An Expansion Technique on Super Edge-Magic Total Graphs

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Abstract

We denote by *(p, q)-graph* G a graph with p vertices and q edges. *An edge-magic total (EMT)* labeling on a (p,q)-graph G is a bijection $\lambda : V(G) \cup E(G) \to \{1, 2, ..., p+q\}$ with the property that, for each edge xy of G, $\lambda(x) + \lambda(xy) + \lambda(y) = k$, for a fixed positive integer k. Moreover, λ is *a super edge-magic total labeling (SEMT)* if it has the property that $\lambda(V(G)) = \{1, 2, \ldots, p\}$. A (p,q)-*graph* G is called *EMT* (*SEMT*) if there exists an EMT (SEMT) labeling of G. In this paper, we propose further properties of the SEMT graph. Based on these conditions, we will give the new theorems how to construct new SEMT (bigger) graphs from old (smaller) ones. We also give the SEMT labeling of $P_n \cup P_{n+m}$ for possible magic constants k and $m = 1, 2$ or 3.

Key words and phrases: *Labeling, EMT, SEMT, Dual Labeling, Magic Constant, Magic Graph.*

1 INTRODUCTION

We consider finite undirected graphs without loops and multiple edges. The notation $V(G)$ and $E(G)$ stand for the vertex set and edge set of graph G, respectively. We denote by T_n a tree on n vertices, $K_{1,n-1}$ a star on n vertices, and P_n a path on n vertices. The general references for graph-theoretic ideas can be found in [12] and [17].

We denote by (p,q)-*graph* G a graph with p vertices and q edges. *An edge-magic total labeling (EMT)* on a (p,q)-graph G is a bijection $\lambda : V(G) \cup E(G) \rightarrow \{1,2,...,p+q\}$ with the property that, for each edge xy of G, $\lambda(x)+\lambda(xy)+\lambda(y) = k$, for a fixed positive integer k. Moreover, λ is a *super edge-magic total (SEMT)* labeling if it has the property that $\lambda(V(G)) = \{1,2,\ldots,p\}$. We shall follow [16] to call $\lambda(x)+\lambda(xy)+\lambda(y)$ the *edge sum* of xy, and k the *magic constant* of the graph G.

A (p,q)-graph G is called *EMT* (resp. *SEMT*) if there exists an EMT (SEMT) labeling on G. EMT graphs were first discussed by Kotzig and Rosa [1] (under the name of graph with magic valuation). The term of a SEMT graph was introduced by Enomoto et al. [6].

A number of classification studies on EMT (resp. SEMT) graphs has been intensively investigated. In [1] and [13] it is proved that every cycle C_n and caterpillar are EMT. Kotzig and Rosa [3] showed that no complete graph K_n with $n > 6$ is EMT and gave some EMT labelings for K_n, $3 \leq n \leq 6$, $n \neq 4$. Wallis et al. [16] showed that all paths P_n and all n-suns are EMT. In [14] and [11], the relation between SEMT labelings and other labelings such as harmonious, cordial, graceful, and anti-magic was studied. Recently, Bača et al. [10] have shown that the friendship graph F_n has a super $(a,0)$-edge-antimagic total labeling (SEMT in our terminology) for $n \in \{1,3,4,5,7\}$.

Some conjectures are still open, namely that all trees are EMT by [1] and SEMT [6]. Enomoto et al. [6] have checked, by a computer, that all trees with less than or equal to 16 vertices are SEMT.

For disconnected graphs, in [1] it was proved that nP_2 is SEMT if and only if n is odd. Kotzig [2] showed that if G is a trichromatic graph and G is EMT then a disjoint union of n (n odd) identical copies of G is also EMT. In particular, if $G = P_3$ then graph nP_3, for n odd, is EMT. Furthermore, Baskoro and Ngurah [4] proved that nP_3 is also SEMT for n even, $n \geq 4$.

Figueroa-Centeno et al. [15] showed that $P_3 \cup nP_2$ is SEMT for every $n \geq 1$; $P_2 \cup P_n$ is SEMT for every $n \geq 3$; $mK_{1,n}$ is SEMT for m odd and for every $n \geq 1$; and graph mP_n is SEMT for any m and for any odd n. The SEMT characterization of the $nP_3 \cup kP_2$ and $K_{1,m} \cup K_{1,n}$ can be found in [9].

