Partial permutation decoding of some binary codes from graphs on triples

J. D. Key*
Department of Mathematical Sciences
Clemson University
Clemson SC 29634, U.S.A.

J. Moori†
School of Mathematical Sciences
University of KwaZulu-Natal
Pietermaritzburg 3209, South Africa

B. G. Rodrigues‡
School of Mathematical Sciences
University of KwaZulu-Natal
Durban 4041, South Africa

Abstract

We show that partial permutation decoding can be used, and give explicit s-PD-sets in the symmetric group, where s is less than the full error-correction capability of the code, for some classes of binary codes obtained from the adjacency matrices of the graphs with vertices the $\binom{n}{3}$ 3-subsets of a set of size n with adjacency defined by the vertices as 3-sets being adjacent if they have a fixed number of elements in common.

1 Introduction

In [KMR04, KMRa] we examined the binary codes obtained from the three uniform-subset graphs having as vertices the 3-subsets of a set of size $n \geq 7$,

*This work was supported by the DoD Multidisciplinary University Research Initiative (MURI) program administered by the Office of Naval Research under Grant N00014-00-1-0565.

†Support of NRF and the University of KwaZulu-Natal (URF) acknowledged.

‡University of KwaZulu-Natal Competitive Research Grant acknowledged.

with two vertices being adjacent if they have, as 3-subsets, intersection of size k, where k is $0,1$ or 2. In [KMRa] we showed that permutation decoding can be used for full error-correction for two of the classes of these codes. Here we look at the remaining two classes of these codes that have minimum weight that is a function of n, thus completing the work of permutation decoding for the better binary codes that arise in this way from these graphs. We prove the following, using the facts established about the codes in [KMR04] (see also Result 2 in Section 3 below, where the coding parameters are given):

Theorem 1 *Let Ω be a set of size n, where $n \geq 7$. Let $\mathcal{P} = \Omega^{\{3\}}$, the set of subsets of Ω of size 3, be the vertex set of the two graphs $A_i(n)$, for $i = 0,1$, with adjacency defined by two vertices (as 3-sets) being adjacent if the 3-sets have intersection of size i. Let $C_i(n)$, for $i = 0,1$ denote the code formed from the row span over $\mathbb{F}_2$ of an adjacency matrix for $A_i(n)$.*

For $n = 4k$, $k \geq 2$, the dual $C_0(n)^\perp$ is a $[\binom{n}{3}, n, \binom{n-1}{2}]_2$ code with

$$\mathcal{I} = \{\{i, n-1, n\} \mid 1 \leq i \leq n-2\} \cup \{\{n-3, n-2, n-1\},\ \{n-3, n-2, n\}\}$$

as information set. For $n \equiv 1 \pmod 4$, $n \geq 13$, $C_1(n)^\perp$ is a $[\binom{n}{3}, n-1, 2\binom{n-2}{2}]_2$ code and $C_1(9)^\perp$ is a $[84, 8, 38]_2$ code, with $\mathcal{I} \setminus \{\{n-2, n-1, n\}\}$ as information set.

Taking the following elements of S_n in their natural action on triples of elements of $\Omega = \{1, 2, \ldots, n\}$:

$$\begin{aligned}
\Sigma_1 &= \{(n,i) \mid 1 \leq i \leq n-2\} \cup \{\imath\};\\
\Sigma_2 &= \{(n-1,i) \mid 1 \leq i \leq n-2\} \cup \{\imath\};\\
\Sigma_3 &= \{(n-2,i) \mid 1 \leq i \leq n-4\} \cup \{\imath\};\\
\Sigma_4 &= \{(n-3,i) \mid 1 \leq i \leq n-4\} \cup \{\imath\},
\end{aligned}$$

where $\imath$ is the identity element of S_n, let $\Sigma_{1,2} = \Sigma_1\Sigma_2 \setminus \{(n,a)(n-1,a) \mid 1 \leq a \leq n-2\}$ and $\Sigma_{3,4} = \Sigma_3\Sigma_4 \setminus \{(n-2,a)(n-3,a) \mid 1 \leq a \leq n-4\}$. Then $\Sigma = \Sigma_{1,2}\Sigma_{3,4}$ is an s-PD-set of size $n^4 - 10n^3 + 37n^2 - 60n + 39$ for $C_0(n)^\perp$ for $s \leq \lceil n^2/6 \rceil - 1$, and for $C_1(n)^\perp$ for $s \leq \lceil n(n-1)/6 \rceil - 1$.

