ON f-DERIVATIONS OF B-ALGEBRAS

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ABSTRACT. In this paper, we introduced the notion of left-right and right-left f-derivations of a B-algebra and investigated some related properties. We studied the notion of f-derivation of a 0-commutative B-algebra and stated some related properties.

1. INTRODUCTION

The study of BCK-algebras and BCI-algebras are introduced as two classes of abstract algebra by Y. Imai and K. Iseki [8, 9]. BCK-algebras is a proper subclass of the class of BCI-algebras. A new class BCH-algebras was introduced by Q. P. Hu and X. Li [6, 7] and was stated that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. The notion of d-algebras which is another generalization of BCK-algebras was introduced by Neggers, J. and Kim, H. S. in [13]. And also, many related properties with d-algebras and BCK-algebras were investigated by them whereas other relations between d-algebras and oriented digraphs were investigated by others. A generalization of BCH, BCI, BCK-algebras called BH-algebra was introduced and the notion of ideals in BH-algebra was given by Y. B. Jun, E. H. Roh and H. S. Kim in [10]. The notion of B-algebra was introduced by J. Neggers and H. S. Kim and some of its related properties in [14] were studied. The notion of derivation in rings and near rings theory was applied to BCI-algebras by Y. B. Jun and Xin and some of related properties were given by them [11]. In [4], the notion of f-derivations of BCI-algebras was introduced by Dudek and Zhang and a characterizations of a p-semisimple BCI-algebra was given by using regular f-derivations in that study. Later, in [15] the notion of a regular derivation in BCI-algebras was applied to BCC-algebras by Prabpayak and Leerawat and also some of its related properties were investigated. Later, in [5] the notion of left-right and right-left-f-derivations of BCC-algebras were introduced and some related related properties were investigated. Also, regular f-derivation and d-invariant on f-ideals in BCC-algebras were considered.

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In [2] the notion of derivation in B-algebra was given and some related properties were stated. In this paper the notion of f-derivation in B-algebra is given and some of its properties are stated. Additionally, this definition in 0-commutative B-algebra is studied and related properties are given.

2. Preliminaries

Definition 2.1. [14] A B-algebra is a non-empty set X with a constant 0 and with a binary operation $*$ satisfying the following axioms for all $x, y, z \in X$:

(I) $x * x = 0$,

(II) $x * 0 = x$,

(III) $(x * y) * z = x * (z * (0 * y))$.

Proposition 2.2. [14] *If $(X, *, 0)$ is a B-algebra, then for all $x, y, z \in X$:*

(1) $(x * y) * (0 * y) = x$,

(2) $x * (y * z) = (x * (0 * z)) * y$,

(3) $x * y = 0$ *implies* $x = y$,

(4) $0 * (0 * x) = x$.

Theorem 2.3. [14] *$(X, *, 0)$ is a B-algebra if and only if it satisfies the following axioms for all $x, y, z \in X$:*

(5) $x * x = 0$,

(6) $0 * (0 * x) = x$,

(7) $(x * z) * (y * z) = x * y$,

(8) $0 * (x * y) = y * x$.

Theorem 2.4. [3] *In any B-algebra, the left and right cancellation laws hold.*

Definition 2.5. [12] A B-algebra $(X, *, 0)$ is said to be 0-commutative if for all $x, y \in X$:

$x * (0 * y) = y * (0 * x)$.

Proposition 2.6. [12] *If* $(X,*,0)$ *is a* 0- *commutative* B*-algebra, then for all* $x,y,z \in X$*:*

(9) $(0*x)*(0*y) = y*x$.

(10) $(z*y)*(z*x) = x*y$.

(11) $(x*y)*z = (x*z)*y$

(12) $[x*(x*y)]*y=0$.

(13) $(x*z)*(y*t) = (t*z)*(y*x)$.