People also consider how to construct a new (bigger) SEMT graphs from some known (smaller) SEMT graphs. These constructions are proposed by inserting some new pendant edges and points, see for instance [5] and [7]. For other results concerning SEMT graphs can be seen in [8].

In this paper, we also propose new constructions of SEMT (bigger) graphs from old (smaller) ones. By using this construction, we can have more classes of SEMT graphs. We also give SEMT labelings for graph $P_n \cup P_{n+m}$, with $n \geq 2$ and $m = 1, 2$, or 3, for all possible magic constants.

2 Necessary Conditions and Duality

Figueroa-Centeno et al. [15] gave some necessary conditions for a graphs being SEMT as in the following lemma.

Lemma 1 *A (p,q)-graph G is SEMT if and only if there exists a bijective function $f : V(G) \to \{1, 2, ..., p\}$ such that the set $S = \{f(u) + f(v) : uv \in E(G)\}$ consists of q consecutive integers. In such a case, f extends to a SEMT labeling of G with the magic constant $k = p + q + s$, where $s = min(S)$ and $S = \{k - (p+1), k - (p+2), \cdots, k - (p+q)\}$.*

Further properties of SEMT graph proposed in the next lemmas will be useful in the next section.

Lemma 2 *Let λ be an EMT labeling on a (p,q)-graph G with the magic constant k. Then, the labeling λ_1 defined:*

$$\lambda_1(x) = \lambda(x) + n, \ \forall x \in V(G), \text{ and}$$
$$\lambda_1(xy) = \lambda(xy) + n, \ \forall xy \in E(G), \text{ for } n \geq 0.$$

has the magic constant $k_1 = k + 3n$.

Let $uv \in E(G)$. Then,
$k_1 = \lambda_1(u) + \lambda_1(uv) + \lambda_1(v)$

$$\begin{aligned} &= \lambda(u) + n + \lambda(uv) + n + \lambda(v) + n \\ &= \lambda(u) + \lambda(uv) + \lambda(v) + 3n \\ &= k + 3n. \quad \square \end{aligned}$$

Lemma 3 *[5] Let λ be a SEMT labeling of a (p,q)-graph G with the magic constant k. Then, the labeling λ' defined:*

$$\lambda'(x) = p + 1 - \lambda(x), \ \forall x \in V(G), \text{ and}$$
$$\lambda'(xy) = 2p + q + 1 - \lambda(xy), \ \forall xy \in E(G)$$

is a SEMT labeling with the magic constant $k' = 4p + q + 3 - k$.

The labeling λ' is called a *dual super labeling of* λ on G.

3 Expansion Technique on SEMT Graphs

The theorems proposed in this section are expansion techniques on SEMT graphs.

Theorem 1 *Let* G *be a (p,q)-graph having a SEMT labeling* λ *with the magic constant* $k \geq 2p + 2$, $p \geq 2$. *Let* G_1 *be a graph constructed from* G *by joining* h *new vertices* x_i *to* z_i *and* z_{i+1} $(1 \leq i \leq h)$, *where* $z_i \in V(G)$ *with* $\lambda(z_i) = k - 2p - 2 + i$. *If* $h \leq 3p + 1 - k$ *then* G_1 *is SEMT with magic constant* $k_1 = k + 3h$.

Proof: Define a vertex labeling λ_1 on G_1 in the following way:

$$\lambda_1(u) = \lambda(u), \text{ for } u \in V(G);$$
$$\lambda_1(x_i) = p + i, \text{ for } 1 \leq i \leq h.$$

Consider the set $S_1 = \{\lambda_1(u) + \lambda_1(v) : uv \in E(G_1)\}$. If $S = \{\lambda(u) + \lambda(v) : uv \in E(G)\}$, then $S_1 = S \cup \{\lambda_1(x_i) + \lambda_1(z_i) : 1 \leq i \leq h\} \cup \{\lambda_1(x_i) + \lambda_1(z_{i+1}) : 1 \leq i \leq h\}$, where $x_i z_i$ and $x_i z_{i+1}$ are the new edges. Lemma 1 guarantees that the set $S = \{k - (p + q), k - (p + q - 1), ..., k - (p + 1)\}$ consists of q consecutive integers. Thus, $S_1 = \{k - (p+q), ..., k - (p+1)\} \cup \{k - (p + 1) + 2i - 1, k - (p + 1) + 2i : 1 \leq i \leq h\}$ contains $q + 2h$ consecutive integers. This implies that G_1 is SEMT with the magic constant $k_1 = k + 3h$, provided the highest label of z_i adjacent to x_h is less then or equal to p, namely $k - 2p - 2 + (h+1) \leq p$. So, $h \leq 3p + 1 - k$. □

