On Multiplicity of triangles in 2-edge colouring of graphs

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Abstract

We denote by $G(n)$, the graph obtained by removing a Hamilton cycle from the complete graph K_n. In this paper, we calculate the lower bound for the minimum number of monochromatic triangles in any 2-edge colouring of $G(n)$ using the weight method. Also, by explicit constructions, we give an upper bound for the minimum number of monochromatic triangles in 2-edge colouring of $G(n)$ and the difference between our lower and upper bound is just two.

1 Introduction and background results

If F and G are graphs, define $M(G,F)$ to be the minimum number of monochromatic G that occur in any 2-colouring of the edges of F. $M(G,F)$ is called the multiplicity of G in F. A.W.Goodman [2] has proved that

$$\begin{aligned} M(K_3,K_n) &= \frac{1}{3}\, u\,(u-1)\,(u-2) && if\ n=2u \\ &= \frac{2}{3}\, u\,(u-1)\,(4u+1) && if\ n=4u+1 \\ &= \frac{2}{3}\, u\,(u+1)\,(4u-1) && if\ n=4u+3 \end{aligned}$$

where u is a nonnegative integer. A cycle in a graph is said to be a Hamilton cycle if it contains all the vertices of the graph. We denote by $G(n)$, the graph obtained by removing a Hamilton cycle from the complete graph K_n. In this paper, we calculate the lower bound for the minimum number of monochromatic triangles in any 2-edge colouring of $G(n)$ using the weight method. Also, by explicit constructions, we give an upper bound for the minimum number of monochromatic triangles in 2-edge colouring of $G(n)$ and the difference between our lower and upper bound is just two. The idea of proof in this paper is similar to the one used in the papers of Goodman [2] and Sauve [4] but somewhat more complicated due to the fact that we are not dealing with complete graphs. We follow the method of weights given in [4]. For the basic definitions and notations used in this paper, we follow [1].

2 Method of Weights

Let $\mathbf{G} = \mathbf{G(n)}$. Our aim is to find out the minimal number of monochromatic triangles when the edges of G are coloured with two colours. For this we give weight to each pair of edges at every vertex p of G. Let $\mathcal{A}(p)$ be the set of all pairs of edges at a vertex p in G. Suppose $a \in \mathcal{A}(p)$. We define $W(a) = 2$, if both the edges are of the same colour and $W(a) = -1$ otherwise. For every vertex p of G we define $W(p)$, the weight at the vertex p, to be $\sum_{a \in \mathcal{A}(p)} W(a)$. Let $W(G) = \sum_{p \in V(G)} W(p)$. where $V(G)$ is the vertex set of G. We define the weight of the graph G as $W(G)$.

Let $\mathcal{B}$ be the set of all subgraphs of G induced by any three of the vertices of the graph G. As any pair of edges at a vertex p of the graph G lies in exactly one subgraph of G induced by three vertices we get $W(G) = \sum_{B \in \mathcal{B}} W(B)$. These subgraphs in $\mathcal{B}$ fall under any one of the following 4 sets.

1. S_1, the set of all subgraphs induced by 3 vertices such that the subgraphs have 3 edges of the same colour.

2. S_2, the set of all subgraphs induced by 3 vertices such that the subgraphs have 3 edges, not of the same colour.

3. S_3, the set of all subgraphs induced by 3 vertices such that the subgraphs have 2 edges of the same colour and a nonedge.

4. S_4, the set of all subgraphs induced by 3 vertices such that the subgraphs have 2 edges of different colours and a nonedge.

