Cographic and global cographic domination number of a graph

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Abstract

In this paper, we prove that for any graph G, there is a dominating induced subgraph which is a cograph. Two new domination parameters γ_{cd} - the cographic domination number and γ_{gcd} - the global cographic domination number are defined. Some properties including complexity aspects are discussed.

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1 Introduction

We consider only finite, simple graphs $G = (V, E)$ with $|V| = n$ and $|E| = m$.

Cographs - complement reducible graphs-are graphs that can be reduced to edgeless graphs by taking complements within components. Various properties of this subclass of perfect graphs [6] are discussed in [3], [4], [5] and [10]. Every cograph is a comparability graph [11] and the distance hereditary graphs form a super class of cographs.

Cographs are recursively defined as follows:

(1) K_1 is a cograph

(2) If G is a cograph, so is its complement G^c and

(3) If G and H are cographs, so is their disjoint union, $G \cup H$.

The join of two graphs G and H, denoted by $G \vee H$ is defined as the graph with $V(G \vee H) = V(G) \cup V(H)$ and $E(G \vee H) = E(G) \cup E(H) \cup \{uv$, where $u \in V(G)$ and $v \in V(H)\}$. Then $G \vee H = (G^c \cup H^c)^c$. Hence the condition (3) in the definition of cographs can as well be replaced by

(3$'$): If G and H are cographs, so is their join.

The following characterizations for a graph G to be a cograph are well known [10].

G does not contain an induced P_4 - the path on 4 vertices.

For every induced subgraph H of G, H or H^c is disconnected.

G.F. Royle [13] has proved that the rank of a cograph is equal to the number of distinct non zero rows of its adjacency matrix. F.Larrión et.al, [9] have studied in detail the clique operator on cographs and proved that a cograph is clique convergent if and only if it is clique Helly. A characterization of cographs whose clique graph is a cograph is also obtained.

In [8], a polynomial time algorithm is given for the Hamiltonian cycle problem for cographs.

In [12] Rao and Vijayakumar have studied two graph operators - the median and the antimedian - of a cograph. Planar and outer planar cographs are also discussed in [12].

The following definitions are from [7]. A set $S \subseteq V$ of vertices in a graph G is called a dominating set if every vertex $v \in V$ is either an element of S or is adjacent to an element of S. A dominating set S is minimal dominating if no proper subset of S is a dominating set. The set of all minimal dominating sets of a graph G is denoted by $MDS(G)$. The domination number $\gamma(G)$ of a graph G is the minimal cardinality of a set in $MDS(G)$, or equivalently, the minimum cardinality of a dominating set in G. The minimum cardinality of an independent dominating set of G is the independent domination number, $\gamma_i(G)$.

In this paper,we prove that for any graph G, there is a dominating induced subgraph which is a cograph. A new domination parameter γ_{cd} - the cographic domination number is defined. In general $\gamma(G) \leq \gamma_{cd}(G) \leq \gamma_i(G)$ and there are graphs which satisfy the strict inequality. We prove that there are no trees which satisfy $\gamma(G) < \gamma_{cd}(G) = \gamma_i(G)$. Another domination parameter γ_{gcd} - the global cographic domination number is introduced and its relationship with cographic domination number for different classes of graphs are studied. Some constructions to illustrate the existence of graphs satisfying the inequalities among the various domination parameters and the complexity of evaluating these parameters are also discussed.

For all graph theoretic notations and preliminaries, we follow [2].

2 The cographic domination number

In [1], the following problem is considered.

Problem : Let **P** be a property of vertex sets in a graph. Characterize all graphs having a dominating set satisfying the property **P**.

Motivated by this, we first prove that,

Theorem 1: For any graph G, there exists a dominating induced subgraph which is a cograph.

Proof: The proof is by induction on n, the number of vertices of G. For $n \leq 3$, the theorem can be easily verified. Assume that it is true for all graphs with at most n vertices.

Let G be a graph with $n+1$ vertices. By induction hypothesis, the graph $G-v$ has a dominating induced subgraph H which is a cograph. If at least one of the vertices in H is adjacent to v, then H is a dominating induced subgraph for G. If not, $H \cup \{v\}$ is a dominating induced subgraph of G which is also a cograph. Therefore by induction, the theorem is true for all graphs.

Definition 2: For a graph G, the cardinality of a minimum dominating set whose vertices induces a cograph is called the cographic domination number, denoted by $\gamma_{cd}(G)$.

