On antimagic total labeling of some families of graphs

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Abstract. This paper deals with two types of graph labelings namely, super (a,d)-edge antimagic total labeling and (a,d)-vertex antimagic total labeling. We provide super (a,d)-edge antimagic total labeling for disjoint unions of Harary graphs and disjoint unions of cycles. We also provide (a,d)-vertex antimagic total labeling for disjoint unions of Harary graphs, disjoint unions of cycles, sun graphs and disjoint unions of sun graphs.

Keywords : Super antimagic total labeling, Harary graphs, Cycles, Sun graphs.

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1 Introduction

All graphs in this paper are finite, simple and undirected. The graph G has the vertex-set $V(G)$ and edge-set $E(G)$. A general reference for graph-theoretic ideas can be seen in [13]. A *labeling* (or *valuation*) of a graph is a map that carries graph elements to numbers (usually to positive or non-negative integers). In this paper the domain will be the set of all vertices and edges and such a labeling is called *a total labeling*. Some labelings use the vertex-set only, or the edge-set only, and we shall call them *vertex-labelings* and *edge-labelings* respectively. Other domains are possible. The most complete recent survey of graph labelings can be seen in [8]. There are many types of graph labelings, for example harmonius, cordial, graceful and antimagic. In this paper, we focus on two types of labelings called (a,d)-edge antimagic total labeling and (a,d)-vertex antimagic total labeling. A graph G is called *(a,d)-edge antimagic total* ((a,d)-EAT) if there exist integers $a > 0$, $d \geq 0$ and a bijection $\lambda : V \cup E \rightarrow \{1, 2, ..., |V| + |E|\}$ such that the elements of the set $W = \{w(xy) : xy \in E(G)\}$ form an arithmetic progression starting from a with common difference d, where $w(xy) = \lambda(x)+\lambda(y)+\lambda(xy)$. W is called the set of *edge-weights* of the graph G. Additionally, if $\lambda(V) = \{1, 2, ..., |V|\}$ then G is super (a,d)-EAT. Similarly, a graph G is called *(a,d)-vertex antimagic total* ((a,d)-VAT) if there exist integers $a > 0$, $d \geq 0$ and a bijection $\lambda : V \cup E \rightarrow \{1, 2, ..., |V| + |E|\}$ such that the elements of the set $W = \{w(x) : x \in V(G)\}$ form an arithmetic progression starting from a with common difference d, where $w(x) = \lambda(x) + \sum_{xy \in E(G)} \lambda(xy)$ where the sum is taken over all vertices y which are adjacent to x. In this case, W is called the set of *vertex-weights* of the graph G. In particular, an (a,d)-VAT labeling λ of graph G is *super* if $\lambda(V) = \{1, 2, ..., |V|\}$.

Let G be a regular graph. Let $\lambda : V \cup E \rightarrow \{1, 2, \cdots, |V| + |E|\}$ be an (a,d)-vertex antimagic total labeling for G. Define a new labeling:

$$\lambda' : V \cup E \rightarrow \{1, 2, \cdots, |V| + |E|\}$$

on G as follows:

$$\lambda'(x) = |V| + |E| + 1 - \lambda(x); x \in V(G)$$
$$\lambda'(xy) = |V| + |E| + 1 - \lambda(xy); xy \in E(G).$$

In [1] it is proved that λ' is also an (a',d)-vertex antimagic total labeling for some a'. This new labeling λ' is called the dual of the labeling λ. The same is true in the case of and (a,d)-edge antimagic total labeling. A number of classification studies on super (a,d)-EAT labeling (resp. (a,d)-EAT) for connected graphs has been extensively investigated. For instance, in [3], Baca et al. showed that a wheel W_n has a super (a,d)-EAT labeling if and only if $d = 1$ and $n \equiv 1 (\mathrm{mod}\ 4)$. A.A.G. Ngurah and E.T. Baskoro in [5] proved that every Petersen graph $P(n,m), n \geq 3, 1 \leq m \leq \frac{n}{2}$, has a super $(4n+2,1)$-EAT labeling.

