On the Domination Number of Generalized Petersen Graphs $P(n,3)$*

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Abstract

Let $G = (V(G), E(G))$ be a graph. A set $S \subseteq V(G)$ is a dominating set if every vertex of $V(G) - S$ is adjacent to some vertices in S. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. In this paper, we study the domination number of generalized Petersen graphs $P(n,3)$ and proved that $\gamma(P(n,3)) = n - 2\lfloor \frac{n}{4} \rfloor (n \neq 11)$.

Keywords: *Domination number*; *Dominating set; Generalized Petersen Graph; Domination number;*

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1 Introduction

We consider only finite undirected graphs without loops or multiple edges.

A graph $G = (V(G), E(G))$ is a set $V(G)$ of vertices and a subset $E(G)$ of the unordered pairs of vertices, called edges. We use [7] for the terminology and notation not defined here.

The open neighborhood and the closed neighborhood of a vertex $v \in V$ are denoted by $N(v) = \{u \in V(G) \ : \ vu \in E(G)\}$ and $N[v] = N(v) \cup \{v\}$, respectively. For a vertex set $S \subseteq V(G)$, $N(S) = \bigcup_{v \in S} N(v)$ and $N[S] = \bigcup_{v \in S} N[v]$. The maximum degree of vertices in $V(G)$ is denoted by $\Delta(G)$.

A set $S \subseteq V(G)$ is a dominating set if for each $v \in V(G)$ either $v \in S$ or v is adjacent to some $w \in S$. That is, S is a dominating set if and only if $N[S] = V(G)$. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G.

The study of domination in graphs was initiated by Ore[11]. Topic on domination number and related parameters have long attracted graph theorists for their strongly practical background and theoretical interest. It has been proved [5] that the decision problem corresponding to the domination number for arbitrary graphs is NP-complete. So much work was done to establish bounds on $\gamma(G)$. There is the well known bounds on $\gamma(G)$ in terms of the number of vertices n and maximum degree $\Delta(G)$.

Theorem 1.1 [1, 12] For any graph G, $\lceil \frac{n}{1+\Delta(G)} \rceil \leq \gamma(G) \leq n - \Delta(G)$.

In 1995, Molloy and Reed [10] studied the dominating number of a random cubic graph and proved $.2636n \leq \gamma(G) \leq .3126n$.

The dominating numbers of very few families of graphs are known exactly. By [7], we have, $\gamma(K_n) = 1$, $\gamma(K_{1,n-1}) = 1 (n \geq 2)$, $\gamma(K_{m,n}) = 2 (m \geq 2, n \geq 2)$, $\gamma(P_n) = \lceil \frac{n}{3} \rceil$, $\gamma(C_n) = \lceil \frac{n}{3} \rceil$.

The Cartesian product of two graphs G and H is the graph denoted $G \Box H$, with $V(G \Box H) = V(G) \times V(H)$ and $((u, u'), (v, v')) \in$

$E(G\Box H)$ if and only if $u' = v'$ and $(u, v) \in E(G)$ or $u = v$ and $(u', v') \in E(H)$. The grid graph $G_{k,n} = P_k \Box P_n$.

In 1983, M. S. Jacobson and L. F. Kinch [8] determined the domination number $\gamma(G_{k,n})$ for $k \leq 4$. In 1993, T. Y. Chang and W. E. Clark [2] determined $\gamma(G_{k,n})$ for $5 \leq k \leq 6$. In 1993, D. C. Fisher determined $\gamma(G_{k,n})$ for $7 \leq k \leq 16$ and given out the following conjecture[4]:

Conjecture 1.2 $\gamma(G_{m,n}) = \lfloor (m+2)(n+2)/5 \rfloor - 4$.

The cross product of two graphs G and H is the graph denoted $G \times H$, with $V(G \times H) = V(G) \times V(H)$ and $((u, u'), (v, v')) \in E(G \times H)$ if and only if $(u, v) \in E(G)$ and $(u', v') \in E(H)$.