Note 3: For any graph G, $\gamma(G) \leq \gamma_{cd}(G) \leq \gamma_i(G)$. However, there are graphs with $\gamma(G) < \gamma_{cd}(G) < \gamma_i(G)$. For e.g:-

G :

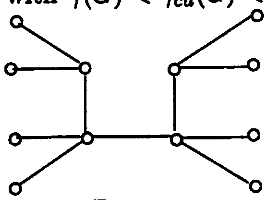


Fig : 1

$\gamma(G) = 4$
$\gamma_{cd}(G) = 5$
$\gamma_i(G) = 6$

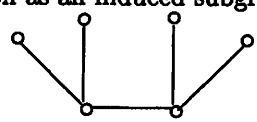
In the following, we finally prove that there are no trees which satisfy $\gamma(G) < \gamma_{cd}(G) = \gamma_i(G)$.

Lemma 4: If T is a tree with $\gamma(T) < \gamma_{cd}(T)$, then T must have the graph G in Fig:1 as an induced subgraph.

Proof: Since $\gamma(T) < \gamma_{cd}(T)$, in every dominating set D with cardinality $\gamma(T)$ there exists an induced $P_4 : u_1u_2u_3u_4$. Since D is minimal dominating and u_i for i = 1,2,3,4 is adjacent to at least one vertex in the dominating set, there exists at least one v_i in the vertex set of T corresponding to each u_i such that v_i is adjacent only to u_i in D for each i = 1,2,3,4. If for one of these 'i', v_i is the only such neighbor of u_i then we can replace u_i by v_i for that i in the dominating set to remove the induced P_4 without changing the cardinality. Therefore, there exists at least one induced P_4 in T such that each of its vertices is adjacent to a pair of vertices. These twelve vertices together induce the required graph.

Corollary 5: For any graph G with less than twelve vertices, $\gamma(G) = \gamma_{cd}(G)$.

Lemma 6: If T is a tree with $\gamma_{cd}(T) < \gamma_i(T)$, then T has the following graph as an induced subgraph.



Proof: Since $\gamma_{cd}(T) < \gamma_i(T)$, every cographic dominating set D with cardinality $\gamma_{cd}(T)$ will have at least one pair of adjacent vertices, say uv. Since u and v are mutually dominating, there exist at least two vertices u_1 and v_1 in T which are adjacent only to u and v, respectively. If these are the only such vertices then we can replace u by u_1 or v by v_1 in T to remove the adjacency in D without affecting the cardinality. Therefore, there exist at least one pair of vertices in D which has at least two neighbors of their own. These six vertices induce the required graph.

Corollary 7: For any graph G with less than six vertices, $\gamma_{cd}(G) = \gamma_i(G)$.

Theorem 8: There is no tree T which satisfies $\gamma(T) < \gamma_{cd}(T) = \gamma_i(T)$.

Proof: If possible assume that there is a tree T which satisfies $\gamma(T) < \gamma_{cd}(T) = \gamma_i(T)$. Let D be a minimal dominating set of cardinality $\gamma(T)$. Since $\gamma(T) < \gamma_{cd}(T)$, by lemma 4, T must contain the graph in Fig:1 as an induced subgraph and the vertices which induce a P_4 in it must be present in D. Also, none of the vertices of this P_4 can be replaced without affecting the domination property and without increasing the cardinality of D. To make D a cographic dominating set, only one vertex is to be replaced, whereas to make D an independent dominating set, two of the vertices are to be replaced. Since D is arbitrary, $\gamma_{cd}(T) < \gamma_i(T)$ which is a contradiction. Hence, the theorem.

3 Global cographic domination number

Theorem 9: Given any graph $G = (V, E)$, there exists a cographic dominating set which dominates G^c also.

Proof: If D is a cographic dominating set in G which dominates G^c also, then there is nothing to prove. Otherwise, there exists at least one vertex, say v_1 which is not adjacent to any vertex of D in G^c. Adjoin v_1 to D.

If $D \cup \{v_1\}$ does not dominate G^c, then there exist a v_2 which is not adjacent to any vertex of $D \cup \{v_1\}$ in G^c. Adjoin v_2 to $D \cup \{v_1\}$. Continue this process until we get a dominating set $D' = D \cup \{v_1, v_2, ..., v_k\}$ which dominates G^c. The process will eventually terminate, since V dominates G^c. The subgraph induced by D' in G is the join of the subgraph induced by D in G with K_p, for some p. Therefore, the subgraph induced by D' is also a cograph by the choice of D and since $D \subseteq D'$, D' dominates G. Therefore, D' is a cographic dominating set in G which dominates G^c also.

