On a Stević integral-type operator from generally weighted Bloch spaces to Bloch-type spaces on the unit ball

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Abstract

Let $g \in H(\mathbb{B})$, $g(0) = 0$ and φ be a holomorphic self-map of the unit ball $\mathbb{B}$ in $\mathbb{C}^n$. The following integral-type operator

$$I_\varphi^g(f)(z) = \int_0^1 \mathcal{R}f(\varphi(tz))g(tz)\frac{dt}{t}, \ f \in H(\mathbb{B}), \ z \in \mathbb{B},$$

was recently introduced by S. Stević and studied on some spaces of holomorphic functions on $\mathbb{B}$, where $\mathcal{R}f(z) = \sum_{k=1}^n z_k \frac{\partial f}{\partial z_k}(z)$ is the radial derivative of f. The boundedness and compactness of this operator from generally weighted Bloch spaces to Bloch-type spaces on $\mathbb{B}$ are investigated in this note.

§1 Introduction

Let $\mathbb{B}$ be as usual the unit ball in $\mathbb{C}^n$, $\mathbb{D}$ the open unit disc in $\mathbb{C}$, $H(\mathbb{B})$ the space of all holomorphic functions in $\mathbb{B}$. For any $z = (z_1, z_2, \dots, z_n)$, $w = (w_1, w_2, \dots, w_n) \in \mathbb{C}^n$, the inner product is defined by $\langle z, w\rangle = \sum_{j=1}^n z_j \overline{w_j}$.

For any $f \in H(\mathbb{B})$ with the Taylor expansion $f(z) = \sum_{|\beta|\geq 0} a_\beta z^\beta$, let $\mathcal{R}f(z) = \sum_{|\beta|\geq 0} |\beta| a_\beta z^\beta$ be the radial derivative of f, where $\beta = (\beta_1, \beta_2, \dots, \beta_n)$ is a multi-index, $|\beta| = \beta_1 + \cdots + \beta_n$ and $z^\beta = z_1^{\beta_1} \cdots z_n^{\beta_n}$. It is easy to see that $\mathcal{R}f(z) = \langle \nabla f(z), \overline{z}\rangle$, where ∇f denotes the complex gradient of f.

For any $0 < \alpha < \infty$, the generally weighted Bloch space ([5]) $B_{\log}^\alpha$ consists of all functions $f \in H(\mathbb{B})$ such that

$$\|f\|_{B_{\log}^\alpha} = |f(0)| + \sup_{z\in\mathbb{B}}(1-|z|^2)^\alpha |\mathcal{R}f(z)| \log\frac{e}{1-|z|^2} < \infty.$$

The little generally weighted Bloch space $B_{\log,0}^\alpha$ is a subspace of $B_{\log}^\alpha$ consisting of those $f \in B_{\log}^\alpha$ such that

$$\lim_{|z|\to 1}(1-|z|^2)^\alpha |\mathcal{R}f(z)| \log\frac{e}{1-|z|^2} = 0.$$

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A positive continuous function μ on the interval [0,1) is called normal ([23]) if there are $\delta \in [0,1)$, a and b, $0 < a < b$ such that

$$\frac{\mu(r)}{(1-r)^a} \text{ is decreasing on } [0,1) \text{ and } \lim_{r\to 1} \frac{\mu(r)}{(1-r)^a} = 0;$$

$$\frac{\mu(r)}{(1-r)^b} \text{ is increasing on } [0,1) \text{ and } \lim_{r\to 1} \frac{\mu(r)}{(1-r)^b} = 0.$$

If a function $\mu : \mathbb{B} \to [0,\infty)$ is normal, we also assume that $\mu(z) = \mu(|z|)$, $z \in \mathbb{B}$.