The proof of the theorem is given in Section 3. Background results and notation are given in Section 2

2 Background and terminology

The notation for designs and codes is as in [AK92]. An incidence structure $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$, with point set $\mathcal{P}$, block set $\mathcal{B}$ and incidence $\mathcal{I}$ is a t-(v,k,λ) design, if $|\mathcal{P}| = v$, every block $B \in \mathcal{B}$ is incident with precisely k points, and every t distinct points are together incident with precisely λ blocks. The design is **symmetric** if it has the same number of points and blocks.

The **code** C_F **of the design** $\mathcal{D}$ over the finite field F is the space spanned by the incidence vectors of the blocks over F. If the point set of $\mathcal{D}$ is denoted by $\mathcal{P}$ and the block set by $\mathcal{B}$, and if $\mathcal{Q}$ is any subset of $\mathcal{P}$, then we will denote the incidence vector of $\mathcal{Q}$ by $v^{\mathcal{Q}}$. Thus $C_F = \langle v^B \mid B \in \mathcal{B} \rangle$, and is a subspace of $V = F^{\mathcal{P}}$, the full vector space of functions from $\mathcal{P}$ to F. For any vector $w \in V$, the coordinate of w at the point $P \in \mathcal{P}$ is denoted by $w(P)$.

The notation $[n,k,d]_q$ will be used for a **linear code** C of length n, dimension k, and minimum weight d over a field of order q. A **generator matrix** for the code is a $k \times n$ matrix made up of a basis for C. The **dual** code $C^{\perp}$ is the orthogonal under the standard inner product $(,)$, i.e. $C^{\perp} = \{v \in F^n | (v,c) = 0 \text{ for all } c \in C\}$. A **check** matrix for C is a generator matrix H for $C^{\perp}$. The all-one vector will be denoted by $\jmath$, and is the constant vector of weight the length of the code. Two linear codes of the same length and over the same field are **isomorphic** if they can be obtained from one another by permuting the coordinate positions. An **automorphism** of a code C is an isomorphism from C to C. The automorphism group will be denoted by $\mathrm{Aut}(C)$.

Terminology for **graphs** is standard: the graphs, $\Gamma = (V, E)$ with vertex set V and edge set E, are undirected and the **valency** of a vertex is the number of edges containing the vertex. A graph is **regular** if all the vertices have the same valency.

Any code is isomorphic to a code with generator matrix in so-called **standard form**, i.e. the form $[I_k \mid A]$; a check matrix then is given by $[-A^T \mid I_{n-k}]$. The first k coordinates are the **information symbols** and the last $n-k$ coordinates are the **check symbols**.

Permutation decoding was first developed by MacWilliams [Mac64] and involves finding a set of automorphisms of a code called a PD-set. The method is described fully in MacWilliams and Sloane [MS83, Chapter 15] and Huffman [Huf98, Section 8]. We extend the definition of PD-sets to s-PD-sets for s-error-correction:

Definition 1 *If C is a t-error-correcting code with information set $\mathcal{I}$ and check set $\mathcal{C}$, then a* **PD-set** *for C is a set $\mathcal{S}$ of automorphisms of C which is such that every t-set of coordinate positions is moved by at least one member of $\mathcal{S}$ into the check positions $\mathcal{C}$.*

*For $s \leq t$ an s-***PD-set** *is a set $\mathcal{S}$ of automorphisms of C which is such that every s-set of coordinate positions is moved by at least one member of $\mathcal{S}$ into $\mathcal{C}$.*

The algorithm for permutation decoding is given in [Huf98] and requires that the generator matrix is in standard form, so an information set needs to be known. The property of having a PD-set will not, in general, be invariant under isomorphism of codes, i.e. it depends on the choice of information set. Furthermore, there is a bound on the minimum size of $\mathcal{S}$ (see [Gor82],[Sch64], or [Huf98]):

Result 1 *If $\mathcal{S}$ is a PD-set for a t-error-correcting $[n,k,d]_q$ code C, and $r = n-k$, then* $|\mathcal{S}| \geq \left\lceil \frac{n}{r} \left\lceil \frac{n-1}{r-1} \left\lceil \dots \left\lceil \frac{n-t+1}{r-t+1} \right\rceil \dots \right\rceil \right\rceil \right\rceil$.