Definition 10: Let $G = (V, E)$ be a graph. A subset V' of V is called a global cographic dominating set if it dominates both G and G^c and the subgraph induced by V' is a cograph. The global cographic domination number, denoted by $\gamma_{gcd}(G)$ is the minimum cardinality of a global cographic dominating set.

Eg: γ_{gcd} of G in Fig-1 is 5.

Note 11: For any graph G, $\gamma_{gcd}(G) \geq max\{\gamma_{cd}(G), \gamma_{cd}(G^c)\}$.

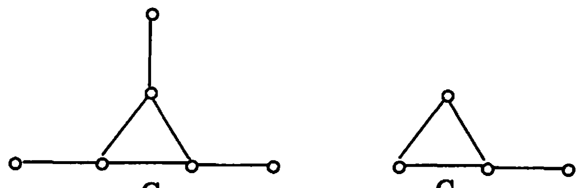
Lemma 12: For any graph G, $\gamma_{gcd}(G) > 1$.

Proof: If $\gamma_{gcd}(G) = 1$, then $\gamma_{cd}(G) = 1$. Then G has a vertex of full degree and so G^c has an isolated vertex. Therefore, $\gamma_{cd}(G^c) > 1$ and so $\gamma_{gcd}(G) < \gamma_{cd}(G^c)$. This is a contradiction and hence $\gamma_{gcd}(G) > 1$.

Theorem 13: If G is a triangle free graph, then $\gamma_{gcd}(G) = \gamma_{cd}(G)$ or $\gamma_{cd}(G) + 1$.

Proof: Let $\gamma_{gcd}(G) \neq \gamma_{cd}(G)$. Let D be a minimum cographic dominating set. Since none of the minimum cographic dominating sets dominate G^c, at least one vertex v of G must be adjacent to all the vertices of D. Consider $D \cup \{v\}$. Since the graph is triangle free, none of the neighbors of the vertices of D are adjacent to v. Since D is dominating, every vertex of G is either in D or is adjacent to a vertex of D. Therefore, the only neighbors of v are those present in D. Hence, in G^c, v dominates all the vertices outside D. Also, $D \cup \{v\}$ induces a cograph. Thus, $D \cup \{v\}$ is a cographic dominating set in G as well as G^c, of cardinality $\gamma_{cd}(G) + 1$.

Note 14: The converse need not be true. For example, in the graphs given below, $\gamma_{gcd}(G_1) = \gamma_{cd}(G_1) = 3$ and $\gamma_{gcd}(G_2) = 2$ and $\gamma_{cd}(G_2) = 1$.



G_1 G_2

Corollary 15: If G is a triangle free graph with $\gamma_{gcd}(G) \neq \gamma_{cd}(G)$, then $\gamma_{cd}(G) = \gamma_i(G)$.

Proof: Let D be a minimum cographic dominating set of G. Since, none of the minimum cographic dominating sets dominate G^c, at least one vertex v of G must be adjacent to all the vertices of D. Since, G is triangle free, no two vertices of D are adjacent. Therefore, D is an independent dominating set. Hence, $\gamma_{cd}(G) = \gamma_i(G)$.

Corollary 16: Every tree T has $\gamma_{gcd}(T) = \gamma_{cd}(T)$ or $\gamma_{cd}(T)+1$. Moreover, $\gamma_{gcd}(T) = \gamma_{cd}(T) + 1$ only if T is a rooted tree of depth two in which every vertex (may be except the root) has at least two children.

Proof: The first statement follows from the above theorem, since trees are triangle free. Assume that $\gamma_{gcd}(T) = \gamma_{cd}(T) + 1$ for a tree T. Then as in the proof of corollary 15, there exists a minimum cographic dominating set D, which is independent and has a common neighbor v. Since D is dominating and T is a tree, v is not adjacent to any other vertex of G. Now, every vertex of D has at least two pendant vertices attached to it. Since, otherwise if $u \in D$ has only one pendant vertex w attached to it, then $(D - \{u\}) \cup \{w\}$ is a global dominating set of cardinality $\gamma_{cd}(T)$, which is a contradiction. Therefore, all the vertices in D have at least two pendant vertices attached to it and so T is a rooted tree of depth two with v as its root in which every vertex has at least two children.

Lemma 17: If G is a disconnected graph, then $\gamma_{cd}(G^c) \leq 2$ and $\gamma_{gcd}(G) = \gamma_{cd}(G)$.

