Some lower bounds for constant weight codes

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Abstract

Fifty-five new or improved lower bounds for $A(n, d, w)$, the maximum possible number of binary vectors of length n, weight w, and pairwise Hamming distance no less than d, are presented.

1 Introduction

A considerable literature exists on the determination of bounds on $A(n, d, w)$, the maximum possible number of binary vectors of length n, weight w, and pairwise Hamming distance no less than d. In particular the authors of [1] and [2] maintain tables of bounds at [3] and [4], and new results continue to appear (e.g. [5-7]). The results presented in [5] were obtained by a variety of methods, but one method, lexicographic in nature and referred to as 'B-Lex' in [5], usually gave good estimates of previously known lower bounds over a fairly wide range of (n, d, w) values. Lexicographic methods are more fully discussed in [8]. In [5] a random vector $(a_1, a_2, \dots a_w)$, where $1 \leq a_i \leq n$, and $a_i \leq a_{i+1}$, was chosen to begin the vector list, and then, proceeding lexicographically, from $(1, 2, \dots w)$ to $(n - w + 1, \dots n)$, adding to the vector list those vectors satisfying the criteria. Sometimes more than one random start vector is preferable. In some cases 'B-Lex' either improved on the bounds in [3], e.g. for $A(n, 4, 5)$, $A(n, 6, 5)$, and $A(n, 6, 6)$, for many n $(31 < n < 63)$, or gave results where none were previously given, e.g. for $A(n, 8, 6)$, $A(n, 8, 7)$, $A(n, 10, 7)$ and $A(n,10, 8)$, for a number of n-values, $(40 < n < 63)$.

Given the attention this $A(n, d, w)$ problem has received, and the many gaps waiting to be filled in the tables, further results are presented, obtained by the lexicographic-style method, in areas of the tables where little has previously been reported. Should these results prove of interest, yet more results could be generated for other areas of the tables. In two instances the vectors themselves are shown, one in binary $(b_1, b_2, \dots b_n)$ form and both in support form $\{ i \mid b_i = 1 \}$.

2 Results

2.1 Lower bounds for $A(n, d, w)$

Results in bold improve on existing bounds given in [3]. All other results fill gaps in the tables.

$d = 10$

n	43	44	45	46	47	48	49	50
$A(n, d, 9)$	**1233**	1398	1582	1782	2014	2263	2528	2826

$d = 12$

n	31	32	33	34	35	36	37	38	39	40	41	42	43	44
$A(n, d, 9)$	**39**	**45**	52	59	69	76	89	99	110	125	140	155	172	194
$A(n, d, 10)$		**80**	96	115	133	153	179	206	236	269				

$d = 14$

n	32	33	34	35	36	37	38	39	40
$A(n, d, 9)$		14	15	16	19	21	22	25	27
$A(n, d, 10)$	**22**	24	28	31	34	38	41	46	52

$d = 16$

n	35	36	37	38	39	40
$A(n,d,11)$	15	18	19	22	26	28

$d = 18$

n	42
$A(n,d,12)$	18

2.2 Some sets of codewords

The 59 codewords for (34,12,9).
First random vector (seed) 1 4 5 7 12 14 17 24 26.
Vector list in support and binary forms:

1	1	2	3	4	5	6	8	9	10	1111110111000000000000000000000000
2	1	2	3	7	11	12	13	15	16	1110001000111011000000000000000000
3	1	2	3	14	17	18	19	20	21	1110000000000100111110000000000000
4	1	2	3	22	23	24	25	26	27	1110000000000000000001111110000000
5	1	2	3	28	29	30	31	32	33	1110000000000000000000000001111110
6	1	4	5	7	12	14	17	24	26	1001101000010100100000010100000000
7	1	4	5	11	13	18	19	22	23	1001100000101000011001100000000000
8	1	4	5	15	16	20	21	25	27	1001100000000011000110001010000000
9	1	4	6	7	11	20	28	29	34	1001011000100000000100000001100001
10	1	4	8	12	13	21	30	31	34	1001000100011000000010000000011001
11	1	4	9	14	15	18	32	33	34	1001000010000110010000000000000111
12	1	5	6	16	17	19	30	32	34	1000110000000001101000000000010101
13	1	5	7	10	22	25	31	33	34	1000101001000000000001001000001011
14	1	6	7	8	12	18	19	25	27	1000011100010000011000001010000000
15	1	6	7	9	13	17	21	23	33	1000011010001000100010100000000010
16	1	6	8	11	14	15	17	22	31	1000010100100110100001000000001000