In 1995, S. Gravier and A. Khelladi [6] determined the domination number $\gamma(P_n \times \overline{P_k})$ for every $n \geq 2$ and $k \geq 4$. In 1999, R. Chérifi, S. Gravier, and X. Lagraula et al [3] determined the domination number $\gamma(P_n \times P_k)$ for $k \leq 8$, $\gamma(P_n \times P_9)$ for $n \geq 8$ and $\gamma(P_n \times P_k)$ for $10 \leq k \leq 33$ and $1 \leq n \leq 40$.

In 1995, S. Klavžar and N. Seifter [9]determined the domination number $\gamma(C_n \Box C_k)$ for $k \leq 5$.

The generalized Petersen graph $P(n, k)$ is defined to be a graph on $2n$ vertices with $V(P(n,k)) = \{v_i, u_i : 0 \leq i \leq n-1\}$ and $E(P(n,k)) = \{v_i v_{i+1}, v_i u_i, u_i u_{i+k} : 0 \leq i \leq n-1$, subscripts modulo $n\}$.

In 2002, Zelinka and Liberecwere [13] studied the domination in $P(n, k)$ and proved the domatic number $d(P(n, k)) = 4$ if and only if $n \equiv 0 \bmod 4$.

In this paper, we consider the domination number of $P(n, 3) (n \geq 4)$ and prove that $\gamma(P(n, 3)) = n - 2\lfloor \frac{n}{4} \rfloor$ $(n \neq 11)$, $\gamma(P(11, 3)) = 6$.

2 The domination number of $P(n, 3)$

Let $m = \lfloor \frac{n}{4} \rfloor$ and $t = n \bmod 4$, then n=4m+t.

Lemma 2.1. $\gamma(P(n,3)) \le n - 2\lfloor \frac{n}{4} \rfloor$.
Proof. Let

$$S = \begin{cases} \{v_{4i}, u_{4i+2} \ : \ 0 \le i \le m-1\}, & t = 0, \\ \{v_{4i}, u_{4i+2} \ : \ 0 \le i \le m-1\} \cup \{u_{4m-1}\}, & t = 1, \\ \{v_{4i}, u_{4i+2} \ : \ 0 \le i \le m-1\} \cup \{v_{4m-1}, u_{4m}\}, & t = 2, \\ \{v_{4i}, u_{4i+2} \ : \ 0 \le i \le m-1\} \cup \{v_{4m}, u_{4m+1}, u_{4m+2}\}, & t = 3. \end{cases}$$

Then $N[S] = V(P(n,3))$, S is a dominating set of $P(n,3)$ with $|S| = n - 2\lfloor \frac{n}{4} \rfloor$. Hence, $\gamma(P(n,3)) \le n - 2\lfloor \frac{n}{4} \rfloor$. □

Lemma 2.2. $\gamma(P(11,3)) = 6$.
Proof. Let $S = \{v_0, u_2, v_4, u_4, u_6, v_8, \}$, then $N[S] = V(P(11,3))$, S is a dominating set of $P(11,3)$ with $|S| = 6$. Hence, $\gamma(P(11,3)) \le 6$. By theorem 1.1, $\gamma(P(11,3)) \ge \lceil \frac{2\times 11}{1+3} \rceil = \lceil \frac{22}{4} \rceil = 6$. Hence $\gamma(P(11,3)) = 6$. □

In Figure 2.1, we show the dominating sets of $P(n,3)$ for $11 \le n \le 15$, where the vertices of S are in dark.