For $t \geq 2$ and $n \geq 4$, a *Harary graph* C_n^t is a graph constructed from a cycle C_n by joining any two vertices at distance t in C_n.

Super antimagic labeling of Harary graphs have been studied by Baskoro et al. [9] . Some of their results are presented as follows:

Theorem 1. *For $n \geq 5$, $k \geq 2$ and $t \geq 2$, $G \simeq kC_n^t$ admits a super $(8nk+3,1)$-vertex-antimagic total labeling, provided $p \neq 2t$.*

Theorem 2. *For $n \geq 4$, $k \geq 2$ and $t \geq 2$, $G \cong kC_n^t$ admits a super $(2nk+2,1)$-edge-antimagic total labeling.*

Super (a,d) antimagic labeling of union of isomorphic copies of cycles were studied by Dafik et al. [6]. Some of their results are presented below.

Theorem 3. *The graph mC_n has a super $(2mn+2,1)$-edge antimagic total labeling for every $m \geq 2$ and $n \geq 3$.*

Theorem 4. *The graph mC_n has a super $(\frac{3mn+5}{2},2)$-edge antimagic total labeling if and only if m and n are odd, $m,n \geq 3$.*

For more results concerning antimagic total labeling, see for instance [5, 11] and a nice survey paper by Gallian [8].

Throughout the paper we will denote $|V(G)|$ by p and $|E(G)|$ by q. In the following section we present our results concerning super $(a,1)$-edge antimagic total labeling of disconnected non isomorphic copies for Harary graphs and cycles respectively.

2 On super (a,d)-EAT labeling

In this section, we give a uniform construction for super $(a,1)$-edge antimagic total labeling for disjoint union of non isomorphic copies of Harary graphs C_n^t and cycles C_n.

Theorem 5. *For $m \geq 2$, $n_i \geq 5$, $t_i \geq 2$, $i = 1,2,\ldots,m$, $G \cong C_{n_1}^{t_1} \cup C_{n_2}^{t_2} \cup \ldots \cup C_{n_m}^{t_m}$ admits super $(2\sum_{k=1}^{m} n_k + 2, 1)$-EAT labeling provided $n_i \neq 2t_i$.*

Proof.
We denote the vertex and edge sets of G as follows.

$$V = \{v_i^j : 1 \leq i \leq n_j, 1 \leq j \leq m\},$$

$$E = \{v_i^j v_{i+1}^j : 1 \leq i \leq n_j, 1 \leq j \leq m\} \cup \{v_i^j v_{i+t}^j \; : \; 1 \leq i \leq n_j, 1 \leq j \leq m\}.$$

Where the index $n_j + 1$ is taken modulo n_j.
Now, we define the labeling $\lambda : V \cup E \to \{1,2,...,p+q\}$ as follows:

$$\lambda(v_i^j) = \sum_{k=1}^{j} n_{k-1} + i, \; 1 \leq i \leq n_j, \; 1 \leq j \leq m$$

For $i = n_j$, $1 \le j \le m$.

$$\lambda(v_i^j v_{i+1}^j) = 3p - \sum_{k=1}^{j} n_{k-1},$$

and

$$\lambda(v_i^j v_{i+1}^j) = 3p - \sum_{k=1}^{j} n_{k-1} - i,\ 1 \le i \le n_j - 1$$

$$\lambda(v_i^j v_{i+t_j}^j) = 2p - \sum_{k=1}^{j} n_{k-1} - i + 1,\ 1 \le i \le n_j.$$

We have two types of edges namely, $E_1 = \{v_i^j v_{i+t}^j : 1 \le i \le n_j,\ 1 \le j \le m\}$ and $E_2 = \{v_i^j v_{i+1}^j : 1 \le i \le n_j,\ 1 \le j \le m\}$. Now we consider all edge-weights. The edge-weights of all edges in E_1 will form consecutive integers sequence $2p+2, 2p+3, ..., 3p+1$. Note that the weight $2p+2$ is attained by the edge $v_{n_1+1-t}^1 v_1^1$. Whereas the edge-weights of all edges in E_2 will form consecutive integers sequence $3p+2, 3p+3, ..., 4p+1$. Therefore, the edge-weights form the consecutive integers sequence $2p+2, 2p+3, ..., 4p+1$. Since all vertices receive the smallest labels hence λ is a super $(2p+2, 1)$-edge antimagic total labeling. □