17	1	6	9	16	18	20	24	26	31	1000010010000001010100010100001000
18	1	6	10	12	13	22	24	28	32	1000010001011000000001010001000100
19	1	6	10	14	21	25	26	29	30	1000010001000100000010001100110000
20	1	7	8	10	14	16	20	23	32	1000001101000101000100100000000100
21	1	7	9	15	19	22	24	29	30	1000001010000010001001010000110000
22	1	8	9	11	13	26	27	29	32	1000000110101000000000000110100100
23	1	8	11	16	19	21	24	28	33	1000000100100001001010010001000010
24	1	10	11	12	17	20	27	30	33	1000000001110000100100000010010010
25	2	4	5	11	24	27	31	32	34	0101100000100000000000001001000110 1
26	2	4	6	7	13	14	22	27	30	0101011000001100000001000010010000
27	2	4	6	11	12	18	21	26	33	0101010000110000010010000100000010
28	2	4	6	15	16	17	23	24	28	0101010000000011100000110001000000
29	2	4	7	8	15	19	20	26	31	0101001100000010001100000100001000
30	2	4	7	9	12	23	25	29	32	0101001010010000000000101000100100
31	2	4	10	13	16	19	29	33	34	0101000001001001001000000000100011
32	2	5	6	12	13	17	20	25	31	0100110000011000100100001000001000
33	2	5	7	8	11	17	18	28	30	0100101100100000110000000001010000
34	2	5	8	12	14	15	23	27	33	0100100100010110000000100010000010
35	2	5	9	11	14	16	19	25	26	0100100010100101001000001100000000
36	2	5	9	13	15	21	22	28	34	0100100010001010000011000001000001
37	2	5	10	18	20	22	26	29	32	0100100001000000010101000100100100
38	2	8	9	20	24	25	30	33	34	0100000110000000000100011000010011
39	2	9	10	16	18	21	23	27	30	0100000011000001010010100010010000
40	2	12	14	16	21	22	24	29	31	0100000000010101000011010000101000
41	3	4	5	14	23	25	28	30	34	0011100000000100000000101001010001
42	3	4	6	19	20	22	25	32	33	0011010000000000001101001000000110
43	3	4	7	8	16	18	22	24	34	0011001100000001010001010000000001
44	3	4	7	10	15	17	21	30	32	0011001001000010100010000000010100
45	3	4	9	11	12	17	19	28	31	0011000010110000101000000001001000
46	3	5	6	7	15	18	23	29	31	0010111000000010010000100000101000
47	3	5	6	13	16	26	27	28	33	0010110000001001000000000111000010
48	3	5	10	12	15	19	20	24	34	0010100001010010001100010000000001
49	3	6	9	14	17	24	27	29	34	0010010010000100100000010010100001
50	3	7	9	10	13	18	20	25	28	0010001011001000010100001001000000
51	3	8	11	20	21	22	23	29	30	0010000100100000000111100000110000
52	3	10	11	13	14	23	24	31	33	0010000001101100000000110000001010
53	3	12	13	17	18	23	26	32	34	0010000000011000110000100100000101
54	4	8	10	14	18	27	28	29	31	0001000101000100010000000011101000
55	5	8	13	14	18	21	24	25	32	0000100100001100010010011000000100
56	7	11	15	22	23	26	28	32	33	0000001000100010000001100101000110
57	8	10	15	16	17	18	25	26	33	0000000101000011110000001100000010
58	10	13	17	19	21	22	26	27	31	0000000001001000101011000110001000
59	14	19	20	24	26	27	28	30	32	0000000000000100001100010111010100

The 27 codewords for (40,14,9) in support form. Seed 4 5 11 16 26 32 33 38 40.

1	1	2	3	4	5	6	7	8	9
2	1	2	10	11	12	13	14	15	16
3	1	2	17	18	19	20	21	22	23
4	1	2	24	25	26	27	28	29	30
5	1	2	31	32	33	34	35	36	37

6	1	3	10	17	24	31	38	39	40
7	3	4	10	11	18	19	25	27	34
8	3	4	12	13	17	20	26	28	35
9	3	4	14	15	21	22	24	29	32
10	3	5	10	12	21	23	30	33	36
11	3	6	11	16	20	22	30	31	37
12	4	5	11	16	26	32	33	38	40
13	4	5	13	14	18	23	31	37	39
14	4	6	15	19	28	30	36	38	39
15	5	6	10	13	19	22	29	35	40
16	5	6	12	14	20	24	25	34	38
17	5	7	10	15	17	20	27	32	37
18	5	8	11	15	21	25	28	31	35
19	5	9	16	17	18	28	29	34	36
20	6	7	11	12	18	21	26	29	39
21	6	7	14	16	23	27	28	33	35
22	6	8	11	13	17	23	24	32	36
23	7	8	12	13	27	30	31	34	40
24	7	8	14	19	22	25	26	36	37
25	8	9	10	14	18	30	32	35	38
26	8	9	15	20	23	26	33	34	39
27	9	13	16	19	21	24	27	37	38

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