Clearly

$$\begin{aligned} W(B) &= 6, && \text{if } B \in S_1 \\ W(B) &= 0, && \text{if } B \in S_2 \\ W(B) &= 2, && \text{if } B \in S_3 \\ W(B) &= -1, && \text{if } B \in S_4 \end{aligned}$$

Hence

$$W(G) = 6|S_1| + 2|S_3| - |S_4|,$$

where for any set X, $|X|$ denotes the cardinality of the set X. Thus

$$|S_1| = \frac{1}{6}\left(W(G) - 2|S_3| + |S_4|\right).$$

Let $S_3(p)$ be the number of elements of the set S_3 where the two edges of the same colour are incident at p and $S_4(p)$ be the number of subgraphs of the set S_4 where the two edges of opposite colours are incident at p. It is easy to see that $|S_3| = \sum_{p\in V(G)} S_3(p)$ and $|S_4| = \sum_{p\in V(G)} S_4(p)$. Therefore we get,

$$|S_1| = \frac{1}{6}\left(\sum_{p\in V(G)} W(p) - 2\sum_{p\in V(G)} S_3(p) + \sum_{p\in V(G)} S_4(p)\right). \tag{1}$$

Also whatever be the colouring of the graph G, $S_3(p) + S_4(p)$ is a constant, as this is precisely the number of pairs of edges $\{pv, pw\}$ such that $vw \notin E(G)$, where $E(G)$ is the edge set of G. Therefore at any vertex p, maximizing $S_3(p)$ is equivalent to minimizing $S_4(p)$. From equation (1) the graph G will have the minimum number of monochromatic triangles if it satisfies the following two conditions.

$(*)_1$ At every vertex p of G, almost equal number of edges of each colour are incident with.

$(*)_2$ At every vertex p of G, whenever v_iv_j is a nonedge, the edges pv_i and pv_j are of the same colour.

If a graph G with 2-colouring of the edges has the minimum number of monochromatic triangles then that colouring of G is said to be a **minimal colouring**. From the forgoing discussion we get the following Proposition.

Proposition 2.1 *Let G be a graph on finite number of vertices. A two colouring of the edges of G will be a minimal colouring if the coloring satisfies the conditions $(*)_1$ and $(*)_2$.*

3 Multiplicity of Triangles in G(n)

Let the vertices of K_n be $v_1, v_2, \ldots, v_n$. Suppose we remove the edges

$$v_1v_2,\ v_2v_3,\ \ldots,\ v_{n-1}v_n,\ v_nv_1$$

to get $G(n)$. We shall colour the edges of $G(n)$ with the two colours red and blue.

Proposition 3.1 $M(K_3, G(n)) = 0$ for all $n \leq 11$.

Proof : Note that $G(n-1)$ is obtained from $G(n)$ by deletion of one vertex and one edge. It then follows that the restriction of the coloring of $G(n)$ gives a colouring of $G(n-1)$. Therefore, it suffices to prove the statement for $G(11)$ alone. Now we give a construction of $G(11)$ which has no monochromatic triangles.

Let $v_1, v_2, \ldots, v_{11}$ be the vertices of $G(11)$. For $1 \leq i, j \leq 11$, v_iv_j is a red edge only if

$$j \equiv i+2,\ i+3,\ i-2,\ i-3\ (mod\ 11).$$

To see that there is no monochromatic triangle in this construction, it suffices to look at the edges incident at v_1. v_1 is adjacent to v_3, v_4, v_{10}, v_9 by red edges and between these vertices there is no red edge. Similarly, there is no triangle of blue colour. So,

$$M(K_3, G(11)) = 0.$$

Hence, $M(K_3, G(n)) = 0$ for all $n \leq 11$. ⊔

Hereafter, in all our discussion in this paper, we consider n ≥ 12.

In $G(n)$, the degree of each vertex will be $n-3$. $S_3(p) + S_4(p) = n-4$ at each vertex p. If condition $(*)_2$ is to be satisfied at a vertex p, then all the edges incident at that vertex should be of the same colour and so condition $(*)_1$ cannot be satisfied at p. So, if condition $(*)_1$ is to be satisfied at any vertex p, then $S_4(p) \geq 1$.