From (12) *and* (3) *we get that, if* $(X,*,0)$ *is a* 0- *commutative* B*-algebra, then:*

(14) $x*(x*y) = y$ *for all* $x,y \in X$.

For a B- algebra X, we denote $x \wedge y = y*(y*x)$ for all $x,y \in X$.

Definition 2.7. [2] Let X be a B-algebra and let d: $X \to X$ be a map. If d satisfies the identity $d(x*y) = (d(x)*y) \wedge (x*d(y))$ for all $x,y \in X$, then d is said to be a (l,r)-derivation of X. If d satisfies the identity $d(x*y) = (x*d(y)) \wedge (d(x)*y)$ for all $x,y \in X$, then d is said to be a (r,l)-derivation of X. Moreover, if d is both a (l,r) and (l,r)-derivation, then d is said to be a derivation of X.

Definition 2.8. [2]A self map d of a B-algebra X is said to be regular if $d(0) = 0$.

3. The f-Derivations of B-Algebras

The following definition introduces the notion of f-derivation for a B-algebra. In what follows, let f be an endomorphism of X unless otherwise specified.

Definition 3.1. Let X be a B-algebra and f be an endomorphism of X. A map d: $X \to X$ is said to be a left-right f-derivation (briefly, a (l,r)-f-derivation) of X if it satisfies the identity $d(x*y) = (d(x)*f(y)) \wedge (f(x)*d(y))$ for all $x,y \in X$.

If d satisfies the identity $d(x*y) = (f(x)*d(y)) \wedge (d(x)*f(y))$ for all $x,y \in X$, then d is said to be a right-left derivation (briefly, (r,l)-f-derivation) of X. Moreover, if d is both (l,r) and (r,l)-f-derivation, then it is said that d is an f-derivation.

Example 3.1. *Let $X = \{0, 1, 2, 3\}$ be a B-algebra with Cayley table as follows.*

*	0	1	2	3
0	0	2	1	3
1	1	0	3	2
2	2	3	0	1
3	3	1	2	0

By [2] *we know that a map $d : X \to X$ for all x in X defined by*

$$d(x) = \begin{cases} 3, & x=0 \\ 2, & x=1 \\ 1, & x=2 \\ 0, & x=3 \end{cases}$$

is a derivation of X. Define an endomorphism f of X by $f(x) = 0$ for all $x \in X$. Then d is not an f-derivation of X since $d(3 * 1) = d(1) = 2$, but $(d(3)*f(1)) \wedge (f(3)*d(1)) = (0* 0) \wedge (0* 2) = 0 \wedge 1 = 1*(1*0) = 1*1 = 0$, and thus $d(3 * 1) \neq (d(3) * f(1)) \wedge (f(3) * d(1))$.

Remark 3.1. *Every derivation of a B-algebra X could be made an f-derivation of X by an identity endomorphism of X.*

Example 3.2. *Let $X = \{0, 1, 2, 3\}$ be a B-algebra with Cayley table as given in Example 3.1.*

Define a map

$$d(x) = \begin{cases} 3, & x=0 \\ 1, & x=1 \\ 2, & x=2 \\ 0, & x=3 \end{cases}$$

Then d is not a derivation of X since $d(2 * 1) = d(3) = 0$, but $(d(2) * 1) \wedge (2 * d(1)) = (2 * 1) \wedge (2 * 1) = 3 \wedge 3 = 3 * (3 * 3) = 3 * 0 = 3$, and thus $d(2 * 1) \neq (d(2) * 1) \wedge (2 * d(1))$.

Now we define an endomorphism f of X for all $x \in X$ by

$$f(x) = \begin{cases} 0, & x=0 \\ 2, & x=1 \\ 1, & x=2 \\ 3, & x=3 \end{cases}$$

Then it is easily checked that d is an f-derivation of X.

