WEIGHTED COMPOSITION OPERATORS FROM LOGARITHMIC BLOCH SPACES TO A CLASS OF WEIGHTED-TYPE SPACES IN THE UNIT BALL

XIANGLING ZHU
Department of Mathematics, JiaYing University
514015, Meizhou, GuangDong, China
E-mail: jyuzxl@163.com

Abstract

The boundedness and compactness of the weighted composition operator from logarithmic Bloch spaces to a class of weighted-type spaces are studied in this paper.

1. INTRODUCTION

Let B be the unit ball of $\mathbb{C}^n$ and $H(B)$ the space of all holomorphic functions in B. Let $\Re f(z) = \sum_{j=1}^n z_j \frac{\partial f}{\partial z_j}(z)$ be the radial derivative of $f \in H(B)$. The Bloch space $\mathcal{B} = \mathcal{B}(B)$, is the space consisting of all $f \in H(B)$ such that $\sup_{z\in B}(1-|z|^2)|\Re f(z)| < \infty$. Let $\mathcal{LB} = \mathcal{LB}(B)$ denote the class of all $f \in H(B)$ such that

$$\beta(f) = \sup_{z\in B}(1-|z|^2)\left(\ln\frac{e}{1-|z|^2}\right)|\Re f(z)| < \infty. \tag{1}$$

It is easy to check that $\mathcal{LB}$ is a Banach space with the norm $\|f\|_{\mathcal{LB}} = |f(0)| + \beta(f)$. $\mathcal{LB}$ is called the logarithmic Bloch space. We would like to point out that the constant e appearing in (1) is introduced in [21], instead of usual constant 2, for avoiding many technical problems (see, e.g., [23]). Namely, the function $h(x) = (1-x^2)\ln\frac{e}{1-x^2}$ is decreasing in x on the interval $[0,1)$, which is not the case with the function $(1-x^2)\ln\frac{2}{1-x^2}$.

Let $\mathcal{LB}_0$ stand for the class of $f \in \mathcal{LB}$ such that

$$\lim_{|z|\to 1}(1-|z|^2)\left(\ln\frac{e}{1-|z|^2}\right)|\Re f(z)| = 0. \tag{2}$$

2000 *Mathematics Subject Classification.* Primary 47B38, Secondary 46E15.
Keywords. Bloch space, logarithmic Bloch space, weighted composition operator.

In [24] (see also [25, Theorem 3.21]) was shown that f is a multiplier of $\mathcal{B}$ if and only if $f \in H^\infty$ and $f \in \mathcal{LB}$. Hence the space $\mathcal{LB}$ is appeared naturally.

A positive continuous function μ on $[0,1)$ is called normal, if there exist positive numbers α and β, $0 < \alpha < \beta$, and $\delta \in [0,1)$ such that (see [14])

$$\frac{\mu(r)}{(1-r)^\alpha} \text{ is decreasing on } [\delta,1) \text{ and } \lim_{r\to 1}\frac{\mu(r)}{(1-r)^\alpha} = 0;$$

$$\frac{\mu(r)}{(1-r)^\beta} \text{ is increasing on } [\delta,1) \text{ and } \lim_{r\to 1}\frac{\mu(r)}{(1-r)^\beta} = \infty.$$

Let μ be a normal function on $[0,1)$. An $f \in H(B)$ is said to belong to the weighted-type space ([3, 17, 18]), denoted by $H_\mu^\infty = H_\mu^\infty(B)$, if

$$\|f\|_{H_\mu^\infty} = \sup_{z\in B}\mu(|z|)\,|f(z)| < \infty.$$

H_μ^∞ is a Banach space with the norm $\|\cdot\|_{H_\mu^\infty}$.

Let $H_{\mu,0}^\infty$ denote the subspace of H_μ^∞ consisting of those $f \in H_\mu^\infty$ for which $\lim_{|z|\to 1}\mu(|z|)|f(z)| = 0$. When $\mu(r) = (1-r^2)^\alpha$, the induced spaces H_μ^∞ and $H_{\mu,0}^\infty$ are the space H_α^∞ and $H_{\alpha,0}^\infty$ respectively. For more information on the spaces H_α^∞ and $H_{\alpha,0}^\infty$ in the unit disk, we refer to [4, 11].