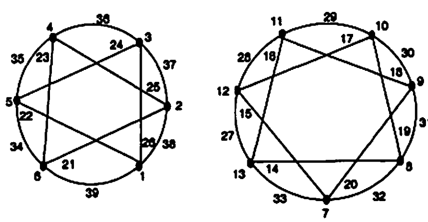

Fig. 1. super (28,1)-EAT labeling of $C_6^2 \cup C_7^2$.

Dafik et al. [6] proved that $G \cong mC_n$ has a super $(2mn+2, 1)$-EAT labeling. In the following theorem we strengthen their result by providing a different labeling scheme with same (a, d) for $G \cong C_{n_1} \cup C_{n_2} \cup \ldots \cup C_{n_m}$.

Theorem 6. *For* $m \ge 2$, $n_i \ge 3$, $i = 1, 2, \ldots, m$, $G \cong C_{n_1} \cup C_{n_2} \cup \ldots \cup C_{n_m}$ *admits super* $(2\sum_{k=1}^{m} n_k + 2, 1)$*-EAT total labeling.*

Proof.
We denote the vertex and edge sets of G as follows.

$$V = \{v_i^j : 1 \le i \le n_j, 1 \le j \le m\},$$

$$E = \{v_i^j v_{i+1}^j : 1 \le i \le n_j, 1 \le j \le m\}.$$

where index $n_j + 1$ is taken modulo n_j.
Now, we define labeling $\lambda : V \cup E \rightarrow \{1, 2, ..., p + q\}$ as follows:

$$\lambda(v_i^j) = \sum_{k=1}^{j} n_{k-1} + i, \;\; 1 \le i \le n_j, \;\; 1 \le j \le m$$

For $i = n_j$, $1 \le j \le m$

$$\lambda(v_i^j v_{i+1}^j) = 2p - \sum_{k=1}^{j} n_{k-1},$$

and

$$\lambda(v_i^j v_{i+1}^j) = 2p - \sum_{k=1}^{j} n_{k-1} - i, \;\; 1 \le i \le n_j - 1.$$

The set of all edge-weights generated by the above scheme forms a consecutive integer sequence $2p+2$, $2p+3$,...,$3p+1$. Since all the vertices receive smallest labels, therefore λ is a super $(2p+2, 1)$-edge antimagic total labeling. Note that the weight $2p+2$ is attained by the edge $v_1^1 v_2^1$. □

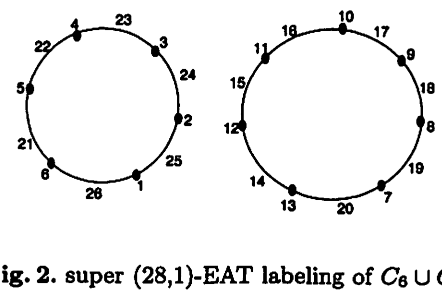

Fig. 2. super (28,1)-EAT labeling of $C_6 \cup C_7$.

A sun graph is constructed from a cycle by attaching a vertex to each vertex of the cycle.
In the following sections we focus on (a, d)-vertex antimagic total labeling for disconnected non isomorphic Harary graph with $d = 1$. We construct super vertex antimagic labeling for non isomorphic copies of cycles for $d = 1$. We provide an $(a, 1)$-vertex antimagic total labeling for sun graphs and an $(a, 4)$-vertex antimagic total labeling for the disjoint union of sun graphs.