This result can be adapted to s-PD-sets for $s \leq t$ by replacing t by s in the formula.

3 The codes and PD-sets

We describe first briefly how the codes are defined from graphs and designs. Let n be any integer and Ω a set of size n; to avoid degenerate cases we take $n \geq 7$. Taking the set $\Omega^{\{3\}}$ to be the set of all 3-element subsets of Ω, we define three non-trivial undirected graphs with vertex set $\mathcal{P} = \Omega^{\{3\}}$, and denote these graphs by $A_i(n)$ where $i = 0,1,2$. The edges of the graph $A_i(n)$ are defined by the rule that **two vertices are adjacent in** $A_i(n)$ if as 3-element subsets they have exactly i elements of Ω in common. For each $i = 0,1,2$ we define from $A_i(n)$ a 1-design $\mathcal{D}_i(n)$, on the point set $\mathcal{P}$ by defining for each point $P = \{a,b,c\} \in \mathcal{P}$ a block $\overline{\{a,b,c\}}_i$ by

$$\overline{\{a,b,c\}}_i = \{\{x,y,z\} \mid |\{x,y,z\} \cap \{a,b,c\}| = i \}.$$

Denote by $\mathcal{B}_i(n)$ the block set of $\mathcal{D}_i(n)$, so that each of these is a symmetric 1-design on $\binom{n}{3}$ points with block size k_i, for $i = 0,1,2$, where $k_0 = \binom{n-3}{3}$, $k_1 = 3\binom{n-3}{2}$, and $k_2 = 3(n-3)$.

In [KMR04] we examined the binary codes of these designs, i.e. , for $i = 0,1,2$, $C_i(n) = \langle v^b \mid b \in \mathcal{B}_i(n) \rangle$, where the span is taken over $\mathbb{F}_2$. Alternatively, this can be regarded as the row span over $\mathbb{F}_2$ of an adjacency matrix of the relevant graph. We obtained the following theorem, which we quote in full to show which codes are worth consideration for permutation decoding:

Result 2 *Let Ω be a set of size n, where $n \geq 7$. Let $\mathcal{P} = \Omega^{\{3\}}$, the set of subsets of Ω of size 3, be the vertex set of the three graphs $A_i(n)$, for $i = 0,1,2$, with adjacency defined by two vertices (as 3-sets) being adjacent if the 3-sets meet in zero, one or two elements, respectively. Let $C_i(n)$ denote the code formed from the row span over $\mathbb{F}_2$ of an adjacency matrix for $A_i(n)$. Then*

1. $n \equiv 0 \pmod 4$*:*

(a) $C_2(n) = \mathbb{F}_2^{\mathcal{P}}$;

(b) $C_0(n) = C_1(n)$ is $[\binom{n}{3}, \binom{n}{3} - n, 4]_2$ and $C_0(n)^\perp$ is $[\binom{n}{3}, n, \binom{n-1}{2}]_2$;

2. $n \equiv 2 \pmod 4$:
$C_i(n) = \mathbb{F}_2^{\mathcal{P}}$ for $i = 0, 1, 2$;

3. $n \equiv 1 \pmod 4$:

(a) $C_0(n) = C_1(n) \cap C_2(n)$;

(b) $C_0(n)$ is $[\binom{n}{3}, \binom{n}{3} - \binom{n}{2}, 8]_2$ and $C_0(n)^\perp$ is $[\binom{n}{3}, \binom{n}{2}, n-2]_2$;
$C_1(9)$ is $[84, 76, 3]_2$ and $C_1(9)^\perp$ is $[84, 8, 38]_2$;
$C_1(n)$ is $[\binom{n}{3}, \binom{n}{3} - n + 1, 4]_2$ and $C_1(n)^\perp$ is $[\binom{n}{3}, n-1, (n-2)(n-3)]_2$ for $n > 9$;
$C_2(n)$ is $[\binom{n}{3}, \binom{n-1}{3}, 4]_2$ and $C_2(n)^\perp$ is $[\binom{n}{3}, \binom{n-1}{2}, n-2]_2$;

4. $n \equiv 3 \pmod 4$:

(a) $C_1(n) = \langle v^P + j \mid P \in \mathcal{P} \rangle$ is $[\binom{n}{3}, \binom{n}{3} - 1, 2]_2$;

(b) $C_0(n) = C_2(n)$ is $[\binom{n}{3}, \binom{n-1}{3}, 4]_2$ and $C_2(n)^\perp$ is $[\binom{n}{3}, \binom{n-1}{2}, n-2]_2$;

For all $n \geq 7, i = 0, 1, 2$, $C_i(n) \cap C_i(n)^\perp = \{0\}$, and the automorphism groups of these codes are S_n or $S_{\binom{n}{3}}$.