Let $\psi \in H(B)$ and φ be a holomorphic self-map of B. The weighted composition operator ψC_φ on $H(B)$, induced by ψ and φ, is defined as follows $(\psi C_\varphi f)(z) = \psi(z)(f\circ\varphi)(z)$. The weighted composition operator is a generalization of the multiplication operator and the composition operator. The book [2] contains much information on this topic. In the setting of the unit ball, some necessary and sufficient conditions for the weighted composition operator to be bounded and compact between the Bloch space and H^∞ are given in [10]. In the setting of the unit polydisk, some necessary and sufficient conditions for the weighted composition operator to be bounded and compact between the Bloch space and H^∞ are given in [7, 8] (see also [15] for the case of composition operators). For some closely related results see also [1, 3, 5, 9, 12, 13, 16, 17, 18, 19, 20, 22, 23, 26] and the references therein.

In this paper, we study the weighted composition operator from $\mathcal{LB}$ and $\mathcal{LB}_0$ to H_μ^∞ and $H_{\mu,0}^\infty$. The sufficient and necessary conditions for the weighted composition operator ψC_φ to be bounded and compact are given.

Throughout the paper, constants are denoted by C, they are positive and may not be the same in every occurrence.

2. Auxiliary results

In this section we state some auxiliary results, which are used in the proofs of our main results.

Lemma 1. *Assume $\psi \in H(B)$, φ is a holomorphic self-map of B. Then $\psi C_\varphi : \mathcal{LB}\,(or\ \mathcal{LB}_0) \to H_\mu^\infty$ is compact if and only if $\psi C_\varphi : \mathcal{LB}\,(or\ \mathcal{LB}_0) \to H_\mu^\infty$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbb{N}}$ in $\mathcal{LB}\,(or\ \mathcal{LB}_0)$ converging to zero uniformly on compacts of B as $k \to \infty$, we have $\lim_{k\to\infty} \|\psi C_\varphi f_k\|_{H_\mu^\infty} = 0$.*

Proof. The result follows by standard arguments similar to those outlined in Proposition 3.11 of [2] or in Lemma 2 in [15]. We omit the details. □

The next lemma is from [3] (one-dimensional case is Lemma 2.1 in [12]).

Lemma 2. *A closed set K in $H_{\mu,0}^\infty$ is compact if and only if it is bounded and satisfies*

$$\lim_{|z|\to 1} \sup_{f\in K} \mu(|z|)|f(z)| = 0.$$

A proof of the following estimate can be found in [21, Lemma 1].

Lemma 3. *Let $f \in \mathcal{LB}$. Then there exists a positive constant C such that*

$$|f(z)| \le |f(0)| + C\|f\|_{\mathcal{LB}} \ln\ln \frac{e}{1-|z|^2}.$$

Lemma 4. *Let $f \in \mathcal{LB}_0$. Then*

$$\lim_{|z|\to 1} \frac{|f(z)|}{\ln\ln \frac{e}{1-|z|^2}} = 0. \tag{3}$$

Proof. We follow the lines of the proof of Lemma 2.3 in [6]. Since $f \in \mathcal{LB}_0$, it follows that for any $\varepsilon > 0$ there is a $\delta \in (1/2, 1)$ such that when $\delta < |z| < 1$

$$(1-|z|)\ln\frac{e}{1-|z|^2}|\Re f(z)| < \varepsilon. \tag{4}$$

From (4) and since the function $h_1(x) = x\ln\frac{e}{x}$ is increasing on the interval $(0,1]$, when $1/2 < \delta < |z| < 1$, we have that