3 On (a, d)-VAT labeling

In this section we construct a super (a, d)-VAT labeling for disjoint union of finite non isomorphic copies of Harary graphs and cycles. We also construct an $(a, 1)$-VAT labeling for sun graphs and an $(a, 4)$-VAT labeling for disjoint union of non isomorphic sun graphs. Before proving our main results let us prove the following fact:

Lemma 1. *Let $t \geq 2$ and $p \geq 5$. If Harary graph $G \cong C_p^t$ is super (a, d)-vertex antimagic total then $d < 9$ for $p \neq 2t$ and $d < 6$ for $p = 2t$.*

Proof. Assume that there exists a bijection

$$\lambda : V(G) \cup E(G) \rightarrow \{1, 2, ..., p + q\}$$

which is super (a, d)-vertex antimagic total and
$W = \{\lambda(u) + \sum \lambda(uv) : uv \in E(G)\}$
$= \{a, a + d, ..., a + (p - 1)d\}$ is the set of vertex-weights.
If $p \neq 2t$, then the minimum possible vertex-weight in a super (a, d)-vertex antimagic total labeling is

$$1 + (p + 1) + (p + 2) + (p + 3) + (p + 4) = 4p + 11.$$

and maximum vertex-weight is no more than

$$p + 3p + (3p - 1) + (3p - 2) + (3p - 3) = 13p - 6.$$

Thus, we have

$$a + (p - 1)d \leq 13p - 6,$$

and

$$d \leq \frac{9p - 17}{p - 1} < 9.$$

If $p = 2t$, then minimum possible vertex-weight in a super (a, d)-vertex antimagic total labeling is

$$1 + (p + 1) + (p + 2) + (p + 3) = 3p + 7.$$

and maximum vertex-weight is no more than

$$p + \frac{5p}{2} + (\frac{5p}{2} - 1) + (\frac{5p}{2} - 2) = \frac{17p}{2} - 3.$$

Thus, we have

$$a + (p - 1)d \leq \frac{17p}{2} - 3,$$

and

$$d \leq \frac{11p-20}{2(p-1)} < 6. \qquad \square$$

Theorem 7. *For $m \geq 2$, $n_i \geq 5$, $t_i \geq 2$, $i = 1, 2, \ldots, m$, $G \cong C_{n_1}^{t_1} \cup C_{n_2}^{t_2} \cup \ldots \cup C_{n_m}^{t_m}$ admits super $(8\sum_{k=1}^{m} n_k + 3, 1)$-VAT labeling provided $n_i \neq 2t_i$.*

Proof.
We denote the vertex and edge sets of G as follows.

$$V = \{v_i^j : 1 \leq i \leq n_j, 1 \leq j \leq m\},$$

$$E = \{v_i^j v_{i+1}^j : 1 \leq i \leq n_j, 1 \leq j \leq m\} \cup \{v_i^j v_{i+t_j}^j \ : \ 1 \leq i \leq n_j, 1 \leq j \leq m\}.$$

where all indices are taken mod n_j.
Now, we define labeling $\lambda : V \cup E \to \{1, 2, ..., p+q\}$ as follows:

$$\lambda(v_i^j) = \sum_{k=1}^{j} n_{k-1} + i, \ \ 1 \leq i \leq n_j, \ 1 \leq j \leq m$$

$$\lambda(v_i^j v_{i+1}^j) = \begin{cases} 2p - \sum_{k=1}^{j} n_{k-1}, & i = n_j, \ 1 \leq j \leq m, \\ 2p - \sum_{k=1}^{j} n_{k-1} - i, & 1 \leq i \leq n_j - 1, \ 1 \leq j \leq m. \end{cases}$$

$$\lambda(v_i^j v_{i+t_j}^j) = \begin{cases} 2p + 1 + \sum_{k=1}^{j} n_{k-1}, & i = n_j, \ 1 \leq j \leq m, \\ 2p + 1 + \sum_{k=1}^{j} n_{k-1} + i, & 1 \leq i \leq n_j - 1, \ 1 \leq j \leq m. \end{cases}$$

