WEIGHTED COMPOSITION OPERATORS FROM BLOCH-TYPE SPACES TO WEIGHTED-TYPE SPACES

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Abstract

Let $H(B)$ denote the space of all holomorphic functions on the unit ball B. Let $u \in H(B)$ and φ be a holomorphic self-map of B. This paper characterizes the boundedness and compactness of the weighted composition operator uC_φ from Bloch-type spaces to weighted-type spaces in the unit ball.

1 Introduction

Let $B^n = B$ be the unit ball of $\mathbb{C}^n$, $D = B^1$ the unit disk, and $H(B)$ the space of all holomorphic functions on B. For $f \in H(B)$, let $\Re f(z) = \sum_{j=1}^n z_j \frac{\partial f}{\partial z_j}(z)$ represent the radial derivative of f. For $\alpha > 0$, recall that the α-Bloch space $\mathcal{B}^\alpha = \mathcal{B}^\alpha(B)$, is the space consisting of all functions $f \in H(B)$ such that

$$b_\alpha(f) = \sup_{z \in B}(1 - |z|^2)^\alpha |\Re f(z)| < \infty. \tag{1}$$

Under the norm $\|f\|_{\mathcal{B}^\alpha} = |f(0)| + b_\alpha(f)$, $\mathcal{B}^\alpha$ is a Banach space. When $\alpha = 1$, we get the classical Bloch space $\mathcal{B}$. For more information of the Bloch space and the α-Bloch space (see, e.g., [1, 12, 15, 22, 34] and the references therein).

Assume that μ is a positive continuous function on $[0, 1)$, and there exist positive numbers s and t, $0 < s < t$, and $\delta \in [0, 1)$ such that

$$\frac{\mu(r)}{(1-r)^s} \text{ is decreasing on } [\delta, 1) \text{ and } \lim_{r \to 1} \frac{\mu(r)}{(1-r)^s} = 0;$$

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$$\frac{\mu(r)}{(1-r)^t} \text{ is increasing on } [\delta, 1) \text{ and } \lim_{r\to 1}\frac{\mu(r)}{(1-r)^t} = \infty,$$

then μ is called a normal function (see [18]).

Let ω be a normal function on $[0, 1)$. An $f \in H(B)$ is said to belong to the Bloch-type space $\mathcal{B}_\omega = \mathcal{B}_\omega(B)$, if

$$\|f\|_{b_\omega} = \sup_{z\in B} \omega(|z|)\,|\Re f(z)| < \infty,$$

(see, e.g., [3, 24, 28, 29, 30]). $\mathcal{B}_\omega$ is a Banach space with the norm $\|\cdot\|_{\mathcal{B}_\omega} = |f(0)| + \|f\|_{b_\omega}$. Note that when $\mu(r) = (1-r^2)^\alpha$, the induced space $\mathcal{B}_\omega$ becomes the α-Bloch space $\mathcal{B}^\alpha$.

Let μ be a normal function on $[0, 1)$. The weighted-type space $H_\mu^\infty = H_\mu^\infty(B)$ is the space of all $f \in H(B)$ for which

$$\|f\|_{H_\mu^\infty} = \sup_{z\in B} \mu(|z|)\,|f(z)| < \infty.$$

(see, e.g., [24, 26, 29]). Under the norm $\|\cdot\|_{H_\mu^\infty}$, H_μ^∞ is a Banach space. We denote by $H_{\mu,0}^\infty$ the subspace of H_μ^∞ consisting of those $f \in H_\mu^\infty$ such that $\lim_{|z|\to 1} \mu(|z|)|f(z)| = 0$.

Let $u \in H(B)$ and φ be a holomorphic self-map of B. For $f \in H(B)$, the weighted composition operator uC_φ is defined by

$$(uC_\varphi f)(z) = u(z)f(\varphi(z)), \quad z \in B. \tag{2}$$

When $u = 1$, the weighted composition operator uC_φ is the composition operator, which is defined by $(C_\varphi f)(z) = f(\varphi(z))$. The main purpose in the study of composition operators is to describe operator theoretic properties of C_φ in terms of function theoretic properties of φ. For more information, see, e.g. [2] and the references therein.

In the setting of the unit ball, Zhu studied the weighted composition operator between Bergman-type spaces and H^∞ in [35]. Stević studied the weighted composition operator between mixed norm spaces and H_α^∞ in [23]. Some necessary and sufficient conditions for the weighted composition operator to be bounded or compact between the Bloch space and H^∞ are given in [13]. See also interesting paper [26] in which operator norm of the weighted composition operator from the Bloch space to weighted-type space on the unit ball was calculated. Some related results can be found, e.g., in [1, 3, 4, 6, 7, 8, 9, 10, 11, 14, 17, 19, 21, 24, 25, 26, 27, 29, 31, 33, 36, 37].

