On Equitable Coloring of Sun let Graph Families

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Abstract. The notion of equitable coloring was introduced by Meyer in 1973. In this paper we obtain interesting results regarding the equitable chromatic number $\chi_=$ for the sun let graphs S_n, line graph of sun let graphs $L(S_n)$, middle graph of sun let graphs $M(S_n)$, total graph of sun let graphs $T(S_n)$.

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1. Introduction

The set of vertices of a graph G can be partitioned into k classes $V_1, V_2, ..., V_k$ such that each V_i is an independent set and the condition $||V_i| - |V_j|| \leq 1$ holds for every pair (i, j), then G is said to be *equitably k-colorable.* The smallest integer k for which G is equitable k-colorable is known as the *equitable chromatic number* [1, 3, 6, 7, 8] of G and denoted by $\chi_=(G)$.

This model of graph coloring has many applications. Everytime when we have to divide a system with binary conflicting relations into equal or almost equal conflict-free subsystems we can model such situation by means of equitable graph coloring. This subject is widely discussed in literature [3, 6, 7, 8]. In general, the problem of optimal equitable coloring, in the sense of the number color used, is NP-hard. So we have to look for simplified structure of graphs allowing polynomial-time algorithms. This paper gives such solution for sun let graph families: sun let graphs, its line, middle and total graphs.

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2. Preliminaries

The $n-$sun let graph on $2n$ vertices is obtained by attaching n pendant edges to the cycle C_n and is denoted by S_n.

The line graph [2, 5] of G, denoted by $L(G)$ is the graph with vertices are the edges of G with two vertices of $L(G)$ adjacent whenever the corresponding edges of G are adjacent.

The middle graph [4] of G, is defined with the vertex set $V(G) \cup E(G)$ where two vertices are adjacent iff they are either adjacent edges of G or one is the vertex and the other is an edge incident with it and it is denoted by $M(G)$.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The total graph [2, 4, 5] of G, denoted by $T(G)$ is defined in the following way. The vertex set of $T(G)$ is $V(G) \cup E(G)$. Two vertices x, y in the vertex set of $T(G)$ are adjacent in $T(G)$ in case one of the following holds: (i) x, y are in $V(G)$ and x is adjacent to y in G. (ii) x, y are in $E(G)$ and x, y are adjacent in G. (iii) x is in $V(G), y$ is in $E(G)$, and x, y are incident in G. Additional graph theory terminology used in this paper can be found in [2, 5, 7].

3. Equitable coloring on sun let graph and its line graph

Theorem 3.1. *If $n \geq 2$ the equitable chromatic number of sun let graph S_n,*

$$\chi_{=}(S_n) = \begin{cases} 2 & \text{if } n \text{ is even} \\ 3 & \text{if } n \text{ is odd}. \end{cases}$$

Proof. Let S_n be the sun let graph on $2n$ vertices. Let $V(S_n)=\{v_1, v_2, \cdots, v_n\} \cup \{u_1, u_2, \cdots, u_n\}$ where v_i's are the vertices of cycles taken in cyclic order and u_i's are pendant vertices such that each v_iu_i is a pendant edge.

Case(i): If n is even.

Now, we partition the vertex set $V(S_n)$ as $V_1 = \{v_1, v_3, \cdots, v_{n-1}\} \cup \{u_2, u_4, \cdots, u_n\}$; $V_2 = \{v_2, v_4, \cdots, v_n\} \cup \{u_1, u_3, \cdots u_{n-1}\}$. Clearly V_1 and V_2 are independent sets of $V(S_n)$. Also $|V_1| = |V_2| = n$, it holds the inequality $||V_i| - |V_j|| \leq 1$ for every pair (i, j). $\chi_{=}(S_n) \leq 2$. Since $\chi(S_n) \geq 2$, $2 \leq \chi(S_n) \leq \chi_{=}(S_n)$, $\chi_{=}(S_n) \geq 2$. Therefore $\chi_{=}(S_n) = 2$.

Case(ii): If n is odd.

Case(ii)a:
If $n = 6k - 3$ for some positive integer k, then set the partition of V as below. $V_1 = \{v_{3i-2} : 1 \leq i \leq 2k-1\} \cup \{u_{3i-1} : 1 \leq i \leq 2k-1\}$; $V_2 = \{v_{3i-1} : 1 \leq i \leq 2k-1\} \cup \{u_{3i} : 1 \leq i \leq 2k-1\}$; $V_3 = \{v_{3i} : 1 \leq i \leq 2k-1\} \cup \{u_{3i-2} : 1 \leq i \leq 2k-1\}$. Clearly V_1, V_2, V_3 are independent sets of $V(S_n)$. Also $|V_1| = |V_2| = |V_3| = 4k-2$, it holds the inequality

$||V_i| - |V_j|| \leq 1$ for every pair (i, j).