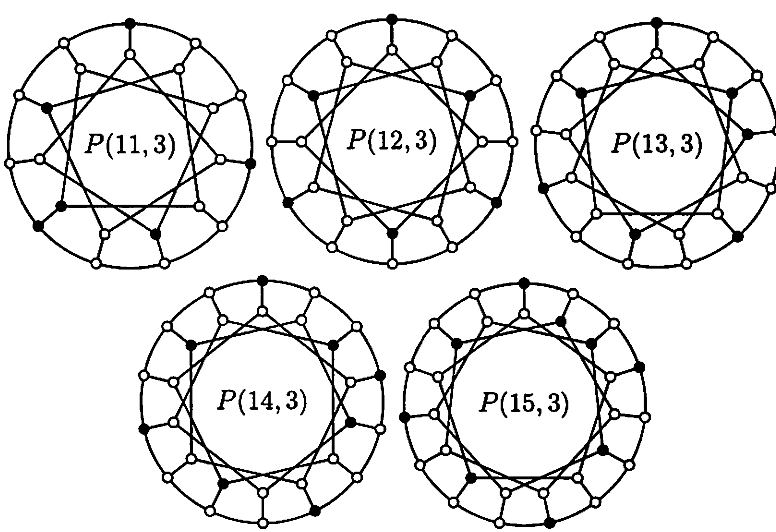

Figure 2.1. The dominating sets of $P(n,3)$ for $11 \le n \le 15$

Let S be an arbitrary dominating set of $P(n,3)$, then for each vertex $v \in V(G)$, $N[v] \cap S \ne \emptyset$, and v is being dominated $|N[v] \cap$

$S| \geq 1$ times. we define the function rd counting the times v is re-dominated as follows:

$$rd(v) = |N[v] \cap S| - 1.$$

For a vertex set $V' \subseteq V(G)$, let $rd(V') = \sum_{v \in V'} rd(v)$, then, for $n \geq 4$ and $n \neq 6$,

$$\begin{aligned} rd(V(P(n,3))) &= \textstyle\sum_{v \in V(G)} rd(v) \\ &= \textstyle\sum_{v \in V(G)} (|N[v] \cap S| - 1) \\ &= 4|S| - 2n. \end{aligned}$$

Lemma 2.3. If $t = 0$, then $\gamma(P(n,3)) \geq n - 2\lfloor \frac{n}{4} \rfloor$.
Proof. Let $|S| = \gamma(P(n,3))$, since $4|S| - 2n = rd(V(P(n,3))) \geq 0$, we have, $4|S| \geq 2n = 8m$, $\gamma(P(n,3)) = |S| \geq 2m = n - 2\lfloor \frac{n}{4} \rfloor$. □

Lemma 2.4. If $t = 1$, then $\gamma(P(n,3)) \geq n - 2\lfloor \frac{n}{4} \rfloor$.
Proof. Let $|S| = \gamma(P(n,3))$, since $4|S| - 2n = rd(V(P(n,3))) \geq 0$, we have, $4|S| \geq 2n = 8m+2$, $\gamma(P(n,3)) = |S| \geq \lceil \frac{8m+2}{4} \rceil = 2m+1 = n - 2\lfloor \frac{n}{4} \rfloor$. □

Let $V'(k,x) = \{v_{k+j}, u_{k+j} : 0 \leq j \leq x-1\}$, we have

Lemma 2.5. If there exists a $V'(k,4)$ with $|S \cap V'(k,4)| \leq 1$, then $rd(V(p(n,3))) \geq 2$ for $n \geq 9$ and $rd(V(p(n,3))) \geq 3$ for $n \geq 14$.
Proof. Suppose that there exists a $V'(k,4)$, say $V'(0,4)$, with $|S \cap V'(0,4)| \leq 1$. Since $N[v_1] \cap S \neq \emptyset$ and $N[v_2] \cap S \neq \emptyset$, we have $S \cap V'(0,4) \in \{v_1, v_2\}$. By symmetry, we can assume that $S \cap V'(0,4) = \{v_1\}$. Since $N[v_3] \cap S \neq \emptyset$, we have $v_4 \in S$. Since $N[u_3] \cap S \neq \emptyset$, we have $u_6 \in S$. Since $N[u_0] \cap S \neq \emptyset$, we have $u_{n-3} \in S$. Since $N[u_2] \cap S \neq \emptyset$, S contains at least one vertex of $\{u_{n-1}, u_5\}$.

Case 1. $u_{n-1} \in S$. If S contains at least one vertex of $\{u_{n-4}, v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}\}$, then $rd(V'(n-4,4)) \geq 2$, else, since $N[v_{n-2}] \cap S \neq \emptyset$ and $N[v_{n-4}] \cap S \neq \emptyset$, we have $u_{n-2} \in S$ and $v_{n-5} \in S$, $rd(V'(n-5,7)) \geq 2$. If $|S \cap V'(4,4)| > 2$, then $rd(V'(4,4)) \geq 1$, else, since $N[v_7] \cap S \neq \emptyset$ and $N[u_5] \cap S \neq \emptyset$, we have $v_8 \in S$ and $u_8 \in S$, $rd(V'(8,1)) \geq 2$ (see Figure 2.2 (1)).