We have vertices, $V = \{v_i^j : 1 \leq i \leq n_j, 1 \leq j \leq m\}$ and we can see that the vertex $v_{t_1}^1$ has the weight $8\sum_{k=1}^{m} n_k + 3$ and set of vertex-weights form a consecutive sequence $8\sum_{k=1}^{m} n_k + 3, 8\sum_{k=1}^{m} n_k + 4, ...,$
$9\sum_{k=1}^{m} n_k + 2$ with $a = 8\sum_{k=1}^{m} n_k + 3$ and $d = 1$. Since all vertices receive the smallest labels hence λ is a super $(8\sum_{k=1}^{m} n_k + 3, 1)$-vertex antimagic total labeling. $\square$

Corollary 1. *For $m \geq 2$, $n_i \geq 3$, $i = 1, 2, \ldots, m$, $G \cong C_{n_1} \cup C_{n_2} \cup \ldots \cup C_{n_m}$ admits super $(3\sum_{k=1}^{m} n_k + 2, 1)$-VAT labeling.*

Proof. Using the dual of the labeling defined in Theorem 6, the result follows. $\square$

Theorem 8. *For* $m \geq 2, n_i \geq 3, i = 1, 2, \ldots, m$, $G \cong C_{n_1} \cup C_{n_2} \cup \ldots \cup C_{n_m}$ *admits a* $(2\sum_{k=1}^{m} n_k + 3, 2)$*-VAT labeling.*

Proof.
We denote the vertex and edge sets of G as follows.

$$V = \{v_i^j : 1 \leq i \leq n_j, 1 \leq j \leq m\},$$

$$E = \{v_i^j v_{i+1}^j : 1 \leq i \leq n_j, 1 \leq j \leq m\}.$$

where index $n_j + 1$ is taken modulo n_j.
Now, we define labeling $\lambda : V \cup E \to \{1, 2, ..., p+q\}$ as follows:

$$\lambda(v_i^j) = 2i - 1 + 2\sum_{k=1}^{j} n_{k-1},\ 1 \leq i \leq n_j,\ 1 \leq j \leq m.$$

$$\lambda(v_i^j v_{i+1}^j) = p + q - (2i-2) - 2\sum_{k=1}^{j} n_{k-1},\ 1 \leq i \leq n_j,\ 1 \leq j \leq m.$$

We have vertices, $V = \{v_i^j : 1 \leq i \leq n_j, 1 \leq j \leq m\}$ and we can see that the vertex v_1^m has the weight $2\sum_{k=1}^{m} n_k + 3$ and set of vertices form a consecutive sequence $2\sum_{k=1}^{m} n_k + 3, 2\sum_{k=1}^{m} n_k + 5, ..., 4\sum_{k=1}^{m} n_k + 1$ with $a = 2\sum_{k=1}^{m} n_k + 3$ and $d = 2$. Hence λ is a $(2\sum_{k=1}^{m} n_k + 3, 2)$-vertex antimagic total labeling. □

Corollary 2. *For* $m \geq 2, n_i \geq 3, i = 1, 2, \cdots, m$, $G \cong C_{n_1} \cup C_{n_2} \cup \ldots \cup C_{n_m}$ *admits* $(\sum_{k=1}^{m} n_k + 2, 2)$*-VAT labeling.*

Proof. Using the dual of the labeling defined in Theorem 8, the result follows. □

Theorem 9. *The sun graph* S_n *on* $2n$ *vertices admits a* $(5n+2, 1)$*-VAT labeling.*

proof. Let S_n be the sun graph on $2n$ vertices. Then $V(S_n) = \{v_1, v_2, \cdots, v_n\} \cup \{a_1, a_2, \cdots, a_n\}$ and $E(S_n) = \{v_i v_{i+1} | 1 \leq i \leq n\} \cup \{v_i a_i | 1 \leq i \leq n\}$. We define a labeling $\lambda : V \cup E \to \{1, 2, \cdots, p+q\}$ as follows;

$$\begin{aligned} \lambda(v_i) &= i \ \ for\ 1 \leq i \leq n \\ \lambda(v_i v_{i+1}) &= p - i + 1,\ \ for\ 1 \leq i \leq n \end{aligned}$$

$$\lambda(v_i a_i) = \begin{cases} p+1, & i = 1; \\ p + \frac{q}{2} - i + 2, & 2 \leq i \leq n. \end{cases}$$

$$\lambda(a_i) = \begin{cases} p+\frac{q}{2}+1, & i=1; \\ p+q-i+2, & 2 \le i \le n. \end{cases}$$