The Bloch-type space B_μ consists of all $f \in H(\mathbb{B})$ such that

$$B_\mu(f) = \sup_{z\in\mathbf{B}} \mu(z)|\mathcal{R}f(z)| < \infty$$

where μ is normal. The Bloch-type space becomes a Banach space with the norm $\|f\|_{B_\mu} = |f(0)| + B_\mu(f)$ ([25, 40]). The little Bloch-type space $B_{\mu,0}$ is a subspace of B_μ consisting of those $f \in B_\mu$ such that $\lim_{|z|\to 1} \mu(z)|\mathcal{R}f(z)| = 0$. Bearing in mind the following asymptotic relation from [40]

$$b_\mu(f) = \sup_{z\in\mathbf{B}} \mu(z)|\nabla f(z)| \asymp \sup_{z\in\mathbf{B}} \mu(z)|\mathcal{R}f(z)|,$$

the little Bloch-type space is equivalent with the subspace of B_μ consisting of those $f \in B_\mu$ such that $\lim_{|z|\to 1} \mu(z)|\nabla f(z)| = 0$.

Let φ be a holomorphic self-map of $\mathbb{B}$. For $f \in H(\mathbb{B})$ the composition operator is defined by

$$C_\varphi f(z) = f(\varphi(z)),\ z \in \mathbb{B}.$$

It is of interest to provide function theoretic characterizations of when φ induces bounded or compact composition operators on spaces of holomorphic functions ([4]). For some recent results, mostly in $\mathbb{C}^n$ or related to Bloch-type spaces, see [3, 6, 7, 19, 20, 21, 22, 25, 33, 42, 44] and the references therein.

Let $g \in H(\mathbb{B})$, $g(0) = 0$, φ be a holomorphic self-map of $\mathbb{B}$ and

$$I_\varphi^g(f)(z) = \int_0^1 \mathcal{R}f(\varphi(tz))g(tz)\frac{dt}{t},\ f \in H(\mathbb{B}),\ z \in \mathbb{B}.$$

The operator I_φ^g was introduced in [35] and studied also in the following papers: [26, 32, 35, 38]. Operator I_φ^g is a generalization of the operator L_g introduced by S. Li and S. Stević and studied in [1, 2, 8, 9, 10, 11, 12, 16, 17]. For one-dimensional case, the case of polydisk and related results see, for example, [13, 14, 15, 18, 24, 27, 28, 29, 30, 31, 34, 36, 37, 39, 41, 43]. In this paper, we study the operator I_φ^g by investigating the boundedness and compactness of the operator from generally weighted Bloch spaces to Bloch-type spaces.

Throughout the remainder of this paper C will denote a positive constant independent of functions, the exact value of which may vary from one appearance to the next.

§2 Main results and its proofs

First we formulate and prove several auxiliary results which are used in the proofs of our main results.

Lemma 2.1 ([26]) *Suppose $f, g \in H(\mathbb{B})$ and $g(0)=0$. Then*

$$\mathcal{R}I_\varphi^g(f)(z)=g(z)\mathcal{R}f(\varphi(z)).$$

Proposition 2.2 *Assume that μ is normal on $\mathbb{B}$, $g\in H(\mathbb{B})$, $g(0)=0$, φ is a holomorphic self-map of $\mathbb{B}$ and $\alpha>0$. Then $I_\varphi^g: B_{\log}^\alpha\to B_\mu$ is compact if and only if for any bounded sequence $(f_j)_{j\in N}$ in $B_{\log}^\alpha$, when $f_j\to 0$ uniformly on compact subsets of $\mathbb{B}$, then $\|I_\varphi^g f_j\|_{B_\mu}\to 0$ as $j\to\infty$.*

The result follows by standard arguments similar to those in Lemma 3 in [11] and [24]. Hence we omit the details.

Now we begin to study the boundedness and compactness of the operator $I_\varphi^g: B_{\log}^\alpha\to B_\mu$ following the ideas from Stević's papers [26, 32, 35, 38].