Lemma 3.2 *Let p be any vertex of the graph $G(n)$. The number of monochromatic triangles in any 2-edge colouring of the graph $G(n)$ is more when $S_4(p)$ is zero than when $S_4(p) = 1$ and condition $(*)_1$ is satisfied at p.*

Proof : Recall the Weight Equation (1.1),

$$|S_1| = \frac{1}{6}\left(\sum_{p\in V(G)} W(p) - 2\sum_{p\in V(G)} S_3(p) + \sum_{p\in V(G)} S_4(p)\right).$$

Let $T(p) = W(p) - 2S_3(p) + S_4(p)$. At any vertex p of $G(n)$,

$$S_3(p) + S_4(p) = n - 4.$$

Case 1 : Let $S_4(p) = 0$. Then all the edges incident at p are of the same colour. Note that in this case condition $(*)_1$ is far from satisfied. Hence,

$$\begin{aligned} T(p) &= 2\binom{n-3}{2} - 2(n-4) \\ &= (n-3)(n-4) - 2(n-4) \end{aligned}$$

Case 2 : Let $(*)_1$ be satisfied at p and $S_4(p) = 1$.

$$T(P) = 2\binom{x}{2} + 2\binom{y}{2} - xy - 2(n-5) + 1$$

where $x = y = \frac{n-3}{2}$ if $n - 3$ is even and
$x = [\frac{n-3}{2}]$ and $y = [\frac{n-5}{2}]$ if $n - 3$ is odd.

From the above two cases it is clear that $T(p)$ in Case 1 is greater than $T(p)$ in Case 2. Hence, the number of monochromatic triangles is more in Case 1 than in Case 2. ⊔

Hence, in any minimal colouring of $G(n)$, $(*)_1$ is satisfied and $S_4(p) \geq 1$ at each vertex p. Let p be a vertex of $G(n)$ such that $(*)_1$ is satisfied. Then from the Weight Equation (1.1) it is easily seen that the number of monochromatic triangles in $G(n)$ is less when $S_4(p) = 1$ than in the case $S_4(p) > 1$. In fact, the number of monochromatic triangles in $G(n)$ increases by half when $S_4(p)$ increases by one at any vertex p.

Notation : We read the subscripts modulo n, i.e.,

$$v_\alpha = v_\beta \text{ if } \alpha \equiv \beta \ (mod\ n).$$

Definition 3.3 *Let $n = 2m$ for some $m \in \mathbf{N}$. Two vertices v_i and v_j are said to be diagonal vertices in $G(n)$ if $i - j \equiv m \ (mod\ n)$.*

Proposition 3.4 *Consider $G(n)$ where $n \geq 12$. Given any 5 vertices in $G(n)$ then,*
(i) there exist three vertices among these five such that any two are adjacent if $n = 2m + 1$ for some $m \in \mathbf{N}$.
(ii) there exist three vertices among these five such that any two are adjacent and no two of them are diagonal vertices if $n = 2m$ for some $m \in \mathbf{N}$.

Proof : Case 1 : $n = 2m + 1$.
Note that in $G(n)$ every vertex is adjacent to all but two vertices. Let H be the subgraph induced by the given five vertices. Then each vertex has degree at least 2 in H. Since $n \geq 12$, at least one vertex has degree at least 3. Clearly there is a cycle in H. If it is a 3-cycle, we are done. Otherwise we have a 5-cycle and since at least one vertex has degree at least 3, we have a chord of this 5-cycle which produces a triangle.
Case 2 : $n = 2m$.
Using the proof given in Case 1, we get three vertices say v_i, v_j, v_k such that any two are adjacent. If no two of these three vertices are diagonal vertices, we are done. Suppose v_i and v_j are diagonal vertices. Now consider the other two vertices say, v_l and v_m. These two vertices are adjacent to at least one of v_i and v_j. Without loss of generality assume that v_l (or v_m) is adjacent to v_i.