Case 2. $u_{n-1} \notin S$, then $u_5 \in S$. Since $N[v_{n-1}] \cap S \neq \emptyset$, S contains at least one vertex of $\{v_{n-2}, v_{n-1}\}$, $rd(V'(n-3,4)) \geq 1$. Since $N[v_7] \cap S \neq \emptyset$, $rd(V'(4,5)) \geq 2$ (see Figure 2.2 (2)).

From cases 1-2, $rd(V'(n-5,14) \geq 3$, hence, $rd(V(p(n,3))) \geq 3$ for $n \geq 14$. If $u_{n-1} \in S$, then $rd(V'(n-5,9)) \geq 2$, else $rd(V'(0,9)) \geq 2$, hence $rd(V(p(n,3))) \geq 2$ for $n \geq 9$. □

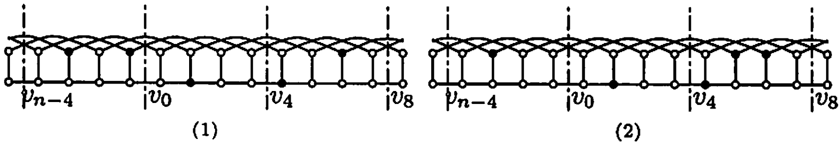

Figure 2.2.

Lemma 2.6. If $t = 2$, then $rd(V(p(n,3))) \geq 1$.
Proof. It is easy to check that the Lemma 2.6 holds when $n = 6$, and we therefore assume $n \geq 10$ in the rest of the proof. By contradiction, suppose $rd(V(p(n,3))) = 0$. If $S \cap \{v_0, v_1, \ldots, v_{n-1}\} = \emptyset$, then $S = \{u_0, u_1, \ldots, u_{n-1}\}$, $rd(V(p(n,3))) = 4|S| - 2n = 2n > 1$, a contradiction with $rd(V(p(n,3))) = 0$. Hence S contains at least one vertex of $\{v_0, v_1, \ldots, v_{n-1}\}$, say v_1. For $0 \leq i \leq m$, by Lemma 2.5, $|S \cap V'(4i,4)| \geq 2$. Since $rd(V(p(n,3))) = 0$, $S \cap V'(0,4) = \{v_1, u_3\}$.

Since $rd(V(p(n,3))) = 0$, we have $S \cap \{v_4, u_4\} = \emptyset$. Since $N[v_4] \cap S \neq \emptyset$, we have $v_5 \in S$. By Lemma 2.5, $|S \cap V'(4,4)| \geq 2$. Since $rd(V(p(n,3))) = 0$, we have $S \cap V'(4,4) = \{v_5, u_7\}$.

Continuing this way, we get $S \cap V'(4i,4) = \{v_{4i+1}, u_{4i+3}\}$ for $0 \leq i \leq m-1$. Then $rd(V(p(n,3))) \geq rd(u_0) \geq 1$, a contradiction with $rd(V(p(n,3))) = 0$(see Figure 2.3). □

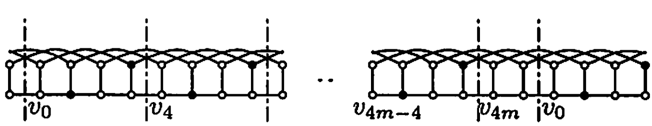

Figure 2.3.

Lemma 2.7. If $t = 2$, then $\gamma(P(n,3)) \geq n - 2\lfloor \frac{n}{4} \rfloor$.
Proof. From Lemma 2.6, $4|S| - 2n = rd(V(P(n,3))) \geq 1$, we have,

$4|S| \geq 2n+1 = 8m+5$, $|S| \geq \lceil \frac{8m+5}{4} \rceil = 2m+2 = n - 2\lfloor \frac{n}{4} \rfloor$. □

Lemma 2.8. For $t = 3$ and $n \geq 15$, if S contains a pair of vertices a, b with $(a,b) \in E(P(n,3))$, then $rd(V(p(n,3))) \geq 3$.
Proof. By contradiction. Suppose that there exists a pair of vertices a, b with $(a,b) \in E(P(n,3))$ and $rd(V(p(n,3))) \leq 2$. By symmetry, we only need to consider the cases $(a,b) \in \{(v_0,u_0),(v_0,v_1),(u_0,u_3)\}$.