Theorem 2.3 *Assume that μ is normal on $\mathbb{B}$, $g\in H(\mathbb{B}), g(0)=0$, φ is a holomorphic self-map of $\mathbb{B}$ and $\alpha\geq 1$. Then the following statements are equivalent.*

(i) $I_\varphi^g: B_{\log}^\alpha\to B_\mu$ is bounded;

(ii) $I_\varphi^g: B_{\log,0}^\alpha\to B_\mu$ is bounded;

(iii)

$$\sup_{z\in\mathbf{B}}\frac{\mu(z)|g(z)||\varphi(z)|}{(1-|\varphi(z)|^2)^\alpha\log\frac{e}{1-|\varphi(z)|^2}}<\infty. \tag{1}$$

Proof (i)⇒ (ii) This is obvious.

(ii)⇒ (iii) Let $f_l(z)=z_l\in B_{\log,0}^\alpha, l\in\{1,2,\ldots,n\}$. By the boundedness of $I_\varphi^g: B_{\log,0}^\alpha\to B_\mu$, we have that for each $l\in\{1,2,\ldots,n\}$

$$\|I_\varphi^g f_l\|_{B_\mu}=\sup_{z\in\mathbf{B}}\mu(z)|g(z)||\varphi_l(z)|\leq\|I_\varphi^g\|_{B_{\log,0}^\alpha\to B_\mu}\|f_l\|_{B_{\log}^\alpha}. \tag{2}$$

Hence

$$\sup_{z\in\mathbf{B}}\mu(z)|g(z)||\varphi(z)|\leq\sum_{l=1}^n\mu(z)|g(z)||\varphi_l(z)|\leq\|I_\varphi^g\|_{B_{\log,0}^\alpha\to B_\mu}\sum_{l=1}^n\|f_l\|_{B_{\log}^\alpha}<\infty.$$

Case $\alpha>1$. Let

$$f_w(z)=\frac{1-|w|^2}{(1-\langle z,w\rangle)^\alpha\log\frac{e}{1-\langle z,w\rangle}}-\frac{(1-|w|^2)^2}{(1-\langle z,w\rangle)^{\alpha+1}\log\frac{e}{1-\langle z,w\rangle}},\quad w\in\mathbb{B}.$$

An easy calculation shows that $f\in B_{\log,0}^\alpha$ and $\sup_{w\in\mathbf{B}}\|f_w\|_{B_{\log}^\alpha}<\infty$. From this and the boundedness of $I_\varphi^g: B_{\log,0}^\alpha\to B_\mu$ it follows that

$$\begin{aligned}C\|I_\varphi^g\|_{B_{\log,0}^\alpha\to B_\mu} &\geq \|I_\varphi^g f_{\varphi(w)}\|_{B_\mu}\\ &= \sup_{z\in\mathbf{B}}\mu(z)|\mathcal{R}f_{\varphi(w)}(\varphi(z))||g(z)|\\ &\geq \frac{\mu(w)|g(w)||\varphi(w)|^2}{(1-|\varphi(w)|^2)^\alpha\log\frac{e}{1-|\varphi(w)|^2}}.\end{aligned}$$

Then

$$\sup_{|\varphi(z)|\geq\frac{1}{2}} \frac{\mu(z)|g(z)||\varphi(z)|}{(1-|\varphi(z)|^2)^\alpha \log\frac{e}{1-|\varphi(z)|^2}} \leq \sup_{|\varphi(z)|\geq\frac{1}{2}} \frac{2\mu(z)|g(z)||\varphi(z)|^2}{(1-|\varphi(z)|^2)^\alpha \log\frac{e}{1-|\varphi(z)|^2}}$$
$$\leq C\|I_\varphi^g\|_{B^\alpha_{\log,0}\to\mathcal{B}_\mu} < \infty.$$

When $|\varphi(z)| \leq \frac{1}{2}$, by (2), we have

$$\frac{\mu(z)|g(z)||\varphi(z)|}{(1-|\varphi(z)|^2)^\alpha \log\frac{e}{1-|\varphi(z)|^2}} \leq \frac{1}{(\frac{3}{4})^\alpha \log\frac{4e}{3}} \sup_{z\in\mathbb{B}} \mu(z)|g(z)||\varphi(z)|$$
$$\leq \frac{1}{(\frac{3}{4})^\alpha \log\frac{4e}{3}} \|I_\varphi^g\|_{B^\alpha_{\log,0}\to\mathcal{B}_\mu} \sum_{l=1}^n \|f_l\|_{B^\alpha_{\log}} < \infty,$$

from which (1) follows in this case.