Example 3.3. *Let $X = \{0,1,2,3\}$ be a B-algebra with Cayley table as in Example 3.1.*

Define a map

$$d(x) = \begin{cases} 0, & x=0 \\ 2, & x=1 \\ 1, & x=2 \\ 3, & x=3 \end{cases}$$

Then d is not a derivation of X since $d(3*2) = d(2) = 1$, but $(d(3)*2) \wedge (3*d(2)) = (3*2) \wedge (3*1) = 2 \wedge 1 = 1*(1*2) = 1*3 = 2$, and thus $d(3*2) \neq (d(3)*2) \wedge (3*d(2))$.

Now we define an endomorphism f of X for all $x \in X$ by

$$f(x) = \begin{cases} 0, & x=0 \\ 2, & x=1 \\ 1, & x=2 \\ 3, & x=3 \end{cases}$$

Then it is easily checked that d is an f-derivation of X.

Definition 3.2. An f-derivation d of a B-algebra X is said to be regular if $d(0) = 0$.

Proposition 3.3. *Let d be a $(l,r) - f$-derivation of B-algebra X. Then*

(i) $d(0) = d(x)*f(x)$ for all $x \in X$.

(ii) If f is $1-1$, then d is $1-1$.

(iii) If d is regular, then $d = f$.

(iv) If there is an element $x \in X$ such that $d(x) = f(x)$, then d is regular.

(v) If there exists $x \in X$ such that $d(y)*f(x) = 0$ or $f(x)*d(y) = 0$ for all $y \in X$, then $d(y) = f(x)$.

Proof: (i) Let x be an element in X. Since $x*x = 0$ we have

$$\begin{aligned} d(0) = d(x*x) &= (d(x)*f(x)) \wedge (f(x)*d(x)) \\ &= (f(x)*d(x))*((f(x)*d(x))*(d(x)*f(x))). \end{aligned}$$

Then, by using (2), (8), (5), (8) respectively we can write

$$\begin{aligned} d(0) &= ((f(x)*d(x))*(0*(d(x)*f(x)))*(f(x)*d(x)) \\ &= ((f(x)*d(x))*(f(x)*d(x)))*(f(x)*d(x)) \\ &= 0*(f(x)*d(x)) \\ &= d(x)*f(x). \end{aligned}$$

Hence we get $d(0) = d(x) * f(x)$ for all $x \in X$.

(ii) Let f be a $1-1$ endomorphism and $d(x) = d(y)$ for $x, y \in X$. Then by (i) we have $d(0) = d(x) * f(x)$ and $d(0) = d(y) * f(y)$. Then we get $d(x) * f(x) = d(x) * f(y)$. By Theorem 2.4 we have $f(x) = f(y)$. Since f is a $1-1$ endomorphism we get $x = y$. Therefore d is $1-1$.

(iii) Let d be a regular map. By part (i) we have $d(0) = d(x)*f(x)$. Since d is regular we have $d(0) = d(x) * f(x) = 0$ and by (3) we get $d(x) = f(x)$.

(iv) Let $d(x) = f(x)$ for some $x \in X$. By (5) we have $d(x) * f(x) = 0$ then we can write by part (i) $d(0) = d(x) * f(x) = f(x) * f(x) = 0$ therefore $d(0) = 0$.

Hence we get that d is regular.

(v) Let x be an element in X such that $d(y) * f(x) = 0$ or $f(x) * d(y) = 0$ for all $y \in X$ then by (3) we get $d(y) = f(x)$.

Proposition 3.4. *Let d be a $(r, l) - f$-derivation of B-algebra X. Then*

(i) $d(0) = f(x) * d(x)$ for all $x \in X$.

(ii) $d(x) = d(x) \wedge f(x)$ for all $x \in X$.

(iii) If f is a $1-1$ endomorphism, then d is $1-1$.

(iv) If d is regular, then $d = f$.

(v) If there is an element $x \in X$ such that $d(x) = f(x)$, then d is regular.