In [KMR04] we also obtained information sets for all these codes. In [KMRa] we found PD-sets (for full error-correction) for $C_2(n)^\perp$ for n odd and for $C_0(n)^\perp$ for $n \equiv 1 \pmod 4$. The sizes of the PD-sets found were of the order of n^3 and n^4, respectively.

Now we consider the codes $C_0(n)^\perp$, where $n = 4k$, $k \geq 2$, correcting $t = n(n-3)/4$ errors, and $C_1(n)^\perp$, for $n \equiv 1 \pmod 4$, correcting 18 errors for $n = 9$, and $(n-1)(n-4)/2$ errors for $n \geq 13$. That the information symbols can be taken as given in the statement of the theorem follows from Lemma 8 and Proposition 2 of [KMR04].

We first need a lemma, and for this and the proof of the theorem, we introduce some notation. Let $\mathcal{I}$ denote the information positions and $\mathcal{C}$ the check positions for $C_0(n)^\perp$, and $\mathcal{I}^*$ and $\mathcal{C}^*$ those for $C_1(n)^\perp$. Let

$$\mathcal{J} = \{\{i, n-1, n\} \mid 1 \leq i \leq n-2\}\}$$

and $P = \{n-3, n-2, n-1\}$, $Q = \{n-3, n-2, n\}$. Thus $\mathcal{I} = \mathcal{J} \cup \{P, Q\}$, and $\mathcal{P} = \mathcal{I} \cup \mathcal{C}$. Also $\mathcal{I}^* = (\mathcal{J} \setminus \{\{n-2, n-1, n\}\}) \cup \{P, Q\}$. Let $\mathcal{T} = \{X_i \mid 1 \leq i \leq s\}$, be a set of $s \geq 0$ points of $\mathcal{P}$. Let $A = \cup_{i=1}^s X_i$ and $\mathcal{Q} = \{\{a, b\} \mid a \neq b, \{a, b\} \subset X_i$ for some $i\}$.

Lemma 1 *With the above notation, if $s < n^2/6$ for n even, and $s < n(n-1)/6$ for n odd, then there is a 2-set $\{a,b\}$, $1 \leq a,b \leq n$, that is not in $\mathcal{Q}$.*

Proof: For each j such that $1 \leq j \leq n$, let z_j be the number of $X \in \mathcal{T}$ such that $j \in X$. Counting the number of elements in the set of ordered pairs $\{< j, X >| \; 1 \leq j \leq n, X \in \mathcal{T}, j \in X\}$ in two ways, we get $\sum_{j=1}^{n} z_j = 3s$. If every j occurs with every k $(1 \leq k \leq n)$ in some $X \in \mathcal{T}$, then $z_j \geq n/2$ for each j in the case n even, and $z_j \geq (n-1)/2$ for n odd. From this we get $3s \geq n^2/2$ for n even, $3s \geq n(n-1)/2$ for n odd. Thus if $s < n^2/6$ for n even and $s < n(n-1)/6$ for n odd, then not all 2-sets occur. ■

Note: For $n = 8$, $C_0(8)^{\perp}$ corrects 10 errors and $10 = \lceil n^2/6 \rceil - 1$, so that in this case the decoding method achieves the full error-correcting capability of the code. In contrast, see Note. 1 below.

Now we can prove the theorem, again using the notation as above.

Proof of Theorem 1:

We assume that $\mathcal{T} \not\subset C$, so that $\mathcal{T} \cap \mathcal{I} \neq \emptyset$. We give the proof only for the case $n = 4k$, since the argument for the other class of codes is virtually identical.