Case 1. $(a,b) = (v_0,u_0)$. Since $rd(V(p(n,3))) \leq 2$, $S \cap \{v_1,v_2,v_3\} = \emptyset$. Since $N[v_2] \cap S \neq \emptyset$, we have $u_2 \in S$. Since $rd(V(p(n,3))) \leq 2$, $S \cap \{v_2,v_3,u_3\} = \emptyset$. Since $N[v_3] \cap S \neq \emptyset$, we have $v_4 \in S$. Since $rd(V(p(n,3))) \leq 2$, S does not contain any vertex of $\{v_3,u_3,u_4,v_5,u_5,v_6,u_6\}$, i.e. $|S \cap V'(3,4)| = 1$, by Lemma 2.5, $rd(V(p(n,3))) \geq 3$, a contradiction with $rd(V(p(n,3))) \leq 2$(see Figure 2.4 (1)).

case 2. $(a,b) = (v_0,v_1)$. Since $rd(V(p(n,3))) \leq 2$, $S \cap \{v_2,v_3,u_3\} = \emptyset$. Since $N[v_3] \cap S \neq \emptyset$, we have $v_4 \in S$. Since $rd(V(p(n,3))) \leq 2$, $S \cap \{u_{n-1},u_2,v_2\} = \emptyset$. Since $N[u_2] \cap S \neq \emptyset$, we have $u_5 \in S$. Then $rd(V(p(n,3))) \geq rd(V'(0,6)) \geq 3$, a contradiction with $rd(V(p(n,3))) \leq 2$(see Figure 2.4 (2)).

Case 3. $(a,b) = (u_0,u_3)$. Since $rd(V(p(n,3))) \leq 2$, $S \cap \{v_0,v_1,v_2,v_3\} = \emptyset$. S contains both vertices u_1 and u_2. Since $rd(V(p(n,3))) \leq 2$, $S \cap V'(4,3) = \emptyset$, $|S \cap V'(3,4)| = 1$, by Lemma 2.5, $rd(V(p(n,3))) \geq 3$, a contradiction with $rd(V(p(n,3))) \leq 2$(see Figure 2.4 (3)). □

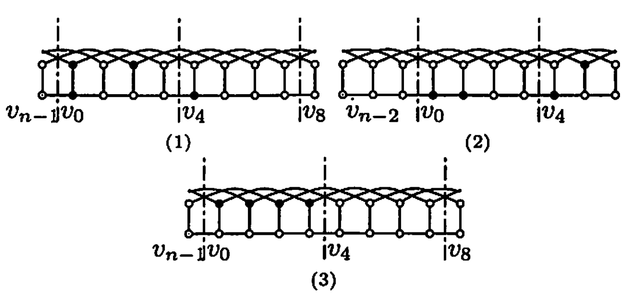

Figure 2.4.

Lemma 2.9. For $t = 3$ and $n \geq 15$, if there exists a set $V'(j,3)(0 \leq j \leq n-1)$ with $|S \cap V'(j,3)| \geq 3$, then $rd(V(p(n,3))) \geq 3$.
Proof. By contradiction. Suppose that there exists a set $V'(j,3)$, say $V'(0,3)$, with $|S \cap V'(0,3)| \geq 3$ and $rd(V(p(n,3))) \leq 2$, then

by Lemma 2.5, $|S \cap V'(4i+3,4)| \geq 2 (0 \leq i \leq m-1)$. Hence, $rd(V(p(n,3))) = 4|S| - 2n \geq 4 \times (2m+3) - 2 \times (4m+3) = 6$, a contradiction with $rd(V(p(n,3))) \leq 2$. □