$$\begin{aligned}
|f(z)| &= \left| f(z/2|z|) + \int_{1/(2|z|)}^{1} \Re f(tz)\frac{dt}{t} \right| \\
&\le M_\infty(f, 1/2) + 2\int_{1/(2|z|)}^{\frac{\delta}{|z|}} |\Re f(tz)|\,|z|dt + 2\int_{\frac{\delta}{|z|}}^{1} |\Re f(tz)|\,|z|dt \\
&\le M_\infty(f,1/2) + 2\|f\|_{\mathcal{LB}} \int_0^{\frac{\delta}{|z|}} \frac{|z|dt}{(1-t|z|)\ln\frac{e}{1-t|z|}} + 2\varepsilon\int_{\frac{\delta}{|z|}}^{1} \frac{|z|dt}{(1-t|z|)\ln\frac{e}{1-t|z|}} \\
&\le M_\infty(f,1/2) + 2\|f\|_{\mathcal{LB}}\ln\ln\frac{e}{1-\delta} + 2\varepsilon\ln\ln\frac{e}{1-|z|} - 2\varepsilon\ln\ln\frac{e}{1-\delta},
\end{aligned}$$

where $M_\infty(f,r)=\sup_{z\in B}|f(rz)|$. Dividing the above inequality by $\ln\ln\frac{e}{1-|z|}$, using the fact that the quantity $M_\infty(f,1/2)$ is finite, and letting $|z|\to 1$, we get

$$\limsup_{|z|\to 1}|f(z)|\Big(\ln\ln\frac{e}{1-|z|}\Big)^{-1}\leq 2\varepsilon$$

from which the lemma follows. □

3. Main results and proofs

In this section, we consider the boundedness and compactness of the weighted composition operator from logarithmic Bloch spaces to weighted-type spaces.

Theorem 1. *Assume that $\psi\in H(B)$, φ is a holomorphic self-map of B and μ is a normal function on $[0,1)$. Then the following statements are equivalent.*

(a) $\psi C_\varphi:\mathcal{LB}\to H_\mu^\infty$ is bounded;
(b) $\psi C_\varphi:\mathcal{LB}_0\to H_\mu^\infty$ is bounded;
(c) $\psi\in H_\mu^\infty$ and

$$M=\sup_{z\in B}\mu(|z|)|\psi(z)|\ln\ln\frac{e}{1-|\varphi(z)|^2}<\infty. \tag{5}$$

Proof. (a) ⇒ (b) is clear.

(b) ⇒ (c). Assume that $\psi C_\varphi:\mathcal{LB}_0\to H_\mu^\infty$ is bounded. Taking the function $f(z)=1$, we see that $\psi\in H_\mu^\infty$. For $a\in B$, set

$$f_a(z)=\ln\ln\frac{e}{1-\langle z,a\rangle}. \tag{6}$$

It is known that $K_1:=\sup_{a\in B}\|f_a\|_{\mathcal{LB}}<\infty$ (see [21]). In addition,

$$\lim_{|z|\to 1}(1-|z|)\ln\frac{e}{1-|z|}|\Re f_a(z)|\leq\lim_{|z|\to 1}(1-|z|)\ln\frac{e}{1-|z|}\frac{1}{\ln(e/2)}\frac{1}{1-|a|}=0.$$

Therefore $f_a\in\mathcal{LB}_0$. For any $\lambda\in B$, we have

$$\begin{aligned}\infty>\|\psi C_\varphi f_{\varphi(\lambda)}\|_{H_\mu^\infty} &= \sup_{z\in B}\mu(|z|)|(\psi C_\varphi f_{\varphi(\lambda)})(z)|\\ &= \sup_{z\in B}\mu(|z|)|\psi(z)||f_{\varphi(\lambda)}(\varphi(z))|\\ &\geq \mu(|\lambda|)|\psi(\lambda)|\ln\ln\frac{e}{1-|\varphi(\lambda)|^2},\end{aligned} \tag{7}$$

which implies (5).

$(c) \Rightarrow (a)$. Assume that $\psi \in H_\mu^\infty$ and (5) holds. For any $f \in \mathcal{LB}$, using Lemma 3 we have

$$\begin{aligned} &\mu(|z|)|(\psi C_\varphi f)(z)| = \mu(|z|)|\psi(z)||f(\varphi(z))| \\ \leq\ & \mu(|z|)|\psi(z)||f(\varphi(0))| + C\mu(|z|)|\psi(z)| \ln\ln \frac{e}{1-|\varphi(z)|^2} \|f\|_{\mathcal{LB}}. \quad (8) \end{aligned}$$