Recall that a linear operator is said to be bounded if the image of a bounded set is a bounded set, while a linear operator is compact if it takes bounded sets to sets with compact closure.

This paper is devoted to studing of the boundedness and compactness of the weighted composition operator from $\mathcal{B}_\omega$ to the space H_μ^∞. Some necessary and sufficient conditions for the weighted composition operator uC_φ to be bounded or compact are given.

Throughout this paper C will denote constants, they are positive and may differ from one occurrence to the other. $a \preceq b$ means that there is a positive constant C such that $a \leq Cb$. If both $a \preceq b$ and $b \preceq a$ hold, then one says that $a \asymp b$.

2 Main results and proofs

In order to prove main results, we need some auxiliary results which are incorporated in the following lemmas. The following lemma can be found in [32].

Lemma 1. *Assume that ω is a normal function on $[0,1)$. If $f \in \mathcal{B}_\omega$, then*

$$|f(z)| \leq C\Big(1 + \int_0^{|z|} \frac{dt}{\omega(t)}\Big)\|f\|_{\mathcal{B}_\omega}$$

for some C independent of f.

Lemma 2. *Assume that μ is a normal function on $[0,1)$. A closed set K in $H_{\mu,0}^\infty$ is compact if and only if it is bounded and satisfies*

$$\lim_{|z|\to 1} \sup_{f\in K} \mu(|z|)|f(z)| = 0.$$

The proof of Lemma 2 is similar to the proof of Lemma 1 in [16], hence we omit it.

By standard arguments similar to those outlined in Proposition 3.11 of [2] (or Lemma 3 in [19, 20]), the following lemma follows. We omit the details.

Lemma 3. *Assume that $u \in H(B)$, φ is a holomorphic self-map of B, ω and μ are normal functions on $[0,1)$. Then $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is compact if and only if $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbf{N}}$ in $\mathcal{B}_\omega$ which converges to zero uniformly on compact subsets of B as $k \to \infty$, we have $\|uC_\varphi f_k\|_{H_\mu^\infty} \to 0$ as $k \to \infty$.*

Lemma 4. ([32]) *Assume that ω is normal and $\int_0^1 \frac{dt}{\omega(t)} < \infty$. Let (f_k) be a bounded sequence in $\mathcal{B}_\omega$ which converges to 0 uniformly on compact subsets of B, then*

$$\lim_{k\to\infty} \sup_{z\in B} |f_k(z)| = 0.$$

Assume ω is normal and δ the constant in the definition of normality. Denote

$$k_0 = \max\Big(0, \big[\log_2 \frac{1}{\omega(\delta)}\big]\Big), \quad r_k = \omega^{-1}(1/2^k), \quad n_k = \big[\frac{1}{1-r_k}\big]$$

for $k > k_0$, where $[a]$ denotes the greatest integer not more than a.

Let (see [5] or [32])

$$g(z) = 1 + \sum_{k>k_0}^{\infty} 2^{n_k} z^{n_k}, \quad z \in D. \tag{3}$$

Lemma 5. ([5, 32]) *Assume that ω is normal. Then $g(z)$ is holomorphic on D, $g(r)$ is increasing on $[0,1)$ and*

$$0 < C_1 = \inf_{r\in[0,1)} \omega(r)g(r) \le \sup_{r\in[0,1)} \omega(r)g(r) \le C_2 < \infty. \tag{4}$$

Now we are in a position to state and prove our main results is this paper.

Theorem 1. *Assume that $u \in H(B)$, φ is a holomorphic self-map of B, μ and ω are normal functions on $[0,1)$. Then $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is bounded if and only if*

$$M := \sup_{z\in B} \mu(|z|)|u(z)|\Big(1 + \int_0^{|\varphi(z)|} \frac{dt}{\omega(t)}\Big) < \infty. \tag{5}$$

Moreover, when $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is bounded, the following relationship holds

$$\|uC_\varphi\|_{\mathcal{B}_\omega \to H_\mu^\infty} \asymp M. \tag{6}$$

Proof. Assume that (5) holds. For any $f \in \mathcal{B}_\omega$, in view of Lemma 1, we have

$$\begin{aligned} \|uC_\varphi f\|_{H_\mu^\infty} &= \sup_{z\in B} \mu(|z|)|(uC_\varphi f)(z)| \\ &= \sup_{z\in B} \mu(|z|)|f(\varphi(z))||u(z)| \\ &\le C\|f\|_{\mathcal{B}_\omega} \sup_{z\in B} \mu(|z|)|u(z)|\Big(1 + \int_0^{|\varphi(z)|} \frac{dt}{\omega(t)}\Big). \end{aligned} \tag{7}$$