Sub-Case (a) : v_l (or v_m) is adjacent to v_k.
(i) v_l (or v_m) and v_k are not diagonal vertices. Then, v_i, v_k, v_l (or v_m) are the required vertices.
(ii) v_l (or v_m) and v_k are diagonal vertices. Then v_m (or v_l) lies between any two of the remaining four vertices. Without loss of generality assume v_m lies between v_i and v_k. Then v_j, v_m, v_l are the required vertices.
Sub-Case (b) : v_l and v_m are both non-adjacent to v_k.
Assume without of generality that in the clockwise order we have

$$v_i, v_l, v_k, v_m, v_j$$

where v_l and v_m are non-adjacent to v_k. Since $n \geq 12$, one of v_iv_l or v_mv_j must be an edge. We then get the triangle $v_iv_lv_m$ or $v_jv_mv_l$. ⊔

Proposition 3.5 *Consider a colouring of $G(n)$ such that $(*)_1$ is satisfied and $S_4(p) \geq 1$ at each vertex p. Then, there are at most 4 vertices p such that $S_4(p) = 1$ in this colouring.*

Proof : Let $v_1, v_2, \ldots v_n$ be the vertices of $G(n)$. The proof is essentially based on the following idea.
For any vertex v_l with $S_4(v_l) = 1$, if we look at the edges v_lv_j incident at v_l then we find a band of red(blue) edges on one side (about half as degree of v_l) and a band of blue(red) edges on the other side (as we move on the Hamilton cycle).
Case 1 : Let $n = 2m + 1$ for some integer $m \geq 6$.
If v_l is any vertex of $G(n)$ such that $(*)_1$ is satisfied and $S_4(v_l) = 1$, then the edges $v_lv_{l+t}, 2 \leq t \leq m$ are of one colour and the edges $v_lv_{l-t}, 2 \leq t \leq m$ are of the other colour.

Suppose there exist 5 vertices p such that $S_4(p) = 1$. Then there exist 3 vertices among these 5 say, v_i, v_j, v_k such that any two are adjacent (by Proposition 3.4). This implies that there exists a vertex $\in \{v_i, v_j, v_k\}$ say v_i such that v_iv_j and v_iv_k are of the same colour.

Let v_iv_j, v_iv_k be red. Without loss of generality we can assume that

$$v_j, v_k \in \{v_{i+t} \mid 2 \leq t \leq m\} \text{ and}$$

$$j - i \ (mod\ n) < k - i \ (mod\ n).$$

Now, $S_4(v_k) = 1$ and v_kv_i is red implies that v_kv_j is also red. So, v_jv_i and v_jv_k both are red. This implies, $S_4(v_j) > 1$, a contradiction.
Case 2 : Let $n = 2m$ for some integer $m \geq 6$.
If v_l is any vertex of $G(n)$ such that $(*)_1$ is satisfied and $S_4(v_l) = 1$, then the edges $v_lv_{l+t}, 2 \leq t \leq m-2$ are of one colour and the edges $v_lv_{l-t}, 2 \leq t \leq m-2$ are of the other colour.

Suppose there exist 5 vertices p such that $S_4(p) = 1$. Then by Proposition 3.4 there exist 3 vertices among these 5 say, v_i, v_j, v_k such that any two are adjacent and

$$i-j,\ j-k,\ k-i \not\equiv m-1\ (mod\ n).$$

This implies that there exists a vertex $\in \{v_i, v_j, v_k\}$ say v_i such that v_iv_j and v_iv_k are of the same colour.

Let v_iv_j, v_iv_k be red. Without loss of generality we can assume that

$$v_j, v_k \in \{v_{i+t} \mid 2 \le t \le m-2\} \text{ and}$$

$$j-i\ (mod\ n) < k-i\ (mod\ n).$$

Now, $S_4(v_k) = 1$ and v_kv_i is red implies that v_kv_j is also red. So, v_jv_i and v_jv_k both are red. This implies, $S_4(v_j) > 1$, a contradiction.
From the above two cases it is clear that $S_4(p) = 1$ in at most 4 vertices. ⊔

From Proposition 3.5, we get the following theorem.