Suppose first that $\{n-1, n\} \in \mathcal{Q}$. By the lemma, there is a 2-set $\{a,b\} \neq \{n-1,n\}$ not in $\mathcal{Q}$. If $n, n-1 \notin \{a,b\}$, then $\sigma = (n,a)(n-1,b)$ will satisfy $\mathcal{T}^{\sigma} \cap \mathcal{I} \subseteq \{P,Q\}$. If $a = n$, $b \neq n-1$, then $\sigma = (b, n-1)$ will have the same consequence, and similarly if $a \neq n$ and $b = n-1$, with $\sigma = (a,n)$. Note that $\sigma \in \Sigma_{1,2}$.

Thus we need to deal with $\mathcal{T} \cap \mathcal{I} \subseteq \{P,Q\}$, using only $\Sigma_{3,4}$. Suppose first that $\mathcal{T} \cap \mathcal{I} = \{Q\} = \{n-3, n-2, n\}$. We want to show that there is a 2-set $\{a,b\}$ with $1 \leq a,b \leq n-4$ such that neither $\{a,b,n\}$ nor $\{a,b,n-1\}$ is in $\mathcal{T}$, for then $\tau = (n-3,a)(n-2,b)$ will work. Suppose that for every pair $\{a,b\}$, $\{a,b,n\}$ or $\{a,b,n-1\}$ appears. This will now account for at least $\binom{n-4}{2} + 1$ points. Since this number is greater than $n^2/6$ for $n > 8$, in the even case, and $n(n-1)/6$ for $n > 9$ in the odd case, as long as $n > 9$ this is impossible, so the 2-set exists.

We need a finer count in the case $n = 8$ or $n = 9$. If there is an i, $1 \leq i \leq n-4$, such that neither of $\{n-3, i, n\}$ nor $\{n-3, i, n-1\}$ are in $\mathcal{T}$, then $(n-2, i) \in \Sigma_3$ will work, or similarly if there is an i, $1 \leq i \leq n-4$, such that neither of $\{i, n-2, n\}$ nor $\{i, n-2, n-1\}$ are in $\mathcal{T}$, then $(n-3, i) \in \Sigma_4$ will do. So suppose for each i such that $1 \leq i \leq n-4$, either $\{n-3, i, n\}$ or $\{n-3, i, n-1\}$ are in $\mathcal{T}$, and either $\{i, n-2, n\}$ or $\{i, n-2, n-1\}$ are in $\mathcal{T}$. Then this accounts for at least $2(n-4)+1$ elements of $\mathcal{T}$. Together with the $\binom{n-4}{2}$ points we obtained above, this gives $2(n-4) + 1 + \binom{n-4}{2} > n^2/6$ (or $n(n-1)/6$) for $n \geq 8$, as required.

Clearly the case where $\mathcal{T} \cap \mathcal{I} = \{P\}$ or $\mathcal{T} \cap \mathcal{I} = \{P,Q\}$ will work in precisely the same way, for the same reason. ■

Note: 1. With $n = 12$, $C_0(12)^\perp$ can correct 27 errors, and our s-PD-set will correct up to 23 errors. Taking the set

```
{{1,2,3},{1,5,9},{1,4,8},{1,6,7},{2,5,8},{2,6,9},{2,4,7},
{3,6,8},{3,5,7},{3,4,9},{4,5,6},{7,8,9},{1,11,12},{2,11,12},
{9,10,11},{1,2,10},{2,10,11},{3,10,11},{3,4,12},{4,10,11},
{5,10,12},{5,6,11},{6,10,12},{7,10,12},{7,8,11},{8,10,12},
{9,10,12}}
```

of size 27, computations with Magma [BC94] showed that, using the information set $\mathcal{I}$, there is no element in the group S_{12} acting on the codes that will move this set into the check positions. Thus this information set will not allow full error-correction by permutation decoding of $C_0(12)^\perp$.

In fact the set

```
{{2,3,6},{3,4,7},{4,5,8},{5,6,9},{6,7,10},{7,8,11},{8,9,12},
{1,9,10},{2,10,11},{3,11,12},{1,4,12},{4,6,10},{6,8,12},
{2,7,9},{3,8,10},{4,9,11},{5,10,12},{1,6,11},{2,7,12},
{1,2,8},{1,3,9},{1,5,7},{2,4,5},{3,5,11}}
```

of size 24 can also not be moved into the check positions, showing that 23 is the best we can do with this information set.[1]

2. Smaller s-PD-sets than those found here can undoubtedly be found, but the argument will most likely involve case-by-case analysis.