Case(ii)b:
If $n = 6k-1$ for some positive integer k, then set the partition of V as below. $V_1 = \{v_{3i-2} : 1 \leq i \leq 2k\} \cup \{u_{3i-1} : 1 \leq i \leq 2k\}$; $V_2 = \{v_{3i-1} : 1 \leq i \leq 2k\} \cup \{u_{3i} : 1 \leq i \leq 2k-1\}$; $V_3 = \{v_{3i} : 1 \leq i \leq 2k-1\} \cup \{u_{3i-2} : 1 \leq i \leq 2k\}$. Clearly V_1, V_2, V_3 are independent sets of $V(S_n)$. Also $|V_1| = 4k$ and $|V_2| = |V_3| = 4k-1$, it holds the inequality $||V_i|-|V_j|| \leq 1$ for every pair (i, j).

Case(ii)c:
If $n = 6k+1$ for some positive integer k, then set the partition of V as below. $V_1 = \{v_{3i-2} : 1 \leq i \leq 2k\} \cup \{u_{3i-1} : 1 \leq i \leq 2k\}$; $V_2 = \{v_{3i-1} : 1 \leq i \leq 2k\} \cup \{v_{6k+1}\} \cup \{u_{3i} : 1 \leq i \leq 2k\}$; $V_3 = \{v_{3i} : 1 \leq i \leq 2k\} \cup \{u_{3i-2} : 1 \leq i \leq 2k+1\}$. Clearly V_1, V_2, V_3 are independent sets of $V(S_n)$. $|V_1| = 4k$ and $|V_2| = |V_3| = 4k+1$, it holds the inequality $||V_i|-|V_j|| \leq 1$ for every pair (i, j).

In all the three subcases of cases (ii), $\chi_=(S_n) \leq 3$. Since $\chi(S_n) \geq 3$, $3 \leq \chi(S_n) \leq \chi_=(S_n)$, $\chi_=(S_n) \geq 3$. Therefore $\chi_=(S_n) = 3$. □

Theorem 3.2. *If $n \geq 3$ the equitable chromatic number on line graph of sun let graph $L(S_n)$, $\chi_=(L(S_n)) = 3$.*

Proof. Let $V(S_n) = \{v_1, v_2, \cdots, v_n\} \cup \{u_1, u_2, \cdots, u_n\}$ and $E(S_n) = \{e'_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n-1\} \cup \{e_n\}$ where e_i is the edge $v_i v_{i+1}$ $(1 \leq i \leq n-1)$, e_n is the edge $v_n v_1$ and e'_i is the edge $v_i u_i$ $(1 \leq i \leq n)$. By the definition of line graph $V(L(S_n)) = E(S_n) = \{u'_i : 1 \leq i \leq n\} \cup \{v'_i : 1 \leq i \leq n-1\} \cup \{v'_n\}$ where v'_i and u'_i represents the edge e_i and e'_i $(1 \leq i \leq n)$ respectively.

Case(i): If n is odd.

Case(i)a:
If $n = 6k-3$ for some positive integer k, then set the partition of V as below. $V_1 = \{v'_{3i-2} : 1 \leq i \leq 2k-1\} \cup \{u'_{3i} : 1 \leq i \leq 2k-1\}$; $V_2 = \{v'_{3i-1} : 1 \leq i \leq 2k-1\} \cup \{u'_{3i-2} : 1 \leq i \leq 2k-1\}$; $V_3 = \{v'_{3i} : 1 \leq i \leq 2k-1\} \cup \{u'_{3i-1} : 1 \leq i \leq 2k-1\}$. Clearly V_1, V_2, V_3 are independent sets of $V(L(S_n))$. Also $|V_1| = |V_2| = |V_3| = 4k-2$, it holds the inequality $||V_i| - |V_j|| \leq 1$ for every pair (i, j).

Case(i)b:
If $n = 6k-1$ for some positive integer k, then set the partition of V as below. $V_1 = \{v'_{3i-1} : 1 \leq i \leq 2k-1\} \cup \{u'_{3i-2} : 1 \leq i \leq 2k\} \cup \{u'_{6k-1}\}$; $V_2 = \{v'_{3i-2} : 1 \leq i \leq 2k\} \cup \{u'_{3i} : 1 \leq i \leq 2k-1\}$; $V_3 = \{v'_{3i} : 1 \leq i \leq 2k-1\} \cup \{v'_{6k-1}\} \cup \{u'_{3i-1} : 1 \leq i \leq 2k-1\}$. Clearly V_1, V_2, V_3 are independent sets of $V(L(S_n))$. Also $|V_1| = 4k$ and $|V_2| = |V_3| = 4k-1$, it holds the inequality $||V_i| - |V_j|| \leq 1$ for every pair (i, j).

Case(i)c:
If $n = 6k + 1$ for some positive integer k, then set the partition of V as below. $V_1 = \{v'_{3i-1} : 1 \leq i \leq 2k\} \cup \{u'_{3i-2} : 1 \leq i \leq 2k\} \cup \{u'_{6k+1}\}$; $V_2 = \{v'_{3i-2} : 1 \leq i \leq 2k\} \cup \{v'_{6k}\} \cup \{u'_{3i} : 1 \leq i \leq 2k-1\}$; $V_3 = \{v'_{3i} : 1 \leq i \leq 2k-1\} \cup \{v'_{6k+1}\} \cup \{u'_{3i-1} : 1 \leq i \leq 2k\} \cup \{u'_{6k}\}$. Clearly V_1, V_2, V_3 are independent sets of $V(L(S_n))$. Also $|V_2| = 4k$ and $|V_1| = |V_3| = 4k+1$, it holds the inequality $||V_i| - |V_j|| \leq 1$ for every pair (i, j).

