A New Lower Bound for $A(17, 6, 6)$

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Abstract

We construct a record-breaking binary code of length 17, minimal distance 6, constant weight 6, and containing 113 codewords.

1 Introduction

Let $A(n, d, w)$ denote the maximum possible number of codewords in a binary code of length n, minimal distance d and constant weight w. The Nordstrom-Robinson code $\mathcal{N}_{16}$ of length 16, minimal distance 6, and containing 256 codewords has weight enumerator $1 + 112x^6 + 30x^8 + 112x^{10} + x^{16}$. Hence, taking all the codewords of weight 6 in $\mathcal{N}_{16}$ gives a constant weight code that shows $A(16, 6, 6) \geq 112$. Since $A(17, 6, 6) \geq A(16, 6, 6)$, we also have $A(17, 6, 6) \geq 112$. This is in fact the best lower bound on $A(17, 6, 6)$ known [2].

In this note, we give the first improvement on the lower bound for $A(17, 6, 6)$ since that implied by the 1967 result of Nordstrom and Robinson [3]. We exhibit a new binary code C of length 17, minimal distance 6, constant weight 6, and containing 113 codewords, showing $A(17, 6, 6) \geq 113$. Our code has no particular structure (its automorphism group is trivial) and is obtained through a combination of search techniques involving simulated annealing [4], length-reduction [1], and local optimization.

The support supp(x) of a codeword $x = (x_1, \ldots, x_n)$ is the set of indices of its non-zero coordinates, that is, $\text{supp}(x) = \{i \mid x_i \neq 0\}$. The supports of the codewords in C are listed in the next section.

2 The Code

0 1 2 3 6 15	0 1 2 4 11 16	0 1 2 7 8 9
0 1 2 10 12 13	0 1 3 4 8 10	0 1 3 5 7 12
0 1 3 9 13 16	0 1 4 6 7 13	0 1 5 6 10 16
0 1 5 8 11 13	0 1 6 9 11 12	0 1 7 10 11 15
0 1 8 12 14 15	0 2 3 4 9 12	0 2 3 5 8 16
0 2 3 7 11 13	0 2 4 5 7 10	0 2 4 8 13 15
0 2 5 6 9 13	0 2 5 11 14 15	0 2 6 7 12 16
0 2 6 8 10 11	0 3 4 5 6 11	0 3 4 7 14 16
0 3 5 10 13 15	0 3 6 7 9 10	0 3 6 8 12 13
0 3 8 9 11 15	0 3 10 11 12 14	0 4 5 12 13 14
0 4 6 8 9 16	0 4 6 10 12 15	0 4 7 8 11 12
0 4 9 10 11 13	0 5 6 7 8 15	0 5 8 9 10 14
0 5 9 12 15 16	0 6 11 13 14 16	0 7 8 10 13 16
0 7 9 13 14 15	1 2 3 4 5 13	1 2 3 7 10 14
1 2 3 8 11 12	1 2 4 6 9 10	1 2 4 7 12 15
1 2 5 6 7 11	1 2 5 8 10 15	1 2 5 12 14 16
1 2 6 8 13 16	1 2 9 11 13 15	1 3 4 6 12 16
1 3 4 7 9 11	1 3 5 6 8 14	1 3 5 11 15 16
1 3 6 10 11 13	1 3 7 8 13 15	1 3 9 10 12 15
1 4 5 7 8 16	1 4 5 9 14 15	1 4 5 10 11 12
1 4 6 8 11 15	1 4 8 9 12 13	1 4 10 13 14 16
1 5 6 12 13 15	1 5 7 9 10 13	1 6 7 8 10 12
1 6 7 9 15 16	1 7 11 12 13 16	1 8 9 10 11 16
2 3 4 6 7 8	2 3 4 10 11 15	2 3 5 6 10 12
2 3 5 7 9 15	2 3 6 9 11 16	2 3 8 9 10 13
2 3 12 13 15 16	2 4 5 6 15 16	2 4 5 8 9 11
2 4 6 11 12 13	2 4 7 9 13 16	2 4 8 10 12 14
2 5 7 8 12 13	2 5 10 11 13 16	2 6 7 10 13 15
2 6 8 9 12 15	2 7 8 11 15 16	2 7 9 10 11 12
2 9 10 14 15 16	3 4 5 8 12 15	3 4 5 9 10 16
3 4 6 13 14 15	3 4 7 10 12 13	3 4 8 11 13 16
3 5 6 7 13 16	3 5 7 8 10 11	3 5 9 11 12 13
3 6 7 11 12 15	3 6 8 10 15 16	3 7 8 9 12 16
4 5 6 7 9 12	4 5 6 8 10 13	4 5 7 11 13 15
4 6 7 10 11 14	4 7 8 9 10 15	4 11 12 14 15 16
5 6 8 11 12 16	5 6 9 10 11 15	5 7 9 11 14 16
5 7 10 12 14 15	5 8 13 14 15 16	6 7 8 9 11 13
6 9 10 12 13 16	8 10 11 12 13 15	

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