If the magic constant k of a (p, q)-graph G is exactly $2p + 2$, then $h \leq p - 1$ (by Theorem 1). In particular for $h = p - 1$ the resulting graph of Theorem 1 has the magic constant $k + 3p - 3$. Thus, the following corollary holds.

Corollary 1 *Let* G *be a (p,q)-graph having a SEMT labeling* λ *with the magic constant* $k \geq 2p + 2$, $p \geq 2$. *Let* G_1 *be a graph constructed from* G *by joining* $p - 1$ *new vertices* x_i *to* z_i *and* z_{i+1} $(1 \leq i \leq p - 1)$, *where* $z_i \in V(G)$ *with* $\lambda(z_i) = i$. *Then,* G_1 *is SEMT with magic constant* $k_1 = k + 3p - 3$.

By the duality in Lemma 3, we have

Corollary 2 *The graph of Theorem 1 has a SEMT labeling with the magic constant* $4p + q + 3h + 3 - k$.

Theorem 2 *Let* G_1 *be a* (p_1, q_1)*-graph having a SEMT labeling* λ_1 *with magic constant* $k_1 \geq 2p_1 + 2$, $p_1 \geq 2$. *Let* G_2 *be a* (p_2, q_2)*-graph having a SEMT labeling* λ_2 *with magic constant* k_2. *Let* G^* *be a graph constructed by joining all vertices of* G_2 *to one vertex* u_0 *of* G_1 *with* $\lambda_1(u_0) = k_1 - 2p_1 - 1$. *If* $k_2 = 2p_2 + q_2 + k_1 - 3p_1$ *then* G^* *is SEMT with the magic constant* $k^* = k_1 + 2p_2 + q_2$.

Proof: Define a vertex labeling λ^* on G^* in the following way:

$$\lambda^*(u) = \lambda_1(u), \text{ for } u \in V(G_1)$$
$$\lambda^*(v) = \lambda_2(v) + p_1, \text{ for } v \in V(G_2).$$

Consider the set $S^* = \{\lambda^*(x) + \lambda^*(y) : xy \in E(G^*)\}$. Clearly, $S^* = S_1 \cup \{\lambda_1(u_0) + \lambda^*(v_i) : 1 \leq i \leq p_2\} \cup S_2$, where $u_0 v_i$ are the new edges in G^*, $S_1 = \{\lambda_1(u) + \lambda_1(x) : ux \in E(G_1)\}$ and $S_2 = \{\lambda^*(v) + \lambda^*(y) : vy \in E(G_2)\}$. Lemma 1 gives $S_1 = \{k_1 - (p_1 + q_1), k_1 - (p_1 + q_1 - 1), \cdots, k_1 - (p_1 + 1)\}$. Since $\lambda_1(u_0) = k_1 - 2p_1 - 1$ then $\{\lambda_1(u_0) + \lambda^*(v_i) : 1 \leq i \leq p_2\} = \{k_1 - (p_1 + 1) + 1, \cdots, k_1 - (p_1 + 1) + p_2\}$. Next, consider $S_2 = \{\lambda^*(v) + \lambda^*(y) : vy \in E(G_2)\}$. Since $\lambda^*(v) = \lambda_2(v) + p_1$, for $v \in V(G_2)$ then $S_2 = \{\lambda_2(v) + \lambda_2(y) + 2p_1 : vy \in E(G_2)\}$. Again, by using Lemma 1 we have $S_2 = \{k_2 + 2p_1 - (p_2 + q_2), k_2 + 2p_1 - (p_2 + q_2 - 1), \cdots, k_2 + 2p_1 - (p_2 + 1)\}$. Since $k_2 = 2p_2 + q_2 + k_1 - 3p_1$ then $S_2 = \{k_1 - (p_1 + 1) + p_2 + 1, k_1 - (p_1 + 1) + p_2 + 2, \cdots, k_1 - (p_1 + 1) + p_2 + q_2\}$. Therefore, the set S^* consists of $q_1 + 2q_2$ consecutive integers. This implies that the λ^* extends to a SEMT labeling of G^* with the magic constant $k^* = k_1 + 2p_2 + q_2$. □

Again, by the duality in Lemma 3 we have the following corollary.