Case $\alpha = 1$. For fixed $w \in \mathbb{B}$, let

$$f_w(z) = \log\log\frac{e}{1-\langle z,w\rangle}, \quad z\in\mathbb{B}.$$

An easy calculation shows that

$$\mathcal{R}f_w(w) = \frac{|w|^2}{(1-|w|^2)\log\frac{e}{1-|w|^2}}, \quad f\in B^\alpha_{\log,0}, \quad \sup_{w\in\mathbb{B}}\|f_w\|_{B^\alpha_{\log}} < \infty.$$

Similar to Case $\alpha > 1$ we get (1).

(iii)⇒ (i) For any $z\in\mathbb{B}$ and $f\in B^\alpha_{\log}$, we have

$$\mu(z)|\mathcal{R}(I_\varphi^g f)(z)| \leq C\|f\|_{B^\alpha_{\log}} \frac{\mu(z)|g(z)||\varphi(z)|}{(1-|\varphi(z)|^2)^\alpha \log\frac{e}{1-|\varphi(z)|^2}}.$$

From this, (1) and since $I_\varphi^g f(0) = 0$ the boundedness of $I_\varphi^g : B^\alpha_{\log} \to \mathcal{B}_\mu$ follows.

Theorem 2.4 *Assume that μ is normal on $\mathbb{B}$, $g \in H(\mathbb{B})$, $g(0) = 0$, φ is a holomorphic self-map of $\mathbb{B}$ and $\alpha \geq 1$. Then the following statements are equivalent.*

(i) $I_\varphi^g : B^\alpha_{\log} \to \mathcal{B}_\mu$ is compact;

(ii) $I_\varphi^g : B^\alpha_{\log,0} \to \mathcal{B}_\mu$ is compact;

(iii) $I_\varphi^g : B^\alpha_{\log,0} \to \mathcal{B}_\mu$ is bounded and

$$\lim_{|\varphi(z)|\to 1} \frac{\mu(z)|g(z)||\varphi(z)|}{(1-|\varphi(z)|^2)^\alpha \log\frac{e}{1-|\varphi(z)|^2}} = 0; \tag{3}$$

(iv) $I_\varphi^g : B^\alpha_{\log} \to \mathcal{B}_\mu$ is bounded and (3) *holds.*

Proof (i)⇒ (ii) is obvious. By Theorem 2.3 (iii) and (iv) are equivalent.

(ii)⇒ (iii) Clearly $I_\varphi^g : B^\alpha_{\log,0} \to \mathcal{B}_\mu$ is bounded. If $\|\varphi\|_\infty < 1$, then condition (3) is vacuously satisfied. Now assume $\|\varphi\|_\infty = 1$. Let $(z_k)_{k\in\mathbb{N}}$ be a sequence in $\mathbb{B}$ such that $|\varphi(z_k)| \to 1$ as $k\to\infty$.