Theorem 3.6 *Let $\alpha(n)$ be the number of monochromatic triangles in $G(n)$ where, $(*)_1$ is satisfied, $S_4(p) = 1$ at four vertices and $S_4(p) = 2$ at all other vertices p. Then, $M(K_3, G(n)) \ge \alpha(n)$ where $\alpha(n)$ is equal to*

$$\begin{array}{ll} \frac{1}{3}\ (u^3 - 7u^2 + 16u - 6) & if\ n = 2u, \\ \frac{1}{3}\ (8u^3 - 30u^2 + 22u + 3) & if\ n = 4u+1, \\ \frac{1}{3}\ (8u^3 - 18u^2 - 2u + 6) & if\ n = 4u+3, \end{array}$$

and u is a non-negative integer.

Proof : Case 1 : $n = 2u$ where u is a non-negative integer.
The degree of each vertex in $G(n)$ is $2u-3$. By Condition $(*)_1$, the weight of each vertex is minimum when its degree pair is $(u-1, u-2)$ or $(u-2, u-1)$. Also, $S_3(p) + S_4(p) = 2u - 4$. From the Weight Equation (1.1),

$$\begin{aligned} 6|S_1| &\ge 2u\ \{2\binom{u-1}{2} + 2\binom{u-2}{2} - (u-1)(u-2)\} + \\ &\quad + (2u-4)\ \{(2u-6)(-2) + 2\} \\ &\quad + 4\{(2u-5)(-2) + 1\} \\ &= 2u^3 - 14u^2 + 32u - 12 \\ \Longrightarrow |S_1| &\ge \frac{1}{3}(u^3 - 7u^2 + 16u - 6) \end{aligned}$$

Case 2 : $n = 4u + 1$ where u is a non-negative integer.
The degree of each vertex in $G(n)$ is $4u - 2$. By Condition $(*)_1$, the weight

of each vertex is minimum when its degree pair is $(2u-1, 2u-1)$. But since the number of vertices is odd, all the vertices cannot have odd degree, i.e. $2u-1$. So, to attain the next possible minimum, one vertex should have degree pair $(2u, 2u-2)$. Also, $S_3(p) + S_4(p) = 4u - 3$. From the Weight Equation (1.1),

$$\begin{aligned}
6|S_1| &\geq 4u\ \{2\binom{2u-1}{2} + 2\binom{2u-1}{2} - (2u-1)^2\} \\
&\quad +1\ \{2\binom{2u}{2} + 2\binom{2u-2}{2} - 2u(2u-2)\} \\
&\quad + (4u-3)\ \{(4u-5)(-2)+2\} \\
&\quad + 4\ \{(4u-4)(-2)+1\} \\
&= 16u^3 - 60u^2 + 44u + 6 \\
\Longrightarrow |S_1| &\geq \frac{1}{3}(8u^3 - 30u^2 + 22u + 3)
\end{aligned}$$

Case 3 : $n = 4u + 3$ where u is a non-negative integer.
The degree of each vertex in $G(n)$ is $4u$. By Condition $(*)_1$, the weight of each vertex is minimum when its degree pair is $(2u, 2u)$. In this case, $S_3(p) + S_4(p) = 4u - 1$. From the Weight Equation (1.1),

$$\begin{aligned}
6|S_1| &\geq (4u+3)\ \{2\binom{2u}{2} + 2\binom{2u}{2} - (2u)^2\} \\
&\quad + (4u-1)\ \{(4u-3)(-2)+2\} \\
&\quad + 4\ \{(4u-2)(-2)+1\} \\
&= 16u^3 - 36u^2 - 4u + 12 \\
\Longrightarrow |S_1| &\geq \frac{1}{3}(8u^3 - 18u^2 - 2u + 6)
\end{aligned}$$

□

4 An upper bound for $\mathbf{M(K_3, G(n))}$

Condition C1 :
A colouring of $G(n)$ is said to satisfy condition C1 if it satisfies condition $(*)_1$ and $S_4(p) = 2$ for all vertices p of the graph $G(n)$.