Corollary 3 *The graph of Theorem 2 has a SEMT labeling with the magic constant* $4p_1 + 3p_2 + q_1 + 3 - k_1$.

4 Construction of SEMT Labelings for $P_n \cup P_{n+m}$

Consider the graph $P_n \cup P_{n+m}$ with $V(P_n \cup P_{n+m}) = \{u_{1,i} | 1 \leq i \leq n\} \cup \{u_{2,j} | 1 \leq j \leq n + m\}$ and $E(P_n \cup P_{n+m}) = \{e_{1,i} | 1 \leq i \leq n - 1\} \cup$

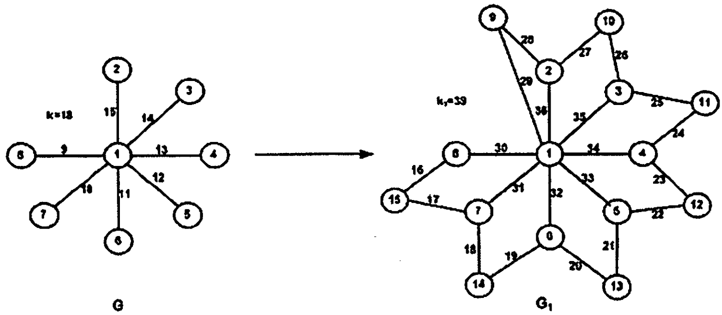

Figure 1: The graph G_1 is formed from the graph G by applying Corollary 1.

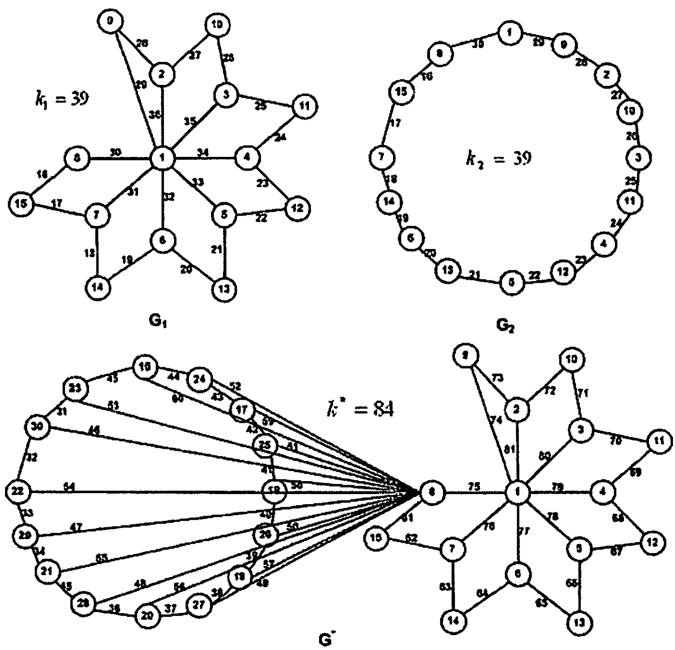

Figure 2: The graph G^* is formed from the graph G_1 and G_2 by applying Theorem 2.

$\{e_{2,j}|1 \leq j \leq n+m-1\}$, where $e_{1,i} = u_{1,i}u_{1,i+1}$, for $1 \leq i \leq n-1$ and $e_{2,j} = u_{2,j}u_{2,j+1}$, for $1 \leq j \leq n+m-1$.
Clearly, $p = 2n+m$, $q = 2n+m-2$, $p+q = 4n+2m-2$. Let λ be a SEMT labeling of $P_n \cup P_{n+m}$ with the magic constant k. Then,

$$k = \frac{10n+5m+5}{2} + \frac{6-B}{2n+m-2} \tag{1}$$

where B is a summation of the labels of all vertices of graph $P_n \cup P_{n+m}$ with degree one, and hence $10 \leq B \leq 8n+4m-6$.

In the next section, we will give the construction of a SEMT labeling of $P_n \cup P_{n+m}$ with $m = 1, 2$ or 3 for all possible values of k.