3. Although our s-PD-sets do not reach the full potential of the code, the ratio of errors corrected to the full potential tends to $2/3$ as n increases. The table shown as Figure 1 shows bounds and sizes of s-PD-sets for $C_0(4k)^\perp$ for $2 \leq k \leq 12$, where $n = 4k$, $N = \binom{n}{3}$, i.e. the length of the code, $t = k(n-3)$, b is the Gordon bound for full error correction for t errors, $s = \lceil n^2/6 \rceil - 1$, bb is the Gordon bound for s errors, S is the size of our s-PD-set, and the final column is the ratio of the number of corrected errors to the number of errors the code can correct.

4. Since the construction of the PD-sets depends only on the information set and the automorphism group, these PD-sets will also apply to the ternary code of $C_0(n)^\perp$, for $n \equiv 1 \pmod 3 \geq 7$, since this code is shown in [KMRb] to be $[\binom{n}{3}, n, \binom{n-1}{2}]_3$ for $n \equiv 4 \pmod 9$ and $[\binom{n}{3}, n-1, \binom{n-1}{2}]_3$ for $n \equiv 1, 7 \pmod 9$, and also has S_n as automorphism group.

5. A simple argument yields that the worst-case time complexity for the decoding algorithm using an s-PD-set of size z on a code of length n and dimension k is $\mathcal{O}(nkz)$, i.e. in terms of the length N of these codes, this would be $\mathcal{O}(N^{2.7})$.

[1]We thank the referee for providing us with this example.

n	N	t	b	s	bb	S	s/t
8	56	10	18	10	18	903	1.0
12	220	27	41	23	31	8103	0.85
16	560	52	75	42	52	33127	0.81
20	1140	85	119	66	78	93639	0.78
24	2024	126	174	95	109	213447	0.75
28	3276	175	241	130	146	422503	0.74
32	4960	232	317	170	188	756903	0.73
36	7140	297	404	215	235	1258887	0.72
40	9880	370	502	266	288	1976839	0.72
44	13244	451	610	322	346	2965287	0.71
48	17296	540	729	383	409	4284903	0.71

Figure 1: Bounds and sizes of s-PD-sets for $C_0(4k)^{\perp}$

Acknowledgement

J. D. Key thanks the School of Mathematical Sciences at the University of KwaZulu-Natal in Pietermaritzburg for their hospitality.

References

[AK92] E. F. Assmus, Jr and J. D. Key. *Designs and their Codes.* Cambridge: Cambridge University Press, 1992. Cambridge Tracts in Mathematics, Vol. 103 (Second printing with corrections, 1993).

[BC94] Wieb Bosma and John Cannon. *Handbook of Magma Functions.* Department of Mathematics, University of Sydney, November 1994. http://magma.maths.usyd.edu.au/magma/.

[Gor82] D. M. Gordon. Minimal permutation sets for decoding the binary Golay codes. *IEEE Trans. Inform. Theory*, 28:541–543, 1982.

[Huf98] W. Cary Huffman. Codes and groups. In V. S. Pless and W. C. Huffman, editors, *Handbook of Coding Theory*, pages 1345–1440. Amsterdam: Elsevier, 1998. Volume 2, Part 2, Chapter 17.

[KMRa] J. D. Key, J. Moori, and B. G. Rodrigues. Binary codes from graphs on triples and permutation decoding. *Ars Combin.* To appear.

[KMRb] J. D. Key, J. Moori, and B. G. Rodrigues. Ternary codes from graphs on triples. In preparation.

[KMR04] J. D. Key, J. Moori, and B. G. Rodrigues. Binary codes from graphs on triples. *Discrete Math.*, 282/1-3:171–182, 2004.

[Mac64] F. J. MacWilliams. Permutation decoding of systematic codes. *Bell System Tech. J.*, 43:485–505, 1964.

[MS83] F. J. MacWilliams and N. J. A. Sloane. *The Theory of Error-Correcting Codes.* Amsterdam: North-Holland, 1983.

[Sch64] J. Schönheim. On coverings. *Pacific J. Math.*, 14:1405–1411, 1964.