Therefore (5) implies that $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is bounded, and moreover

$$\|uC_\varphi\|_{\mathcal{B}_\omega \to H_\mu^\infty} \le CM. \tag{8}$$

Conversely, suppose that $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is bounded. Then for the test function given by $f(z) \equiv 1 \in \mathcal{B}_\omega$, we obtain $u \in H_\mu^\infty$ and moreover

$$\|u\|_{H_\mu^\infty} = \|uC_\varphi(1)\|_{H_\mu^\infty} \le \|uC_\varphi\|_{\mathcal{B}_\omega \to H_\mu^\infty}. \tag{9}$$

For $b \in B$, define

$$f_b(z) = \int_0^{\langle z,b\rangle} g(t)dt, \tag{10}$$

where $g(z)$ is as in Lemma 5. From [5] or [32] we see that $f_b \in \mathcal{B}_\omega$, moreover there is a positive constant C such that $\sup_{b\in B} \|f_b\|_{\mathcal{B}_\omega} \le C$. Therefore

$$\begin{aligned} \sup_{z\in B} \mu(|z|)|f_{\varphi(w)}(\varphi(z))u(z)| &= \sup_{z\in B} \mu(|z|)|(uC_\varphi f_{\varphi(w)})(z)| \\ &= \|uC_\varphi f_{\varphi(w)}\|_{H_\mu^\infty} \le C\|uC_\varphi\|_{\mathcal{B}_\omega \to H_\mu^\infty} \end{aligned} \tag{11}$$

for every $w \in B$, from which we get

$$\sup_{w\in B} \mu(|w|)|u(w)| \int_0^{|\varphi(w)|^2} \frac{dt}{\omega(t)} \le C\|uC_\varphi\|_{\mathcal{B}_\omega \to H_\mu^\infty} < \infty. \tag{12}$$

We derive from Lemma 5 that (see e.g. [32])

$$\int_{|z|^2}^{|z|} \frac{dt}{\omega(t)} \le Cg(|z|^2)(1-|z|) \le C\int_{|z|^4}^{|z|^2} \frac{dt}{\omega(t)}.$$

Hence

$$\int_0^{|\varphi(w)|^2} \frac{dt}{\omega(t)} \le \int_0^{|\varphi(w)|} \frac{dt}{\omega(t)} \le C\int_0^{|\varphi(w)|^2} \frac{dt}{\omega(t)}. \tag{13}$$

From (12), (13) and the fact that $u \in H_\mu^\infty$, (5) follows. From (8), (9), (12) and (13), we see that (6) holds. The proof of this theorem is finished. □

Theorem 2. *Assume that $u \in H(B)$, φ is a holomorphic self-map of B, μ and ω are normal functions on $[0,1)$, and $\int_0^1 \frac{dt}{\omega(t)} < \infty$. Then $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is compact if and only if $u \in H_\mu^\infty$.*

Proof. First assume that $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is compact. Then $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is bounded. By taking $f(z) \equiv 1$, we obtain $u \in H_\mu^\infty$, as desired.

Now suppose that $u \in H_\mu^\infty$. Since $\int_0^1 \frac{dt}{\omega(t)} < \infty$, then

$$\sup_{z\in B} \mu(|z|)|u(z)| \int_0^{|\varphi(z)|} \frac{dt}{\omega(t)} \le \sup_{z\in B} \mu(|z|)|u(z)| \int_0^1 \frac{dt}{\omega(t)} < \infty. \tag{14}$$

From (14), we have that for every $f \in \mathcal{B}_\omega$

$$\begin{aligned} \mu(|z|)|(uC_\varphi f)(z)| &= \mu(|z|)|f(\varphi(z))||u(z)| \\ &\le C\|f\|_{\mathcal{B}_\omega} \sup_{z\in B} \mu(|z|)|u(z)|\Big(1+\int_0^{|\varphi(z)|} \frac{dt}{\omega(t)}\Big) \le C\|f\|_{\mathcal{B}_\omega}\|u\|_{H_\mu^\infty} \end{aligned} \tag{15}$$