(vi) If there exists $x \in X$ such that $d(y) * f(x) = 0$ or $f(x) * d(y) = 0$ for all $y \in X$, then $d(y) = f(x)$, i.e., d is constant.

Proof: (i) Let x be an element in X. Since $x * x = 0$ we have

$$\begin{aligned} d(0) = d(x * x) &= (f(x) * d(x)) \wedge (d(x) * f(x)) \\ &= (d(x) * f(x)) * ((d(x) * f(x)) * (f(x) * d(x))). \end{aligned}$$

Then, by using (2), (8), (5), (8) respectively we can write

$$\begin{aligned} d(0) &= [(d(x) * f(x)) * (0 * (f(x) * d(x)))] * (d(x) * f(x)) \\ &= [(d(x) * f(x)) * (d(x) * f(x))] * (d(x) * f(x)) \\ &= 0 * (d(x) * f(x)) \\ &= f(x) * d(x). \end{aligned}$$

Hence we get $d(0) = f(x) * d(x)$ for all $x \in X$.

(ii) Let $x \in X$ then we have $x * 0 = x$ and

$$\begin{aligned} d(x) = d(x*0) &= (f(x) * d(0)) \wedge (d(x) * f(0)) \\ &= (d(x) * f(0)) * ((d(x) * f(0)) * (f(x) * d(0))) \\ &= (d(x)*f(0))*((d(x)*f(0))*(f(x)*(f(x)*d(x)))) \text{ by part (i)} \\ &= d(x) * (d(x) * (f(x) * (f(x) * d(x)))) \text{ then we get} \end{aligned}$$

$d(x) = d(x) * 0 = d(x) * (d(x) * (f(x) * (f(x) * d(x))))$ by Theorem 2.4 we have $d(x)*(f(x) * (f(x)*d(x))) = 0$. By (3) we have $d(x) = f(x)* (f(x)* d(x))$ this means that $d(x) = d(x) \wedge f(x)$.

(iii) Let f be a $1-1$ endomorphism and $d(x) = d(y)$ for $x, y \in X$. Then by (i) we have $d(0) = f(x) * d(x)$ and $d(0) = f(y) * d(y)$. Then we get $d(0) = f(x) * d(x) = f(y) * d(x)$. By Theorem 2.4 we have $f(x) = f(y)$. Since f is a $1-1$ endomorphism we get $x = y$. Therefore d is $1-1$.

(iv) Let d be a regular map. By (i) we have $d(0) = f(x) * d(x) = 0$ and by (3) we get $d(x) = f(x)$.

(v) Let $d(x) = f(x)$ for some $x \in X$. By (3) we have $f(x) * d(x) = 0$ and by (i) we have $d(0) = 0$.

(vi) Let x be an element in X such that $d(y)*f(x) = 0$ or $f(x)*d(y) = 0$ for all $y \in X$ then by (3) we get $d(y) = f(x)$.

4. The f-Derivations of 0-Commutative B-Algebras

Here we will study the notion of f-derivation of 0-commutative B-algebra and state some related properties.

Example 4.1. *Let $X = \{0, 1, 2\}$ be a 0-commutative B-algebra with Cayley table as follows.*

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Define a map

$$d(x) = \begin{cases} 2, & x{=}0 \\ 1, & x{=}1 \\ 0, & x{=}2 \end{cases}$$

Then d is not a derivation of X since $d(2*1) = d(1) = 1$, *but* $(d(2)*1)) \wedge (2*d(1)) = (0*1) \wedge (2*1) = 2 \wedge 1 = 1*(1*2) = 1*2 = 2$, *and thus* $d(2*1) \neq (d(2)*1) \wedge (2*d(1))$.