Lemma 2.10. For $t = 3$ and $n \geq 15$, if there exists a set $V'(i,2) (0 \leq i \leq n-1)$ with $|S \cap V'(i,2)| \geq 2$, then $rd(V(p(n,3))) \geq 3$.
Proof. By contradiction. Suppose that there exists a set $V'(i,2)$, say $V'(1,2)$, with $|S \cap V'(1,2)| \geq 2$ and $rd(V(p(n,3))) \leq 2$, then by Lemma 2.9, $|S \cap V'(1,2)| = 2$. By symmetry, we only need to consider the cases $S \cap V'(1,2) \in \{\{v_1,u_1\},\{v_1,u_2\},\{v_1,v_2\},\{u_1,u_2\}\}$. By Lemma 2.8, $V'(1,2) \neq \{v_1,u_1\}$ and $V'(1,2) \neq \{v_1,v_2\}$. By Lemma 2.9, $S \cap V'(0,4) \in \{\{v_1,u_2\},\{u_1,u_2\}\}$.

Case 1. $S \cap V'(0,4) = \{u_1,u_2\}$. Since $N[v_3] \cap S \neq \emptyset$, we have $v_4 \in S$. Since $N[u_3] \cap S \neq \emptyset$, we have $u_6 \in S$. Since $N[v_0] \cap S \neq \emptyset$, we have $v_{4m+2} \in S$, $rd(V'(4m+2,6)) \geq 2$. Since $rd(V(p(n,3))) \leq 2$, S does not contain any vertex of $\{u_4,v_5,u_5,v_6,u_7,v_7\}$, we have $S \cap V'(4,4) = \{v_4,u_6\}$. Continuing this way, we have $S \cap V'(4l,4) = \{v_{4l},u_{4l+2}\}$ for $1 \leq l \leq m$. Then $S \cap V'(4m,3) = \{v_{4m},u_{4m+2}\}$, $rd(V'(4m,3)) \geq 3$, contradicting that $rd(V(p(n,3))) \leq 2$(see Figure 2.5 (1)).

Case 2. $S \cap V'(0,4) = \{v_1,u_2\}$. Since $N[v_3] \cap S \neq \emptyset$, we have $v_4 \in S$. Since $N[u_3] \cap S \neq \emptyset$, we have $u_6 \in S$. Since $N[u_0] \cap S \neq \emptyset$, we have $u_{4m} \in S$. Since $N[v_{4m+1}] \cap S \neq \emptyset$, we have $rd(V'(4m,6)) \geq 2$. Since $rd(V(p(n,3))) \leq 2$, S does not contain any vertex of $\{u_4,v_5,u_5,v_6,u_7,v_7\}$, we have $S \cap V'(4,4) = \{v_4,u_6\}$. Continuing this way, we have $S \cap V'(4l,4) = \{v_{4l},u_{4l+2}\}$ for $1 \leq l \leq m$. Then $S \cap V'(4m,4) = \{v_{4m},u_{4m+2}\}$, $rd(V'(4m,6)) \geq 3$, contradicting that $rd(V(p(n,3))) \leq 2$(see Figure 2.5 (2)). □

Lemma 2.11. If $t = 3$ and $n \geq 15$, then $rd(V(p(n,3))) \geq 3$.
Proof. By contradiction. Suppose $rd(V(p(n,3))) \leq 2$. If $S \cap \{v_0,v_1,\ldots,v_{n-1}\} = \emptyset$, then $S = \{u_0,u_1,\ldots,u_{n-1}\}$, $rd(V(p(n,3))) = 4|S| - 2n = 2n \geq 30$, a contradiction. Hence, S contains at least one vertex of $\{v_0,v_1,v_2,\ldots,v_{n-1}\}$, say v_1. Since $rd(V(p(n,3))) \leq 2$, by Lemma 2.10, $|S \cap V'(0,3)| = 1$. By Lemma 2.5, $|S \cap V'(0,4)| \geq 2$, by Lemma 2.9, $|S \cap V'(1,3)| \leq 2$, hence $|S \cap V'(0,4)| = 2$ and $S \cap V'(0,4) \in \{\{v_1,v_3\},\{v_1,u_3\}\}$.