From the above inequality we see that $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is bounded. Let $(f_k)_{k\in\mathbf{N}}$ be any bounded sequence in $\mathcal{B}_\omega$ and $f_k \to 0$ uniformly on compact subsets of B as $k \to \infty$. Employing Lemma 4 we have

$$\|uC_\varphi f_k\|_{H_\mu^\infty} = \sup_{z\in B} \mu(|z|)|f_k(\varphi(z))u(z)| \le \|u\|_{H_\mu^\infty} \sup_{z\in B} |f_k(\varphi(z))| \to 0,$$

as $k \to \infty$. Then the result follows from Lemma 3. □

Theorem 3. *Assume that $u \in H(B)$, φ is a holomorphic self-map of B, μ and ω are normal functions on $[0,1)$, and $\int_0^1 \frac{dt}{\omega(t)} = \infty$. Then $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is compact if and only if $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is bounded and*

$$\lim_{|\varphi(z)|\to 1} \mu(|z|)|u(z)|\Big(1+\int_0^{|\varphi(z)|} \frac{dt}{\omega(t)}\Big) = 0. \tag{16}$$

Proof. First assume that $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is bounded and the condition in (16) holds. Assume that $(f_k)_{k\in\mathbf{N}}$ is a sequence in $\mathcal{B}_\omega$ such that $\sup_{k\in\mathbf{N}} \|f_k\|_{\mathcal{B}_\omega} \le L$ and $f_k \to 0$ uniformly on compact subsets of B as $k \to \infty$. According to Lemma 3 it suffices to show that $\|uC_\varphi f_k\|_{H_\mu^\infty} \to 0$ as $k \to \infty$.

From (16), we have that for every $\varepsilon > 0$, there is a constant $\delta \in (0,1)$, such that

$$\mu(|z|)|u(z)|\Big(1+\int_0^{|\varphi(z)|} \frac{dt}{\omega(t)}\Big) < \varepsilon$$

when $\delta < |\varphi(z)| < 1$. By Lemma 1, we have

$$\begin{aligned} \|uC_\varphi f_k\|_{H_\mu^\infty} &= \sup_{z\in B} \mu(|z|)|(uC_\varphi f_k)(z)| = \sup_{z\in B} \mu(|z|)|u(z)||f_k(\varphi(z))| \\ &\le \sup_{\varphi(z)\in\overline{B(0,\delta)}} \mu(|z|)|u(z)||f_k(\varphi(z))| \\ &\quad + C \sup_{\varphi(z)\in B\setminus\overline{B(0,\delta)}} \mu(|z|)|u(z)|\Big(1+\int_0^{|\varphi(z)|} \frac{dt}{\omega(t)}\Big)\|f_k\|_{\mathcal{B}_\omega} \\ &\le \|u\|_{H_\mu^\infty} \sup_{\varphi(z)\in\overline{B(0,\delta)}} |f_k(\varphi(z))| + CL\varepsilon. \end{aligned}$$

Using the fact that $f_k \to 0$ uniformly on compact subsets of B as $k \to \infty$, we obtain

$$\limsup_{k\to\infty} \sup_{\varphi(z)\in\overline{B(0,\delta)}} |f_k(\varphi(z))| = 0.$$

Therefore $\limsup_{k\to\infty} \|uC_\varphi f_k\|_{H_\mu^\infty} \le CL\varepsilon$. Since ε is an arbitrary positive number we have that $\lim_{k\to\infty} \|uC_\varphi f_k\|_{H_\mu^\infty} = 0$, and therefore, $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is compact by Lemma 3.

Conversely, suppose $uC_\varphi : \mathcal{B}_\omega \to H_\mu^\infty$ is compact. To prove (16), we only need to prove

$$\lim_{|\varphi(z)|\to 1} \mu(|z|)|u(z)| \int_0^{|\varphi(z)|} \frac{dt}{\omega(t)} = 0, \tag{17}$$

since they are equivalent. Let $(z_k)_{k\in\mathbb{N}}$ be a sequence in B such that $|\varphi(z_k)| \to 1$ as $k \to \infty$ (if such a sequence does not exist then condition (16) is vacuously satisfied). For $k \in \mathbb{N}$, set

$$f_k(z) = \Big(\int_0^{|\varphi(z_k)|^2} g(t)dt\Big)^{-1} \Big(\int_0^{\langle z,\varphi(z_k)\rangle} g(t)dt\Big)^2.$$