Now we define an endomorphism f of X for all $x \in X$ *by*

$$f(x) = \begin{cases} 0, & x{=}0 \\ 2, & x{=}1 \\ 1, & x{=}2 \end{cases}$$

Then it is easily checked that d is $(l,r)-f$*-derivation of X.*

Example 4.2. *Let* $X = 0,1,2,3$ *be a* 0*-commutative B-algebra with Cayley table as follows:*

*	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

Define a map

$$d(x) = \begin{cases} 2, & x{=}0 \\ 1, & x{=}1 \\ 0, & x{=}2 \\ 3, & x{=}3 \end{cases}$$

Then d is not a derivation of X since $d(1*0) = d(1) = 1$, *but* $(1*d(0)) \wedge (d(1)*0) = (1*2) \wedge (1*0) = 3 \wedge 1 = 1*(1*3) = 1*2 = 3$, *and thus* $d(1*0) \neq (1*d(0)) \wedge (d(1)*0)$.

Now we define an endomorphism f of X for all $x \in X$ *by*

$$f(x) = \begin{cases} 0, & x{=}0 \\ 3, & x{=}1 \\ 2, & x{=}2 \\ 1, & x{=}3 \end{cases}$$

Then it is easily checked that d is an f-derivation of X.

Example 4.3. *Let $X = 0, 1, 2, 3$ be a 0-commutative B-algebra with Cayley table as in Example 4.2.*

Define a map

$$d(x) = \begin{cases} 0, & x=0 \\ 3, & x=1 \\ 2, & x=2 \\ 1, & x=3 \end{cases}$$

Then d is not a derivation of X since $d(1 * 3) = d(2) = 2$, but $(d(1) * 3) \wedge (1 * d(3)) = (3 * 3) \wedge (1 * 1) = 0 \wedge 0 = 0 * (0 * 0) = 0 * 0 = 0$, and thus $d(1 * 3) \neq (d(1) * 3) \wedge (1 * d(3))$.

Now we define an endomorphism f of X for all $x \in X$ by

$$f(x) = \begin{cases} 0, & x=0 \\ 3, & x=1 \\ 2, & x=2 \\ 1, & x=3 \end{cases}$$

Then it is easily checked that d is an f-derivation of X.

Proposition 4.1. *Let $(X, *, 0)$ be a 0-commutative B-algebra and d be a $(l, r) - f$-derivation of X. Then for all $x, y \in X$*

(i) $d(x * y) = d(x) * f(y)$.

(ii) $d(x) * d(y) = f(x) * f(y)$.

Proof: (i) Let $x, y \in X$, then we have

$$\begin{aligned} d(x * y) &= (d(x) * f(y)) \wedge (f(x) * d(y)) \\ &= (f(x) * d(y)) * ((f(x) * d(y)) * (d(x) * f(y))) \end{aligned}$$

Then by (14) we have $d(x * y) = d(x) * f(y)$

(ii) Let $x, y \in X$, then from Proposition 3.3 (i) we can write

$d(0) = d(x) * f(x)$ and $d(0) = d(y) * f(y)$. From here we get $d(x) * f(x) = d(y) * f(y)$

and we have $(d(y) * f(y)) * (d(x) * f(x)) = 0$ and we can also write by (13) $(f(x) * f(y)) * (d(x) * d(y)) = 0$. Hence, by (3) we have $d(x) * d(y) = f(x) * f(y)$.

Proposition 4.2. *Let $(X, *, 0)$ be a 0-commutative B-algebra and d be a $(r, l) - f$-derivation of X. Then for all $x, y \in X$*

(i) $d(x * y) = f(x) * d(y)$.

(ii) $d(x) * d(y) = f(x) * f(y)$.

Proof: (i) Let $x, y \in X$, then we can write

$$d(x * y) = (f(x) * d(y)) \wedge (d(x) * f(y))$$
$$= (d(x) * f(y)) * ((d(x) * f(y)) * (f(x) * d(y))).$$

Then by (14) we have $d(x * y) = f(x) * d(y)$.