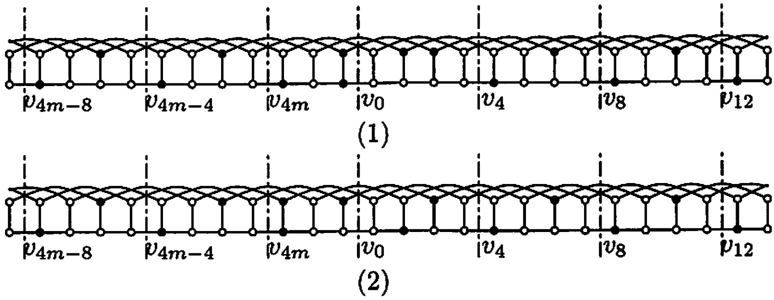

Figure 2.5.

Case 1. $S \cap V'(0,4) = \{v_1, v_3\}$. By Lemma 2.10, $|S \cap V'(3,2)| = 1$, since $N[u_4] \cap S \neq \emptyset$, hence $u_7 \in S$. By Lemma 2.10, $|S \cap V'(6,2)| = 1$, since $N[v_6] \cap S \neq \emptyset$, hence $v_5 \in S$. Since $N[u_6] \cap S \neq \emptyset$, we have $u_9 \in S$. Since $N[v_8] \cap S \neq \emptyset$, hence $|V'(7,3)| \geq 3$, contradicting Lemma 2.9(see Figure 2.6 (1)).

Case 2. $S \cap V'(0,4) = \{v_1, u_3\}$. By Lemma 2.10, $|S \cap V'(3,2)| = 1$. Since $N[v_4] \cap S \neq \emptyset$ and $N[u_4] \cap S \neq \emptyset$, hence $v_5 \in S$ and $u_7 \in S$. By Lemma 2.10, S does not contain any vertex of $\{v_4, u_4, u_5, v_6, u_6, v_7\}$, hence $S \cap V'(4,4) = \{v_5, u_7\}$. Continuing this way, we have $S \cap V'(4i,4) = \{v_{4i+1}, u_{4i+3}\}$ for $1 \leq i \leq m$. Then, $u_{4m+3} = u_0 \in S$, contradiction with $S \cap V'(0,4) = \{v_1, u_3\}$(see Figure 2.6 (2)). □

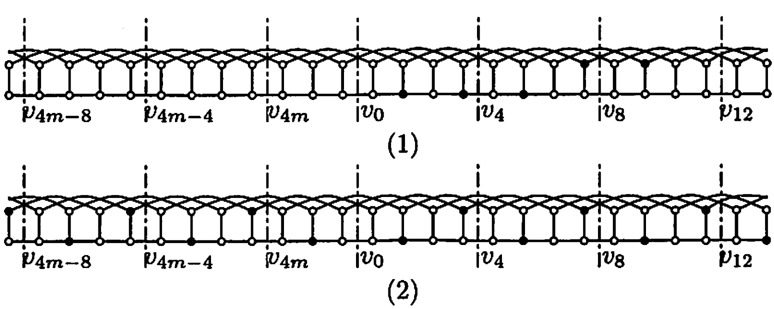

Figure 2.6.

Lemma 2.12. If $t = 3$, then $\gamma((P(n,3)) \geq n - 2\lfloor \frac{n}{4} \rfloor (n \neq 11)$.
Proof. We leave for reader to verify that $\gamma((P(n,3)) \geq 5 = n - 2\lfloor \frac{n}{4} \rfloor$ for $n = 7$. For $n \geq 15$, by Lemma 2.11, $4|S| - 2n = rd(V(P(n,3))) \geq$

3, we have, $4|S| \geq 2n+3 = 8m+9$, $|S| \geq \lceil \frac{8m+9}{4} \rceil = 2m+3 = n-2\lfloor \frac{n}{4} \rfloor$.
□

From Lemmas 2.1-2.4,2.7,2.12, we have

Theorem 3.1.

$$\gamma(P(n,3)) = \begin{cases} 6, & n = 11, \\ n - 2\lfloor \frac{n}{4} \rfloor, & n \neq 11. \end{cases}$$

□

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