In view of $\psi \in H_\mu^\infty$ and (5), the boundedness of $\psi C_\varphi : \mathcal{LB} \to H_\mu^\infty$ follows by taking the supremum in (8) over B. The proof is completed. □

Theorem 2. *Assume that $\psi \in H(B)$, φ is a holomorphic self-map of B and μ is a normal function on $[0,1)$. Then the following statements are equivalent.*

(a) $\psi C_\varphi : \mathcal{LB} \to H_\mu^\infty$ is compact;

(b) $\psi C_\varphi : \mathcal{LB}_0 \to H_\mu^\infty$ is compact;

(c) $\psi \in H_\mu^\infty$ and

$$\lim_{|\varphi(z)| \to 1} \mu(|z|)|\psi(z)| \ln\ln \frac{e}{1-|\varphi(z)|^2} = 0. \quad (9)$$

Proof. $(a) \Rightarrow (b)$ is clear.

$(b) \Rightarrow (c)$. Assume that $\psi C_\varphi : \mathcal{LB}_0 \to H_\mu^\infty$ is compact. Then $\psi \in H_\mu^\infty$ by the boundedness of $\psi C_\varphi : \mathcal{LB}_0 \to H_\mu^\infty$. Let $(z_k)_{k\in\mathbb{N}}$ be a sequence in B such that $\lim_{k\to\infty} |\varphi(z_k)| = 1$. Take

$$f_k(z) = \left(\ln\ln \frac{e}{1-\langle z, \varphi(z_k)\rangle}\right)^2 \left(\ln\ln \frac{e}{1-|\varphi(z_k)|^2}\right)^{-1}, \quad k \in \mathbb{N}. \quad (10)$$

It is not difficult to see that $f_k \in \mathcal{LB}_0$, $\sup_k \|f_k\|_{\mathcal{LB}} < \infty$. Moreover, $f_k \to 0$ uniformly on compact subsets of B as $k \to \infty$. By Lemma 1,

$$\lim_{k\to\infty} \|\psi C_\varphi f_k\|_{H_\mu^\infty} = 0. \quad (11)$$

On the other hand,

$$\|\psi C_\varphi f_k\|_{H_\mu^\infty} = \sup_{z\in B} \mu(|z|)|f_k(\varphi(z))\psi(z)| \geq \mu(|z_k|) \ln\ln \frac{e}{1-|\varphi(z_k)|^2} |\psi(z_k)|, \quad (12)$$

which together with (11) imply that

$$\lim_{k\to\infty} \mu(|z_k|)|\psi(z_k)| \ln\ln \frac{e}{1-|\varphi(z_k)|^2} = 0.$$

This proves that (9) holds.

$(c) \Rightarrow (a)$. Assume that $\psi \in H_\mu^\infty$ and (9) holds. Let $(f_k)_{k\in\mathbb{N}}$ be a sequence in $\mathcal{LB}$ with $\sup_{k\in\mathbb{N}} \|f_k\|_{\mathcal{LB}} \leq L$ and suppose $f_k \to 0$ uniformly on compact subsets of B as $k \to \infty$.

By (9) we have that if given $\varepsilon > 0$, there is a constant δ $(0 < \delta < 1)$, such that when $\delta < |\varphi(z)| < 1$ we have

$$\mu(|z|)|\psi(z)| \ln\ln \frac{e}{1-|\varphi(z)|^2} < \varepsilon. \tag{13}$$

Inequality (13) along with the fact that $\psi \in H_\mu^\infty$ shows that

$$\begin{aligned} \|\psi C_\varphi f_k\|_{H_\mu^\infty} &= \sup_{z\in B} \mu(|z|)|(\psi C_\varphi f_k)(z)| = \sup_{z\in B} \mu(|z|)|\psi(z) f_k(\varphi(z))| \\ \le & \Big(\sup_{|\varphi(z)|\le\delta} + \sup_{\{z\in B:\ \delta\le|\varphi(z)|<1\}} \Big) \mu(|z|)|\psi(z)||f_k(\varphi(z))| \\ \le & \|\psi\|_{H_\mu^\infty} \sup_{|w|\le\delta} |f_k(w)| + C \sup_{\{z\in B:\ \delta\le|\varphi(z)|<1\}} \mu(|z|)|\psi(z)| \ln\ln \frac{e}{1-|\varphi(z)|^2} \|f_k\| \\ \le & \|\psi\|_{H_\mu^\infty} \sup_{|w|\le\delta} |f_k(w)| + CL\varepsilon. \end{aligned}$$