4.1 $P_n \cup P_{n+1}$

In 2005, Sudarsana et al. [7] showed that $P_n \cup P_{n+1}$ is SEMT with the magic constants $5n+2$ and $5n+4$ for every odd n. Now, we can complete these results by showing the following theorem.

Theorem 3 *Let n be odd. Graph $P_n \cup P_{n+1}$ is SEMT with the magic constant k if and only if $k = 5n+2$, $5n+3$ or $5n+4$.*

Proof: Eq. (1) gives $k = 5n+5+\frac{6-B}{2n-1}$, where $10 \leq B \leq 8n-2$. Now, we will prove that $-3 \leq \frac{6-B}{2n-1} \leq -1$. By a contrary, assume $\frac{6-B}{2n-1} \geq 0$ or $\frac{6-B}{2n-1} \leq -4$. This implies that $B \leq 6$ or $B \geq 8n+2$, respectively. This is a contradiction. Therefore, $5n+2 \leq k \leq 5n+4$.

On the other hand, consider a vertex labeling $f_1 : V(P_n \cup P_{n+1}) \rightarrow \{1, 2, ..., 2n+1\}$ defined as follows:

$$f_1(u_{1,i}) = \begin{cases} \frac{2n+i+1}{2}, & \text{for odd } i, \\ \frac{i+2}{2}, & \text{for even } i. \end{cases}$$

$$f_1(u_{2,j}) = \begin{cases} \frac{3n+j+2}{2}, & \text{for odd } j, \\ 1, & \text{for } j = n+1, \\ \frac{n+j+1}{2}, & \text{otherwise.} \end{cases}$$

Let $S = \{f_1(u) + f_1(v)|uv \in E(P_n \cup P_{n+1})\}$, it can be easily verified that the set $S = \{n+2+i|1 \leq i \leq n-1\} \cup \{2n+2+j|0 \leq j \leq n-1\}$ consists of $2n-1$ consecutive integers. By Lemma 1, f_1 extends to a SEMT labeling of $P_n \cup P_{n+1}$ with the magic constant $k = 5n+3$. The proof is now complete. □

4.2 $P_n \cup P_{n+2}$

Theorem 4 *For $n \geq 2$, if graph $P_n \cup P_{n+2}$ is SEMT with the magic constant k, then $5n+4 \leq k \leq 5n+7$.*

Proof: Eq. (1) gives $k = 5n + 7 + \frac{n+6-B}{2n}$, where $10 \leq B \leq 8n+2$. By similar argument with the proof of Theorem 3, we can show that $-3 \leq \frac{n+6-B}{2n} \leq 0$ and hence $5n+4 \leq k \leq 5n+7$. □

Theorem 5 *For $n \geq 2$, the graph $P_n \cup P_{n+2}$ has a SEMT labeling with the magic constant $k = 5n+6$.*

Proof: Define a vertex labeling f_2 of $P_n \cup P_{n+2}$ for odd n in the following way:

$$f_2(u_{1,i}) = \begin{cases} \frac{i+1}{2}, & \text{for odd } i, \\ n+2+\frac{i}{2}, & \text{for even } i. \end{cases}$$

$$f_2(u_{2,j}) = \begin{cases} \frac{3(n+1)+j+1}{2}, & \text{for odd } j < n+2, \\ n+2, & \text{for } j = n+2, \\ \frac{n+j+1}{2}, & \text{otherwise.} \end{cases}$$

Next, consider the labeling f_3 of the vertices of $P_n \cup P_{n+2}$ for even n as follows:

$$f_3(u_{1,i}) = \begin{cases} \frac{i+1}{2}, & \text{for odd } i, \\ n+2+\frac{i}{2}, & \text{for even } i. \end{cases}$$

$$f_3(u_{2,j}) = \begin{cases} \frac{n+j+1}{2}, & \text{for odd } j, \\ n+2, & \text{for } j = n+2, \\ \frac{3n+4+j}{2}, & \text{otherwise.} \end{cases}$$

By this definition, it can be verified that f_2 and f_3 are SEMT labeling of $P_n \cup P_{n+2}$ with the magic constant $k = 5n+6$ for both cases. □

By Lemma 3, we have the following corollary.