Now, using induction we prove that for all $n \geq 12$, there exists a colouring of $G(n)$ satisfying Condition C1.

Condition C2 :
Let u be a non-negative integer. A colouring of $G(n)$, where $n = 4u$, is said to satisfy condition C2 if $2u+2$ vertices of $G(n)$ have red degree $2u-2$ and the remaining $2u-2$ vertices have red degree $2u-1$.

We give only the red edges in all the constructions below.

Lemma 4.1 *When $n = 12$, there exists a colouring of $G(n)$ such that conditions C1 and C2 are satisfied.*

Proof : Let $n = 12$, which is of the form $4u$, where $u = 3$.
Let $v_1, v_2, \ldots, v_{12}$ be the vertices of the graph $G(n)$. The degree of each vertex is 9. So, to satisfy condition $(*)_1$, the red degree at each vertex should be 4 or 5. Consider the construction of $G(12)$ given in Figure 1, only with the red edges present.

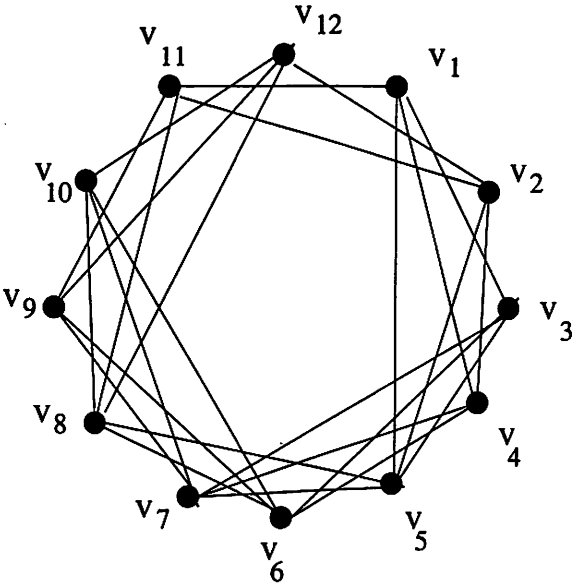

Figure 1:

In this construction, $2u+2 = 8$ vertices have degree 4 and the remaining $2u - 2 = 4$ vertices have degree 5. Also at each vertex $p, S_4(p) = 2$. So, both the conditions C1 and C2 are satisfied. ⊔

Lemma 4.2 *Let u be a non-negative integer. If there exists a construction of $G(n)$, where $n = 4u$ and $u \geq 4$ such that conditions C1 and C2 are satisfied, then*
(a) There exists a construction of $G(4u + 1)$ satisfying condition C1.
(b) There exists a construction of $G(4u + 2)$ satisfying condition C1.
(c) There exists a construction of $G(4u + 3)$ satisfying condition C1.
(d) There exists a construction of $G(4u + 4)$ satisfying conditions C1 and C2.

Proof : Step 0 : Assume that there exists a construction of $G(4u)$, where u is an integer ≥ 4, satisfying conditions C1 and C2. This implies that $2u+2$ vertices say,

$$v_1, v_2, \ldots, v_{u+1} \text{ and } v_{3u}, v_{3u+1}, \ldots, v_{4u}$$

have degree $2u-2$ and the remaining $2u-2$ vertices have degree $2u-1$. Also at each vertex p, $S_4(p) = 2$.

Step 1 : $n = 4u+1$.

The degree at any vertex is $4u-2$ and so to satisfy condition $(*)_1$, red degree at any vertex should be $2u-1$. Since the number of vertices is odd, all the vertices cannot have red degree $2u-1$. So to attain the next possible minimum let one vertex have red degree $2u$ or $2u-2$.

Add the vertex v_{4u+1} to the construction obtained in Step 0 and join the vertices as below.