The assumption gives that $\lim_{k\to\infty} \sup_{|w|\le\delta} |f_k(w)| = 0$. By letting $k \to \infty$ in the last inequality, we obtain $\limsup_{k\to\infty} \|\psi C_\varphi f_k\|_{H_\mu^\infty} \le CL\varepsilon$. Since ε is an arbitrary positive number, it follows that the last limit is equal to zero. Therefore, $\psi C_\varphi : \mathcal{LB} \to H_\mu^\infty$ is compact by Lemma 1. □

Theorem 3. *Assume that $\psi \in H(B)$, φ is a holomorphic self-map of B and μ is a normal function on $[0, 1)$. Then the following statements hold.*

(a) $\psi C_\varphi : \mathcal{LB} \to H_{\mu,0}^\infty$ is compact;

(b) $\psi C_\varphi : \mathcal{LB}_0 \to H_{\mu,0}^\infty$ is compact;

(c) $\psi \in H_{\mu,0}^\infty$ and

$$\lim_{|z|\to 1} \mu(|z|)|\psi(z)| \ln\ln \frac{e}{1-|\varphi(z)|^2} = 0. \tag{14}$$

Proof. $(a) \Rightarrow (b)$. It is obvious.

$(b) \Rightarrow (c)$. By the assumption, it is clear that $\psi C_\varphi : \mathcal{LB}_0 \to H_{\mu,0}^\infty$ is bounded. Taking $f(z) = 1$ we obtain $\psi \in H_{\mu,0}^\infty$. If $\|\varphi\|_\infty < 1$, we have

$$\lim_{|z|\to 1} \mu(|z|)|\psi(z)| \ln\ln \frac{e}{1-|\varphi(z)|^2} \le \ln\ln \frac{e}{1-\|\varphi\|_\infty^2} \lim_{|z|\to 1} \mu(|z|)|\psi(z)| = 0, \tag{15}$$

which implies (14).

Now we assume that $\|\varphi\|_\infty = 1$. By the assumption we see that $\psi C_\varphi : \mathcal{LB}_0 \to H_\mu^\infty$ is compact. From Theorem 2 we have

$$\lim_{|\varphi(z)|\to 1} \mu(|z|)|\psi(z)| \ln\ln \frac{e}{1-|\varphi(z)|^2} = 0. \tag{16}$$

According to (16), for every $\varepsilon > 0$, there exists an $r \in (0, 1)$, such that

$$\mu(|z|)|\psi(z)| \ln\ln \frac{e}{1-|\varphi(z)|^2} < \varepsilon$$

when $r < |\varphi(z)| < 1$. By the fact that $\psi \in H^\infty_{\mu,0}$, there exists a $\delta \in (0,1)$, such that

$$\mu(|z|)|\psi(z)| \le \frac{\varepsilon}{\ln\ln\frac{e}{1-r^2}}$$

when $\delta < |z| < 1$. Therefore, if $\delta < |z| < 1$ and $r < |\varphi(z)| < 1$, we obtain

$$\mu(|z|)|\psi(z)| \ln\ln\frac{e}{1-|\varphi(z)|^2} < \varepsilon. \tag{17}$$

If $\delta < |z| < 1$ and $|\varphi(z)| \le r$, we have

$$\mu(|z|)|\psi(z)| \ln\ln\frac{e}{1-|\varphi(z)|^2} < \ln\ln\frac{e}{1-r^2}\mu(|z|)|\psi(z)| < \varepsilon. \tag{18}$$

Combining (17) with (18), we get (14).