Corollary 4 *For $n \geq 2$, graph $P_n \cup P_{n+2}$ has a SEMT labeling with the magic constant $k = 5n+5$.*

Note that it is still not known whether a $P_n \cup P_{n+2}$ has or not a SEMT labeling with magic constant $5n+4$ or $5n+7$.

4.3 $P_n \cup P_{n+3}$

Theorem 6 *For $n \geq 2$, the graph $P_n \cup P_{n+3}$ is SEMT with the magic constant $k = 5n+7$, $5n+8$ or $5n+9$.*

Proof: Eq. (1) gives $k = 5n + 10 + \frac{6-B}{2n+1}$, where $10 \leq B \leq 8n+6$. Now, we will prove that $-3 \leq \frac{6-B}{2n+1} \leq -1$. By a contrary, assume $\frac{6-B}{2n+1} \geq 0$ or $\frac{6-B}{2n+1} \leq -4$. This implies that $B \leq 6$ or $B \geq 8n+10$, respectively, a contradiction. Therefore, $k = 5n+7$, $5n+8$ or $5n+9$.

Now, we will give the construction of SEMT labelings of $P_n \cup P_{n+3}$ for $k = 5n+8$, $5n+9$ or $5n+7$.

For odd n, consider a vertex labeling $g_1 : V(P_n \cup P_{n+3}) \rightarrow \{1, 2, ..., 2n+3\}$ defined as follows:

$$g_1(u_{1,i}) = \begin{cases} \frac{i+3}{2}, & \text{for odd } i, \\ n+1+\frac{i}{2}, & \text{for even } i. \end{cases}$$

$$g_1(u_{2,j}) = \begin{cases} \frac{n+j}{2}+1, & \text{for } j = 3, 5, 7, ..., n \\ \frac{3n+5}{2}-1, & \text{for } j = 1, \\ 1, & \text{for } j = n+2, \\ \frac{3(n+2)+j-3}{2}, & \text{for } j = 2, 4, ..., n+3. \end{cases}$$

For even n, consider a vertex labeling $g_2 : V(P_n \cup P_{n+3}) \rightarrow \{1, 2, ..., 2n+3\}$ defined as follows:

$$g_2(u_{1,i}) = \begin{cases} \frac{i+1}{2}, & \text{for odd } i, \\ n+2+\frac{i}{2}, & \text{for even } i. \end{cases}$$

$$g_2(u_{2,j}) = \begin{cases} \frac{n+j}{2}, & \text{for even } j < n+3, \\ n+2, & \text{for } j = n+3, \\ \frac{3(n+1)+j+2}{2}, & \text{otherwise.} \end{cases}$$

By Lemma 1, it is easy to show that g_1 (n odd) or g_2 (n even) extends to a SEMT labeling of $P_n \cup P_{n+3}$ with the magic constant $5n+8$. Furthermore, by Lemma 3 g_1 and g_2 are each self-dual.

Next, define the labeling g_3 of the vertices of $P_n \cup P_{n+3}$ for odd n in the following way:

$$g_3(u_{1,i}) = \begin{cases} \frac{i+3}{2}, & \text{for odd } i, \\ n+2+\frac{i}{2}, & \text{for even } i. \end{cases}$$

$$g_3(u_{2,j}) = \begin{cases} \frac{n+j+3}{2}, & \text{for even } j, \\ 1, & \text{for } j = n+3, \\ \frac{3n+4+j}{2}, & \text{otherwise.} \end{cases}$$

and consider labeling g_4 of the vertices of $P_n \cup P_{n+3}$ for even n as follows.

$$g_4(u_{1,i}) = \begin{cases} \frac{i+3}{2}, & \text{for odd } i, \\ n+2+\frac{i}{2}, & \text{for even } i. \end{cases}$$

$$g_4(u_{2,j}) = \begin{cases} \frac{3(n+1)+j+1}{2}, & \text{for even } j, \\ 1, & \text{for } j = n+3, \\ \frac{n+j+3}{2}, & \text{otherwise.} \end{cases}$$

Again, it is a routine procedure to verify that g_3 (n odd) or g_4 (n even) is a SEMT labeling of $P_n \cup P_{n+3}$ with the magic constant $5n + 9$. Now, by the duality in Lemma 3 the graph $P_n \cup P_{n+3}$ has a SEMT labeling with the magic constant $5n + 7$. The proof is now complete. □

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