From [32], we see that $f_k \in \mathcal{B}_\omega$ for every $k \in \mathbb{N}$, moreover $\sup_{k\in\mathbb{N}} \|f_k\|_{\mathcal{B}_\omega} \le C$. Beside this f_k converges to 0 uniformly on compact subsets of B as $k \to \infty$. Since uC_φ is compact, by Lemma 3 we have that $\|uC_\varphi f_k\|_{H_\mu^\infty} \to 0$ as $k \to \infty$. Thus

$$\begin{aligned}
\mu(|z_k|)|u(z_k)| \int_0^{|\varphi(z_k)|^2} g(t)dt &= \mu(|z_k|)|u(z_k)|\ |f_k(\varphi(z_k))| \\
&\le \sup_{z\in B} \mu(|z|)|f_k(\varphi(z))||u(z)| \\
&= \sup_{z\in B} \mu(|z|)|(uC_\varphi f_k)(z)| = \|uC_\varphi f_k\|_{H_\mu^\infty} \to 0
\end{aligned}$$

as $k \to \infty$, which along with (13) implies

$$\mu(|z_k|)|u(z_k)| \int_0^{|\varphi(z_k)|} g(t)dt \to 0$$

as $k \to \infty$. From this we obtain (16), finishing the proof of the theorem. □

Theorem 4. *Assume that $u \in H(B)$, φ is a holomorphic self-map of B, μ and ω are normal functions on $[0, 1)$. Then $uC_\varphi : \mathcal{B}_\omega \to H_{\mu,0}^\infty$ is compact if and only if*

$$\lim_{|z|\to 1} \mu(|z|)|u(z)|\Big(1 + \int_0^{|\varphi(z)|} \frac{dt}{\omega(t)}\Big) = 0. \tag{18}$$

Proof. Suppose that (18) holds. From Lemma 2, we see that $uC_\varphi : \mathcal{B}_\omega \to H^\infty_{\mu,0}$ is compact if and only if

$$\lim_{|z|\to 1} \sup_{\|f\|_{\mathcal{B}_\omega}\le 1} \mu(|z|)|(uC_\varphi f)(z)| = 0. \tag{19}$$

On the other hand, applying Lemma 1, we have that

$$\mu(|z|)|(uC_\varphi f)(z)| \le C\mu(|z|)|u(z)|\Big(1+\int_0^{|\varphi(z)|}\frac{dt}{\omega(t)}\Big)\|f\|_{\mathcal{B}_\omega}. \tag{20}$$

Taking the supremum in (20) over the the unit ball in the space $\mathcal{B}_\omega$, then letting $|z|\to 1$ and applying (18) the result follows.

Conversely, suppose that $uC_\varphi : \mathcal{B}_\omega \to H^\infty_{\mu,0}$ is compact. Taking $f(z)\equiv 1$, then employing the boundedness of $uC_\varphi : \mathcal{B}_\omega \to H^\infty_{\mu,0}$, we get

$$\lim_{|z|\to 1} \mu(|z|)|u(z)| = 0. \tag{21}$$

If $\int_0^1 \frac{dt}{\omega(t)} < \infty$, then by (21) we easily get (18).

Now we assume that $\int_0^1 \frac{dt}{\omega(t)} = \infty$. By the assumption, it is clear that $uC_\varphi : \mathcal{B}_\omega \to H^\infty_\mu$ is compact. Hence by Theorem 2, we have

$$\lim_{|\varphi(z)|\to 1} \mu(|z|)|u(z)|\Big(1+\int_0^{|\varphi(z)|}\frac{dt}{\omega(t)}\Big) = 0. \tag{22}$$

By (22), we have that for every $\varepsilon > 0$, there exists a $\delta \in (0,1)$, such that

$$\mu(|z|)|u(z)|\Big(1+\int_0^{|\varphi(z)|}\frac{dt}{\omega(t)}\Big) < \varepsilon$$

when $\delta < |\varphi(z)| < 1$. By (21), for above chosen ε, there is an $r\in(0,1)$, so that

$$\mu(|z|)|u(z)| < \Big(1+\int_0^{\delta}\frac{dt}{\omega(t)}\Big)^{-1}\varepsilon$$

when $r < |z| < 1$.

Therefore, when $r < |z| < 1$ and $\delta < |\varphi(z)| < 1$, we have that

$$\mu(|z|)|u(z)|\Big(1+\int_0^{|\varphi(z)|}\frac{dt}{\omega(t)}\Big) < \varepsilon. \tag{23}$$

On the other hand, if $|\varphi(z)| \le \delta$ and $r < |z| < 1$, we obtain

$$\mu(|z|)|u(z)|\Big(1+\int_0^{|\varphi(z)|}\frac{dt}{\omega(t)}\Big) \le \Big(1+\int_0^{\delta}\frac{dt}{\omega(t)}\Big)\mu(|z|)|u(z)| < \varepsilon. \tag{24}$$

Combing (23) with (24) we get (18), as desired. □

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