Proof: Since G is disconnected, G^c is connected and any two vertices in the two different components of G dominates G^c. So, $\gamma_{cd}(G^c) \leq 2$. Also, in any cographic dominating set of G, there will be at least one vertex from each component. Therefore any cographic dominating set of G is a cographic dominating set of G^c also. Hence $\gamma_{gcd}(G) = \gamma_{cd}(G)$.

Note 18: This lemma holds for the domination number and the global domination number [14] of a disconnected graph also.

Theorem 19: A cograph G without a vertex of full degree has $\gamma_{gcd}(G) = \gamma_{cd}(G)$ if and only if there exists two vertices u and v such that $N(u)$ and $N(v)$ partitions $V(G)$ or $V(G) - \{u, v\}$, where $N(u)$ and $N(v)$ denotes the set of all vertices which are adjacent to u and v respectively.

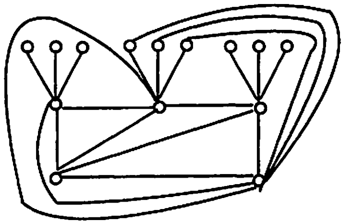
Proof: If $N(u)$ and $N(v)$ partitions $V(G)$ or $V(G) - \{u, v\}$, the cographic domination number of G is 2. In G^c, $\{u, v\}$ itself dominates. Therefore, $\gamma_{gcd}(G) = \gamma_{cd}(G) = 2$.

Conversely, assume that $\gamma_{gcd}(G) = \gamma_{cd}(G)$. Since $\gamma_{gcd}(G) > 1$ and $\gamma_{cd}(G) \leq 2$, we have $\gamma_{gcd}(G) = \gamma_{cd}(G) = 2$. Therefore, there exist two vertices u and v such that $\{u, v\}$ dominates both G and G^c. Since, neighbors of u in G will not be adjacent to u in G^c, they must be adjacent to v in G^c. Hence, no vertex in $N(u)$ is adjacent to v in G and vice versa. Also, since $\{u, v\}$ dominates, $N(u) \cup N(v) = V(G)$ or $V(G) - \{u, v\}$. Therefore, $N(u)$ and $N(v)$ partitions $V(G)$ or $V(G) - \{u, v\}$.

Theorem 20: If G is a planar graph with $\gamma_{cd}(G) \geq 3$, then $\gamma_{gcd}(G) \leq \gamma_{cd}(G) + 2$.

Proof: If possible assume that $\gamma_{gcd}(G) > \gamma_{cd}(G) + 2$. Let u_1, u_2, u_3 be three vertices in any γ_{cd} set D of G. Since $\gamma_{gcd}(G) > \gamma_{cd}(G) + 2$, D cannot dominate G^c and at least three more vertices are to be added to D to make it a global dominating set. Therefore, there exist at least three vertices v_1, v_2, v_3 which are adjacent to each other and to every vertex of D. Then the subgraph induced by these six vertices will be $K_6, K_6 - \{e_1\}, K_6 - \{e_1, e_2\}$ or $K_6 - \{e_1, e_2, e_3\}$ where $e_1, e_2, e_3 \in E(G)$ and are adjacent to each other. Each of the above graph contains $K_{3,3}$ as a subgraph, which is a contradiction to the planarity of G. Hence the theorem.

Note 21: The bound $\gamma_{gcd}(G) \leq \gamma_{cd}(G) + 2$ is strict.



For example, the above plane graph has $\gamma_{cd} = 3$ and $\gamma_{gcd} = 5$.

4 Two constructions

Theorem 22: Given three positive integers a, b and c satisfying a$\leq$ b $\leq$ c, there is a graph G such that $\gamma(G)$ = a, $\gamma_{cd}(G)$ = b, $\gamma_i(G)$ = c.

Proof: We shall prove the theorem by constructing the required graph and by analyzing the following cases.

Case 1 : a = b = c
Let $G = P_n$ or C_n where n = 3a. Then, $\gamma(G) = \gamma_{cd}(G) = \gamma_i(G)$ = a.

Case 2 : a = b < c
Let G be the graph P_n where n = 3(a - 1) together with (c - a + 1) pendant vertices each attached to an end vertex of P_n and its neighbor. Then, $\gamma(G) = \gamma_{cd}(G)$ = a and $\gamma_i(G)$ = c.