(ii) Let $x, y \in X$. By Proposition 3.4 (i) we can write $d(0) = f(x) * d(x)$ and $d(0) = f(y) * d(y)$ and we have $f(x) * d(x) = f(y) * d(y)$ and we can also write by (13) $(d(x) * d(y)) * (f(x) * f(y)) = 0$. Hence, by (3) we have $d(x) * d(y) = f(x) * f(y)$.

Definition 4.3. Let X be a B-algebra and d_1, d_2 be two self maps of X. We define $d_1 o d_2 : X \to X$ for all $x \in X$ as $d_1 o d_2(x) = d_1(d_2(x))$.

Proposition 4.4. *Let X be a 0-commutative B-algebra and d_1 be $(l, r)-$ f_1-derivation, d_2 be $(l, r) - f_2$-derivation of X. Then $d_1 o d_2$ is a $(l, r) -$ $f_1 o f_2$-derivation of X.*

Proof: Let X be a 0-commutative B-algebra and d_1 be $(l, r) - f_1$- derivation, d_2 be $(l, r) - f_2$-derivation of X. Then for all $x, y \in X$ we have

$d_1 o d_2(x * y) = d_1(d_2(x * y))$

$= d_1[(d_2(x) * f_2(y)) \wedge (f_2(x) * d_2(y))]$

$= d_1[(f_2(x) * d_2(y)) * ((f_2(x) * d_2(y)) * (d_2(x) * f_2(y)))]$

$= d_1(d_2(x) * f_2(y))$, by (14)

$= d_1(d_2(x)) * f_1(f_2(y))$, by Proposition 4.1 (i)

$= (f_1(f_2(x)) * d_1(d_2(y))) * [(f_1(f_2(x)) * d_1(d_2(y))) * (d_1(d_2(x)) * f_1(f_2(y)))]$

$= (d_1 o d_2(x) * f_1 o f_2(y)) \wedge (f_1 o f_2(x) * d_1 o d_2(y))$

This means that $d_1 o d_2$ is a $(l, r) - f_1 o f_2$-derivation of X.

Proposition 4.5. *Let X be a 0-commutative B-algebra and d_1 be $(r, l)-$ f_1-derivation, d_2 be $(r, l) - f_2$-derivation of X. Then $d_1 o d_2$ is a $(r, l) -$ $f_1 o f_2$-derivation of X.*

Proof: Proof is similar with the previous one.

Corollary 4.6. *Let X be a 0-commutative B-algebra and d_1 be f_1- derivation, d_2 be f_2-derivation of X. Then $d_1 o d_2$ is also a $f_1 o f_2$- derivation of X.*

Theorem 4.7. *Let X be a 0-commutative B-algebra and d_1 be f_1- derivation, d_2 be f_2-derivation and f_1, f_2 be endomorphisms of X such that $f_2 o d_1 = d_1 o f_2$ and $d_2 o f_1 = f_1 o d_2$ then $d_1 o d_2 = d_2 o d_1$.*

Proof: Let X be a 0-commutative B-algebra and d_1 be f_1- derivation, d_2 be f_2-derivation and f_1, f_2 be endomorphisms of X such that

$f_2od_1 = d_1of_2$ and $d_2of_1 = f_1od_2$. Then for all $x, y \in X$ we have

$$\begin{aligned} d_1od_2(x*y) &= d_1(d_2(x*y)) = d_1(d_2(x)*f_2(y)), \text{ by Proposition 4.1 (i)} \\ &= f_1(d_2(x))*d_1(f_2(y)) \text{ by Proposition 4.2 (i)} \end{aligned}$$

Similarly, for all $x, y \in X$ we get

$$\begin{aligned} d_2od_1(x*y) &= d_2(d_1(x*y)) = d_2(f_1(x)*d_1(y)) \\ &= d_2(f_1(x))*f_2(d_1(y)) \\ &= f_1(d_2(x))*d_1(f_2(y)), \text{ by hypothesis.} \end{aligned}$$

Hence we get $d_1od_2 = d_2od_1$.

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