Case $\alpha > 1$. Let $k \in \mathbb{N}$,

$$f_k(z) = \frac{1-|\varphi(z_k)|^2}{(1-\langle z,\varphi(z_k)\rangle)^\alpha \log\frac{e}{1-\langle z,\varphi(z_k)\rangle}} - \frac{(1-|\varphi(z_k)|^2)^2}{(1-\langle z,\varphi(z_k)\rangle)^{\alpha+1}\log\frac{e}{1-\langle z,\varphi(z_k)\rangle}}.$$

Then $\sup_{w\in\mathbb{B}}\|f_k\|_{B^\alpha_{\log}} < \infty$ and f_k converges to zero uniformly on compacts of $\mathbb{B}$ as $k\to\infty$. By Proposition 2.2, then $\lim_{k\to\infty}\|I^g_\varphi f_k\|_{\mathcal{B}_\mu} = 0$. Hence

$$\begin{aligned} \|I^g_\varphi f_k\|_{\mathcal{B}_\mu} &= \sup_{z\in\mathbf{B}}\mu(z)|\mathcal{R}(I^g_\varphi f_k)(z)| \\ &\geq \mu(z_k)|\mathcal{R}(I^g_\varphi f_k)(z_k)| \\ &= \frac{\mu(z_k)|g(z_k)||\varphi(z_k)|^2}{(1-|\varphi(z_k)|^2)^\alpha\log\frac{e}{1-|\varphi(z_k)|^2}}. \end{aligned}$$

By use of the fact that $|\varphi(z_k)|\to 1$ as $k\to\infty$, we get

$$\lim_{k\to\infty}\frac{\mu(z_k)|g(z_k)||\varphi(z_k)|}{(1-|\varphi(z_k)|^2)^\alpha\log\frac{e}{1-|\varphi(z_k)|^2}} = 0,$$

from which (3) follows.

Case $\alpha = 1$. Let $k\in\mathbb{N}$,

$$f_k(z) = \frac{1-|\varphi(z_k)|^2}{(1-\langle z,\varphi(z_k)\rangle)\log\frac{e}{1-\langle z,\varphi(z_k)\rangle}} - \frac{(1-|\varphi(z_k)|^2)^2}{(1-\langle z,\varphi(z_k)\rangle)^2\log\frac{e}{1-\langle z,\varphi(z_k)\rangle}}.$$

Similar to Case $\alpha > 1$ we get (3).

(iii)$\Rightarrow$ (i) Since $I^g_\varphi : B^\alpha_{\log,0}\to\mathcal{B}_\mu$ is bounded and by (2), then

$$\sup_{z\in\mathbb{B}}\mu(z)|g(z)||\varphi(z)| < \infty.$$

Let $(f_k)_{k\in\mathbb{N}}$ be a bounded sequence in $B^\alpha_{\log}$ and f_k converges to zero uniformly on compacts of $\mathbb{B}$ as $k\to\infty$. By (3) for every $\varepsilon > 0$, there exists a $\delta\in(0,1)$ such that

$$\frac{\mu(z)||g(z)||\varphi(z)|}{(1-|\varphi(z)|^2)^\alpha\log\frac{e}{1-|\varphi(z)|^2}} < \varepsilon$$

whenever $\delta < |\varphi(z)| < 1$. Hence

$$\begin{aligned} \|I^g_\varphi f_k\|_{\mathcal{B}_\mu} &= \sup_{z\in\mathbf{B}}\mu(z)|\mathcal{R}(I^g_\varphi f_k)(z)| \\ &\leq \sup_{|\varphi(z)|\leq\delta}\mu(z)|g(z)||\mathcal{R}f_k(\varphi(z))| + \sup_{\delta<|\varphi(z)|<1}\mu(z)|g(z)||\mathcal{R}f_k(\varphi(z))| \\ &\leq \sup_{z\in\mathbf{B}}\mu(z)|g(z)||\varphi(z)|\cdot\sup_{|w|\leq\delta}|\nabla f_k(w)| \\ &\quad + C\|f_k\|_{B^\alpha_{\log}}\cdot\sup_{\delta<|\varphi(z)|<1}\frac{\mu(z)|g(z)||\varphi(z)|}{(1-|\varphi(z)|^2)^\alpha\log\frac{e}{1-|\varphi(z)|^2}} \\ &\leq C\sup_{|w|\leq\delta}|\nabla f_k(w)| + C\varepsilon. \end{aligned}$$