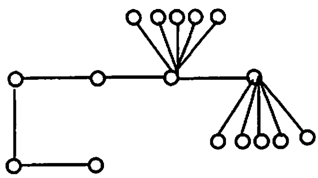
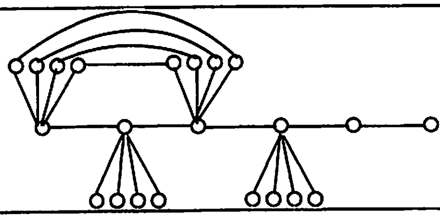
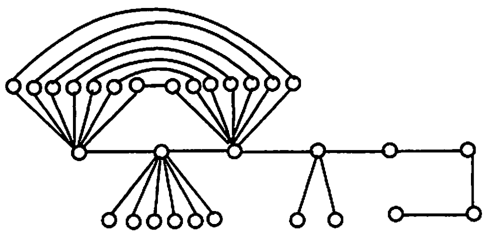
Case 3 : a < b = c
Let G be $P_n : v_1v_2v_3....v_n$, where n = 3a - 7 together with p = b - a + 2 vertices, $u_{i1}, u_{i2}, ..u_{ip}$, made adjacent to each v_i for i = 1,2,3 and 4 and u_{1j} made adjacent to u_{3j} for each j = 1,2,....,p.

Then, the vertices v_1, v_2, v_3 and v_4 dominate all u_{ij} s and v_5. To dominate the remaining (3a - 12) vertices of the path, (a - 4) vertices are required. Therefore, $\gamma(G)$ = a. At least one vertex among v_1, v_2, v_3 and v_4 must be replaced to get a cographic dominating set. Remove v_1 and include all the (b - a + 2) vertices. But, then v_3 is also not required in the dominating set so that $\gamma_{cd}(G)$ = a - 2 + b - a + 2 = b. This set is also independent and hence $\gamma_i(G)$ = b.

Case 4 : a < b < c
Let G be $P_n : v_1v_2v_3....v_n$, where n = 3a - 7 together with (b - a) vertices made adjacent to v_4, (c - a + 1) vertices made adjacent to v_2 and (c - a + 2) vertices each made adjacent to v_1 and v_3 and to each other.

Then, the vertices v_1, v_2, v_3 and v_4 dominate all pendant vertices attached to them and v_5. To dominate the remaining (3a - 12) vertices of the path, (a - 4) vertices are required. Therefore, $\gamma(G)$ = a. At least one vertex among v_1, v_2, v_3 and v_4 must be replaced to get a cographic dominating set. If we remove v_4, the (b - a) pendant vertices adjacent to it and v_5 are to be adjoined to get a cographic dominating set of cardinality a - 1 + b - a + 1 = b. If we remove v_1, the (c - a + 2) pendant vertices adjacent to it are to be adjoined. But, then v_3 also can be removed from the dominating set to get an independent dominating set of cardinality (a - 2 + c - a + 2) = c. Therefore, $\gamma_{cd}(G)$ = b and $\gamma_i(G)$ = c.

Illustration

Case 1	a = b = c = 2	
Case 2	a = b = 3, c = 7	
Case 3	a = 5, b = c = 7	
Case 4	a = 5, b = 7, c = 10	

Theorem 23: Given two positive integers a and b satisfying $a \leq b$ and $b > 1$, there is a graph G such that $\gamma_{cd}(G) = a, \gamma_{gcd}(G) = b$.

Proof: We shall prove the theorem by constructing the required graph and by analyzing the following cases.

Case 1 : $a = b > 1$.

G is the graph obtained by taking any graph of order a and attaching one pendant vertex to each of the vertices.

Case 2 : $a = 1$ and $a < b$.

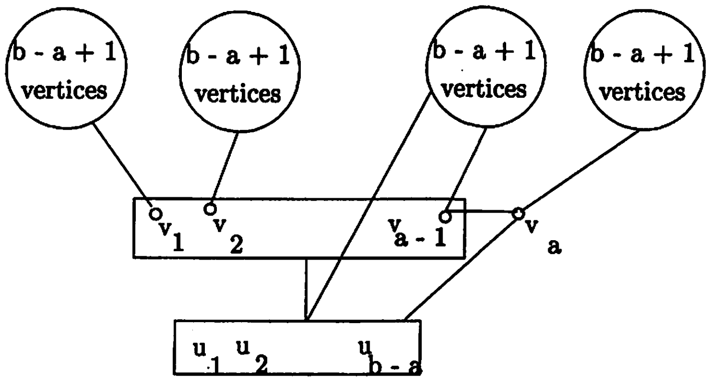
$G = K_b$.

Case 3 : $a = 2$ and $a < b$.