$(c) \Rightarrow (a)$. Suppose that $f \in \mathcal{LB}$. From Lemma 3 it follows that

$$\mu(|z|)|(\psi C_\varphi f)(z)| \le \mu(|z|)|\psi(z)||f(\varphi(0))| + C\mu(|z|)|\psi(z)| \ln\ln\frac{e}{1-|\varphi(z)|^2}\|f\|_{\mathcal{LB}}.$$

Taking the supremum in this inequality over all $f \in \mathcal{LB}$ such that $\|f\|_{\mathcal{LB}} \le 1$, by (14) and $\psi \in H^\infty_{\mu,0}$, letting $|z| \to 1$, we obtain

$$\lim_{|z|\to 1} \sup_{\|f\|_{\mathcal{LB}}\le 1} \mu(|z|)|(\psi C_\varphi(f))(z)| = 0.$$

By Lemma 2 it follows that $\psi C_\varphi : \mathcal{LB} \to H^\infty_{\mu,0}$ is compact, as desired. □

Theorem 4. *Assume that $\psi \in H(B)$, φ is a holomorphic self-map of B and μ is a normal function on $[0,1)$. Then $\psi C_\varphi : \mathcal{LB}_0 \to H^\infty_{\mu,0}$ is bounded if and only if $\psi C_\varphi : \mathcal{LB}_0 \to H^\infty_\mu$ is bounded and $\psi \in H^\infty_{\mu,0}$.*

Proof. Assume that $\psi C_\varphi : \mathcal{LB}_0 \to H^\infty_{\mu,0}$ is bounded. It is clear that $\psi C_\varphi : \mathcal{LB}_0 \to H^\infty_\mu$ is bounded. Taking $f(z) = 1$, we see that $\psi \in H^\infty_{\mu,0}$.

Conversely, assume that $\psi C_\varphi : \mathcal{LB}_0 \to H^\infty_\mu$ is bounded and $\psi \in H^\infty_{\mu,0}$. Suppose $f \in \mathcal{LB}_0$, then by Lemma 4 we have that for any $\varepsilon > 0$ there exists an $r_0 \in (0,1)$, such that

$$\frac{|f(\varphi(z))|}{\ln\ln\frac{e}{1-|\varphi(z)|^2}} < \varepsilon$$

whenever $|\varphi(z)| > r_0$. Hence

$$\mu(|z|)|(\psi C_\varphi f)(z)| = \mu(|z|)|\psi(z)| \ln\ln\frac{e}{1-|\varphi(z)|^2}\frac{|f(\varphi(z))|}{\ln\ln\frac{e}{1-|\varphi(z)|^2}} \le M\varepsilon, \tag{19}$$

whenever $|\varphi(z)| > r_0$, where M is defined in Theorem 1. On the other hand, when $|\varphi(z)| \le r_0$, by the fact that $\psi \in H^\infty_{\mu,0}$, we have

$$\begin{aligned} & \mu(|z|)|(\psi C_\varphi f)(z)| \\ \le\ & \mu(|z|)|\psi(z)|\Big(|f(\varphi(0))| + C\|f\|_{\mathcal{LB}} \ln\ln \frac{e}{1-|\varphi(z)|^2}\Big) \\ \le\ & \Big(|f(\varphi(0))| + C\|f\|_{\mathcal{LB}} \ln\ln \frac{e}{1-|r_0|^2}\Big)\mu(|z|)|\psi(z)| \to 0 \qquad (20) \end{aligned}$$

as $|z| \to 1$. From (19) an (20) we have that $\psi C_\varphi f \in H^\infty_{\mu,0}$. Hence $\psi C_\varphi(\mathcal{LB}_0) \subseteq H^\infty_{\mu,0}$, which implies the boundedness of $\psi C_\varphi : \mathcal{LB}_0 \to H^\infty_{\mu,0}$, as desired. □

References

[1] D. Clahane and S. Stević, Norm equivalence and composition operators between Bloch/ Lipschitz spaces of the unit ball, *J. Inequal. Appl.* Vol. 2006, Article ID 61018, (2006), 11 pages.

[2] C. C. Cowen and B. D. MacCluer, *Composition Operators on Spaces of Analytic Functions*, Studies in Advanced Math., CRC Press, Boca Raton, 1995.

[3] X. Fu and X. Zhu, Weighted composition operators on some weighted spaces in the unit ball, *Abstr. Appl. Anal.* Vol. 2008, Article ID 605807, (2008), 8 pages.