Join v_1 and v_{4u}. Join v_{4u+1} with the vertices

$$v_2, v_3, \ldots, v_{u+1} \text{ and } v_{3u}, \ldots, v_{4u-1}.$$

Now all the vertices except v_{4u+1} have degree $2u-1$ and the vertex v_{4u+1} has degree $2u$. Also at each vertex $p, S_4(p) = 2$. So, condition C1 is satisfied.

Step 2 : $n = 4u+2$.

The degree at any vertex is $4u-1$ and so to satisfy condition $(*)_1$, red degree at any vertex should be $2u-1$ or $2u$.

Add the vertex v_{4u+2} to the construction obtained in Step 1 and join the vertices as below. Join v_{4u+2} with the vertices

$$v_2, v_3, \ldots, v_{u-1} \text{ and } v_{3u}, v_{3u+1}, \ldots, v_{4u}.$$

Now the vertices

$$v_2, v_3, \ldots, v_{u-1} \text{ and } v_{3u}, v_{3u+1}, \ldots, v_{4u+1}$$

have degree $2u$ and all the remaining vertices have degree $2u-1$. Also, at each vertex $p, S_4(p) = 2$. So, condition C1 is satisfied.

Step 3 : $n = 4u+3$.

The degree at any vertex is $4u$ and so to satisfy condition $(*)_1$, red degree at any vertex should be $2u$.

Add the vertex v_{4u+3} to the construction obtained in Step 2 and join the vertices as below. Join v_1 and v_{4u+2}. Join v_{4u+3} with the vertices

$$v_u, v_{u+1}, \ldots, v_{3u-1}.$$

Now all the vertices have degree $2u$ and also at each vertex $p, S_4(p) = 2$. So, condition C1 is satisfied.

Step 4 : $n = 4u + 4$.
The degree at any vertex is $4u + 1$ and so to satisfy condition $(*)_1$, red degree at any vertex should be $2u$ or $2u + 1$.

Add the vertex v_{4u+4} to the construction obtained in Step 3 and join the vertices as below. Join v_{4u+4} with the vertices $v_{u+3}, v_{u+4}, \ldots, v_{3u+2}$. Now the vertices $v_{u+3}, v_{u+4}, \ldots, v_{3u+2}$ have degree $2u + 1$ and all other vertices have degree $2u$.

So we have at $2(u + 1) + 2 = 2u + 4$ vertices the degree as $2u$ and at $2(u + 1) - 2 = 2u$ vertices the degree as $2u + 1$. Also, at each vertex $p, S_4(p) = 2$. This construction satisfies both the conditions C1 and C2. ⊔

From Lemma 4.1 and Lemma 4.2 and by induction hypothesis, we get the following result.

Theorem 4.3 *For all $n \geq 12$, there exists a colouring of $G(n)$ satisfying Condition C1.*

Let $\beta(n)$ be the number of monochromatic triangles in $G(n)$ when Condition C1 is satisfied. It is easy to see that for any n, $\beta(n)$ is exactly two more than the number $\alpha(n)$ given in Theorem 3.6 (since the number of monochromatic triangles in $G(n)$ increases by half when $S_4(p)$ increases by one at a vertex p). Hence from Theorems 3.6 and 4.3 we get the following result.

Theorem 4.4

$$M(K_3, G(n)) = 0 \text{ for all } n \leq 11 \text{ and}$$

$$\beta(n) - 2 \leq M(K_3, G(n)) \leq \beta(n) \text{ for all } n \geq 12$$

where $\beta(n)$ is the number of monochromatic triangles in $G(n)$ such that $(*)_1$ is satisfied and $S_4(p) = 2$ at all vertices p and $\beta(n)$ is equal to

$$\begin{array}{ll} \frac{1}{3}\,(u^3 - 7u^2 + 16u) & if\ n = 2u, \\ \frac{1}{6}\,(16u^3 - 60u^2 + 44u + 18) & if\ n = 4u + 1, \\ \frac{1}{6}\,(16u^3 - 36u^2 - 4u + 24) & if\ n = 4u + 3. \end{array}$$

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