G is K_{2b} minus a perfect matching.

Case 4 : $a > 2$ and $a < b$.

The required graph G is obtained as per the following constructions based on the figure.



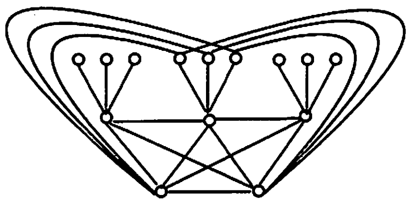
In the figure, the vertices inside each of the circles are independent and the vertices inside both the rectangles form complete graphs. Every vertex v_i for $i = 1, 2, ..., a$ is made adjacent to every vertex inside the circle to which an edge is drawn. All the vertices of the smaller rectangle are made adjacent to, all the vertices in the bigger rectangle, all the vertices inside the circle to which an edge is drawn and to v_a. Further, v_{a-1} is made adjacent to v_a. The graph so obtained is G.

Now, if we choose one vertex from each of the circles, we get an independent set of cardinality a which has no common neighbors. Therefore, any dominating set must contain at least a vertices and $\{v_1, v_2, ..., v_a\}$ is a cographic dominating set. So $\gamma_{cd}(G) = a$.

The set $\{v_1, v_2, ..., v_a\}$ will not dominate u_is in G^c. If we remove any one of the v_is from this cographic dominating set, then all the $b - a + 1$ vertices in the corresponding circle must be included to retain the set as a cographic dominating set. Therefore, the cardinality becomes $a - 1 + b - a + 1 = b$.

If we keep all the v_is, then a vertex from any of the circles, except the one corresponding to v_{a-1} cannot be introduced , since otherwise an induced P_4 will be present. A vertex from the circle corresponding to v_{a-1} cannot dominate u_is in the complement. Also, a u_i cannot dominate u_j for $i \neq j$. Therefore to get a global cographic dominating set all the u_is must be included. Then the cardinality becomes $a+b-a=b$. In any case, $\gamma_{gcd}(G) = b$.

Illustration of case 4 : $a = 3$, $b = 5$.



5 Complexity aspects

In this section we discuss the complexity aspects of the newly defined parameters.

Theorem 24: Determining the cographic domination number of a graph is NP-complete.

Proof: We prove this by reducing in polynomial time the dominating set problem in general to the cographic dominating set problem.

Claim: Given a graph G, we can construct a graph H in polynomial time such that G has a dominating set of size k if and only if H has a cographic dominating set of size $k+1$.

Let $G = (V, E)$ where $V = \{v_1, v_2, ..., v_n\}$ be the given graph. Construct H as follows:
Let $V(H) = \{v_1, v_2, ..., v_n\} \cup \{v'_1, v'_2, ..., v'_n\} \cup \{x, y\}$. Make v_i adjacent to v'_j if $v_iv_j \epsilon E(G)$ or $i = j$; x is adjacent to v'_j for every j and x is adjacent to y in H.

Let $\{v_{i_1}, v_{i_2}, ..., v_{i_k}\}$ be a minimal dominating set of cardinality k in G. Then $\{v'_{i_1}, v'_{i_2}, ..., v'_{i_k}, x\}$ is a minimal dominating set in H. Since there is no induced P_4 in this set, it is a minimal cographic dominating set in H of cardinality $k+1$.

Conversely, let $\{u_1, u_2, ..., u_{k+1}\}$ be a cographic dominating in H. (By construction of H, any minimal dominating set is cographic). One of these u_i's must be x or y. Remove that u_i. All other u_i's must be either v_j or v_k'. Keep each v_j unchanged and replace each v_k' by v_k. This new set of cardinality k will be a minimal dominating set of G. Since H can be computed from G in time polynomial in size of G, our claim holds.

Corollary 25: The problem of determining the cographic domination number is NP-complete for the class of bipartite graphs.

Proof: In the proof above, the graph H constructed from G is bipartite.

Theorem 26: Determining the global cographic domination number of a graph is NP-complete.

Proof:
Claim: Given a graph G, we can construct a graph H in polynomial time such that G has a cographic dominating set of size k if and only if H has a global cographic dominating set of size $k+1$.

Given a graph G define $H = G \cup K_1$. Clearly, a minimum cographic dominating set in G together with the isolated vertex is a minimal global cographic dominating set in H.

Conversely, any minimal global cographic dominating set in H will contain the isolated vertex and the remaining vertices is a minimal cographic dominating set in G. Since H can be computed from G in time polynomial in size of G, our claim holds.

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