[4] H. Hedenmalm, B. Korenblum and K. Zhu, *Theory of Bergman Spaces*, Graduate Texts in Mathematics, 199. Springer-Verlag, New York, 2000.

[5] S. Li and S. Stević, Composition followed by differentiation between Bloch type spaces, *J. Comput. Anal. Appl.* **9** (2) (2007), 195-206.

[6] S. Li and S. Stević, Volterra type operators on Zygmund space, *J. Inequal. Appl.* Vol. 2007, Article ID 32124, (2007), 10 pages.

[7] S. Li and S. Stević, Weighted composition operators from H^∞ to the Bloch space on the polydisc, *Abstr. Appl. Anal.* Vol. 2007, Article ID 48478, (2007), 13 pages.

[8] S. Li and S. Stević, Weighted composition operators from α-Bloch space to H^∞ on the polydisc, *Numer. Funct. Anal. Optimization.* **28** (7-8) (2007), 911-925.

[9] S. Li and S. Stević, Generalized composition operators on Zygmund spaces and Bloch type spaces, *J. Math. Anal. Appl.* **338** (2008), 1282-1295.

[10] S. Li and S. Stević, Weighted composition operators between H^∞ and α-Bloch spaces in the unit ball, *Taiwanese J. Math.* **12** (7) (2008),1625-1639.

[11] J. A. Metzger, Bounded mean oscillation and Riemann surfaces, Bounded Mean Oscillation in Complex Analysis, Joensuu University, 1989, 79-100.

[12] A. Montes-Rodriguez, Weighted composition operators on weighted Banach spaces of analytic functions. *J. London Math. Soc.* **61** (3) (2000), 872-884.

[13] S. Ohno, Weighted composition operators between H^∞ and the Bloch space, *Taiwanese J. Math.* **5** (2001), 555-563.

[14] A. L. Shields and D. L. Williams, Bounded projections, duality, and multipliers in spaces of analytic functions, *Trans. Amer. Math. Soc.* **162** (1971), 287-302.

[15] S. Stević, Composition operators between H^∞ and the α-Bloch spaces on the polydisc, *Z. Anal. Anwendungen* **25** (4) (2006), 457-466.

[16] S. Stević, Weighted composition operators between mixed norm spaces and H_α^∞ spaces in the unit ball, *J. Inequal. Appl.* Vol. 2007, Article ID 28629, (2007), 9 pages.

[17] S. Stević, Essential norms of weighted composition operators from the α-Bloch space to a weighted-type space on the unit ball, *Abstr. Appl. Anal.* Vol. 2008, Article ID 279691, (2008), 11 pages.

[18] S. Stević, Norm of weighted composition operators from Bloch space to H_μ^∞ on the unit ball, *Ars. Combin.* **88** (2008), 125-127.

[19] S. Stević, Generalized composition operators from logarithmic Bloch spaces to mixed-norm spaces, *Util. Math.* **77** (2008) (to appear).

[20] S. Stević, Essential norms of weighted composition operators from the Bergman space to weighted-type spaces on the unit ball, *Ars. Combin.* (2009) (to appear).

[21] S. Stević, On a new integral operator from logarithmic Bloch space to the Bloch-type space on the unit ball, *Appl. Math. Comput.* (2008), doi:10.1016/j.amc.2008.09.002.

[22] S. I. Ueki and L. Luo, Compact weighted composition oprators and multiplication operators between Hardy spaces, *Abstr. Appl. Anal.* Vol. 2008, Article ID 196498, (2008), 11 pages.

[23] S. Ye, Weighted composition operators between the little α-Bloch space and the logarithmic Bloch, *J. Comput. Anal. Appl.* **10** (2) (2008), 243-252.

[24] K. Zhu, Multipliers of BMO in the Bergman metric with applications to Toeplitz operators, *J. Funct. Anal.* **87** (1989), 31-50.

[25] K. Zhu, *Spaces of Holomorphic Functions in the Unit Ball*, Graduate Texts in Mathematics, Springer (2005).

[26] X. Zhu, Products of differentiation, composition and multiplication from Bergman type spaces to Bers type space, *Integ. Tran. Spec. Funct.* **18** (2007), 223-271.