

Let h and s be nonnegative integers such that $\sigma = ht + s$ with $s < t$. Then $h < r$. Moreover, B is constructed so that $\deg_B(u_j) = h + 1$ for $j = 1, 2, \ldots, s$ and $\deg_B(u_j) = h$ for $j = s + 1, \ldots, t$. The following description of H is erroneously omitted in [1, p. 132[5]]:

"Let ${}^p\tilde{K}_t$ be the *complete X-graph* of order t, with edge multiplicity p, and with p loops at each vertex if $X = \mathcal{P}$ (presence of loops being indicated by tilde). A required H is a factor of ${}^p\tilde{K}_t$ such that $F = G \cup B \cup H$ is an r-regular X-graph. The construction can involve 2-factors (of which some comprise either loops if $X = \mathcal{P}$ or 2-cycles of ${}^p\tilde{K}_t$ if the allowed edge multiplicity is $p > 1$). In particular, a Hamilton cycle can be used to construct H in case $t > s \geq 2$. Note that ${}^p\tilde{K}_t$ can be represented as the union of 2-factors and a single 1-factor (or a perfect matching, say $\tilde{M}$) in case t is even and p is odd. Otherwise, if t is odd or p is even, ${}^p\tilde{K}_t$ is the union of 2-factors. This is easily seen if $t = 2$. Otherwise $t \geq 3$. Then (as proved by Walecki, see Lucas [2, Ch. 6, pp. 162–167]) the complete graph K_t is decomposable into Hamilton cycles if t is odd and into a matching M and Hamilton cycles if t is even. Hence the complete multigraph ${}^2K_t$ with even t is decomposable into Hamilton cycles because then ${}^2K_t = K_t \oplus K_t$ and, moreover, the two perfect matchings, one each from either K_t, can be chosen so as to make up a Hamilton cycle of ${}^2K_t$. In case $X = \mathcal{P}$ loops of ${}^p\tilde{K}_t$ make up p 2-factors. Hence all remaining 2-factors in the decomposition can be assumed to be Hamilton cycles of ${}^p\tilde{K}_t$ (if $X = \mathcal{M}, \mathcal{P}$).

Recall that in part U the graph B has s vertices of degree $h+1$ and $t-s$ vertices of degree h. Notice that the parity requirement in Theorem of [1] (namely that $(n+t)r$ be even) implies that $tr - \sigma$, which is to be the sum of vertex degrees in H, is even. Hence if $r-h$ is odd then $t-s$ is even since $t - s = t + th - \sigma = t(h - r + 1) + (tr - \sigma)$ is the sum of even summands. On the other hand, if $r-h$ is even then $s = t(r-h) - (tr - \sigma)$ is even, too.

Recall that $r - h \leq p(t-1)$ for $X = \mathcal{M}$ and $r - h \leq p(t+1)$ for $X = \mathcal{P}$. Let $r-h$ be odd. We now get H as the union of a complete matching on the $t-s$ vertices in U of degree h in B and of $\frac{1}{2}(r-h-1)$ 2-factors of ${}^p\tilde{K}_t$. Let $r-h$ be even. If $s = 0$ then H comprises $\frac{1}{2}(r-h)$ 2-factors of the ${}^p\tilde{K}_t$. Otherwise, for $s \geq 2$, let H include a path, P, on $t-s+2$ vertices such that all $t-s$ vertices in U of degree h in B are inner vertices of P. We next add a complete matching on $s-2$ remaining vertices in U of degree $h+1$ in B so that all degrees in U could become $h+2$. We assume that the path P and the matching in question are extracted from a Hamilton cycle of ${}^p\tilde{K}_t$ where $t > s \geq 2$. We now add $\frac{1}{2}(r-h-2)$ 2-factors of the ${}^p\tilde{K}_t$ in order to complete H."

References

[1] J. Górska, Z. Skupień, Inducing regularization of graphs, multigraphs and pseudographs, Ars Combin. 65 (2002) 129–133.

[2] E. Lucas, Récréations Mathématiques, vol. II, Gauthier-Villars, Paris, 1883.