By the uniform convergence of $(f_k)_{k\in\mathbb{N}}$ on compacts of $\mathbb{B}$ and Cauchy's estimate, then $(|\nabla f_k|)_{k\in\mathbb{N}}$ also converges to zero on compacts of $\mathbb{B}$ as $k\to\infty$. Hence $\lim_{k\to\infty}\sup_{|w|\le\delta}|\nabla f_k(w)|=0$. Hence for every $\varepsilon>0$,

$$\limsup_{k\to\infty}\|I_\varphi^g f_k\|_{\mathcal{B}_\mu}\le C\varepsilon.$$

Then $\lim_{k\to\infty}\|I_\varphi^g f_k\|_{\mathcal{B}_\mu}=0$. By Proposition 2.2, the operator $I_\varphi^g: B_{\log}^\alpha\to\mathcal{B}_\mu$ is compact.

Theorem 2.5 *Assume that μ is normal on $\mathbb{B}$, $g\in H(\mathbb{B})$, $g(0)=0$, φ is a holomorphic self-map of $\mathbb{B}$ and $\alpha\ge 1$. Then the following statements are equivalent.*

(i) $I_\varphi^g: B_{\log,0}^\alpha\to\mathcal{B}_{\mu,0}$ is bounded;

(ii)$I_\varphi^g: B_{\log,0}^\alpha\to\mathcal{B}_\mu$ is bounded and

$$\lim_{|z|\to 1}\mu(z)|g(z)||\varphi(z)|=0.$$

Proof (i)⇒ (ii) Since $I_\varphi^g: B_{\log,0}^\alpha\to\mathcal{B}_{\mu,0}$ is bounded, then $I_\varphi^g: B_{\log,0}^\alpha\to\mathcal{B}_\mu$ is bounded. Since $f_l(z)=z_l\in B_{\log,0}^\alpha, l\in\{1,2,\ldots,n\}$, we have $I_\varphi^g f_l\in\mathcal{B}_{\mu,0}$, hence

$$\mu(z)|\mathcal{R}(I_\varphi^g f_l)(z)|=\mu(z)|g(z)||\varphi_l(z)|\to 0,$$

as $|z|\to 1$. For every $l\in\{1,2,\ldots,n\}$, and consequently

$$\lim_{|z|\to 1}\mu(z)|g(z)||\varphi(z)|=0.$$

(ii)⇒ (i) For every polynomial p, which obviously belongs to $B_{\log,0}^\alpha$, then

$$\begin{aligned}&\mu(z)|\mathcal{R}(I_\varphi^g p)(z)|\\ \le\;&\mu(z)|\mathcal{R}p(\varphi(z))||g(z)|\\ \le\;&\mu(z)|g(z)||\varphi(z)|\cdot\|\nabla p(\varphi(z))\|_\infty\to 0,\ |z|\to 1.\end{aligned}$$

Hence $I_\varphi^g p\in\mathcal{B}_{\log,0}$. Since the set of all polynomials is dense in $B_{\log,0}^\alpha$, for every $f\in B_{\log,0}^\alpha$, there is a sequence of polynomials $(p_k)_{k\in\mathbb{N}}$ such that $\|f-p_k\|_{B_{\log}^\alpha}\to 0$, as $k\to\infty$. By the boundedness of $I_\varphi^g: B_{\log,0}^\alpha\to\mathcal{B}_\mu$, then we have

$$\|I_\varphi^g f-I_\varphi^g p_k\|_{\mathcal{B}_\mu}\le\|I_\varphi^g\|_{B_{\log,0}^\alpha\to\mathcal{B}_\mu}\cdot\|f-p_k\|_{B_{\log}^\alpha}\to 0$$

as $k\to\infty$. Then $I_\varphi^g(B_{\log,0}^\alpha)\subseteq\mathcal{B}_{\mu,0}$. Hence $I_\varphi^g: B_{\log,0}^\alpha\to\mathcal{B}_{\mu,0}$ is bounded.

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