Now by direct computation we see that the vertex a_1 has the weight $5n+2$ and the set of weights of vertices $\{5n+2, 5n+3, \cdots, 7n+1\}$ forms an arithmetic sequence with $a = 5n+2$ and common difference $d = 1$. Hence the labeling λ is a $(5n+2, 1)$-VAT labeling. □

Theorem 10. *If $t_j \ge 3$ for every $j = 1, 2, \cdots, n$ and $n \ge 1$ the disjoint unions of sun graphs $G \cong S_{t_1} \cup S_{t_2} \cup \cdots \cup S_{t_n}$ admits a $(2\sum_{k=1}^{j} t_k + 3, 4)$-VAT labeling.*

Proof. Let $G \cong S_{t_1} \cup S_{t_2} \cup \cdots \cup S_{t_n}$. Then for $t_j \ge 3$, $j = 1, 2, \cdots, n$ and $n \ge 1$, $V(G) = \{v_i^{t_j} | 1 \le i \le t_j\} \cup \{a_i^{t_j} | 1 \le i \le t_j\}$ and $E(G) = \{v_i^{t_j} v_{i+1}^{t_j} | 1 \le i \le t_j\} \cup \{v_i^{t_j} a_i^{t_j} | 1 \le i \le t_j\}$. Define a labeling $\lambda : V \cup E \to \{1, 2, \cdots, p+q\}$ as follows;

$$\lambda(v_i^{t_j}) = 2\sum_{k=1}^{j-1} t_k + 4\sum_{k=j}^{n} t_k - 2i + 2 \ \ ; \ i = 1, 2, \cdots t_j \ \textit{and} \ j = 1, 2, \cdots, n$$

$$\lambda(a_i^{t_j}) = \begin{cases} 2\sum_{k=1}^{j} t_k & \text{for } i = 1 \\ 2\sum_{k=1}^{j-1} t_k + 2i - 2 & \text{for } i = 2, 3, \cdots, t_j \end{cases}$$

$$\lambda(v_i^{t_j} v_{i+1}^{t_j}) = \begin{cases} 2\sum_{k=1}^{n} t_k - 2\sum_{k=1}^{j} t_k + 1 & \text{for } i = t_j \\ 2\sum_{k=1}^{n} t_k - 2\sum_{k=1}^{j-1} t_k - 2i + 1 & \text{for } i = 1, 2, \cdots, t_j - 1 \end{cases}$$

$$\lambda(v_i^{t_j} a_i^{t_j}) = \begin{cases} 2\sum_{k=1}^{n} t_k + 2\sum_{k=1}^{j} t_k - 1 & \text{for } i = 1 \\ 2\sum_{k=1}^{n} t_k + 2\sum_{k=1}^{j-1} t_k + 2i - 3 & \text{for } i = 2, 3, \cdots, t_j \end{cases}$$

Now using direct computation, we see that the vertex $a_{t_n}^n$ has the weight $2\sum_{k=1}^{n} t_k + 3$ and the set of weights of vertices $\{2\sum_{k=1}^{n} t_k + 3, 2\sum_{k=1}^{n} t_k + 7, \cdots, 10\sum_{k=1}^{n} t_k - 1\}$ form an arithematic sequence with $a = 2\sum_{k=1}^{n} t_k + 3$ and a common difference $d = 4$. Hence the labeling λ is a $(2\sum_{k=1}^{n} t_k + 3, 4)$-VAT labeling. □

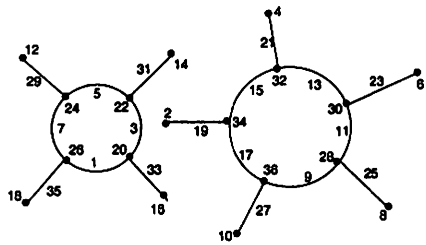

Fig. 3. (21,4)-VAT labeling of $S_4 \cup S_5$.

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