Case(ii): If n is even.

Case(ii)a:
If $n = 6k-2$ for some positive integer k, then set the partition of V as below. $V_1 = \{v'_{3i-1} : 1 \leq i \leq 2k-1\} \cup \{u'_{3i-2} : 1 \leq i \leq 2k-1\} \cup \{u'_{6k-2}\}$; $V_2 = \{v'_{3i-2} : 1 \leq i \leq 2k-1\} \cup \{v'_{6k-3}\} \cup \{u'_{3i} : 1 \leq i \leq 2k-2\}$; $V_3 = \{v'_{3i} : 1 \leq i \leq 2k-2\} \cup \{v'_{6k-2}\} \cup \{u'_{3i-1} : 1 \leq i \leq 2k-1\} \cup \{u'_{6k-3}\}$. Clearly V_1, V_2, V_3 are independent sets of $V(L(S_n))$. Also $|V_2| = 4k-2$ and $|V_1| = |V_3| = 4k-1$, it holds the inequality $||V_i|-|V_j|| \leq 1$ for every pair (i, j).

Case(ii)b:
If $n = 6k$ for some positive integer k, then set the partition of V as below. $V_1 = \{v'_{3i-2} : 1 \leq i \leq 2k\} \cup \{u'_{3i} : 1 \leq i \leq 2k\}$; $V_2 = \{v'_{3i-1} : 1 \leq i \leq 2k\} \cup \{u'_{3i-2} : 1 \leq i \leq 2k\}$; $V_3 = \{v'_{3i} : 1 \leq i \leq 2k\} \cup \{u'_{3i-1} : 1 \leq i \leq 2k\}$. Clearly V_1, V_2, V_3 are independent sets of $V(L(S_n))$. Also $|V_1| = |V_2| = |V_3| = 4k$, it holds the inequality $||V_i| - |V_j|| \leq 1$ for every pair (i, j).

Case(ii)c:
If $n = 6k + 2$ for some positive integer k, then set the partition of V as below. $V_1 = \{v'_{3i-1} : 1 \leq i \leq 2k\} \cup \{u'_{3i-2} : 1 \leq i \leq 2k+1\} \cup \{u'_{6k+2}\}$; $V_2 = \{v'_{3i-2} : 1 \leq i \leq 2k+1\} \cup \{v'_{6k+1}\} \cup \{u'_{3i-1} : 1 \leq i \leq 2k\}$; $V_3 = \{v'_{3i} : 1 \leq i \leq 2k\} \cup \{v'_{6k+2}\} \cup \{u'_{3i-1} : 1 \leq i \leq 2k\}$. Clearly V_1, V_2, V_3 are independent sets of $V(L(S_n))$. Also $|V_1| = 4k+2$ and $|V_2| = |V_3| = 4k+1$, it holds the inequality $||V_i| - |V_j|| \leq 1$ for every pair (i, j).

In all the six subcases of case (i) and case (ii), $\chi_=(L(S_n)) \leq 3$. Since $L(S_n)$ contains a clique of order 3, $\chi(L(S_n)) \geq 3$, $3 \leq \chi(L(S_n)) \leq \chi_=(L(S_n))$, $\chi_=(L(S_n)) \geq 3$. Therefore $\chi_=(L(S_n)) = 3$. □

4. Equitable coloring on middle graph and total graph of sun let graph

Theorem 4.1. *If $n \geq 3$ the equitable chromatic number on middle graph of sun let graph $M(S_n)$, $\chi_=(M(S_n)) = 4$.*

Proof. Let $V(S_n) = \{v_1, v_2, \cdots, v_n\} \cup \{u_1, u_2, \cdots, u_n\}$ and $E(S_n) = \{e'_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n-1\} \cup \{e_n\}$ where e_i is the edge $v_i v_{i+1}$ $(1 \leq i \leq n-1)$,

e_n is the edge v_nv_1 and e'_i is the edge v_iu_i $(1 \le i \le n)$. By the definition of middle graph $V(M(S_n)) = V(S_n) \cup E(S_n) = \{v_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n\} \cup \{v'_i : 1 \le i \le n\} \cup \{u'_i : 1 \le i \le n\}$ where v'_i and u'_i represents the edge e_i and e'_i $(1 \le i \le n)$ respectively.

Case(i): If n is even.

Now, we partition the vertex set $V(M(S_n))$ as $V_1 = \{v_i : 1 \le i \le n\}$; $V_2 = \{v'_{2i-1} : 1 \le i \le \frac{n}{2}\} \cup \{u_{2i-1} : 1 \le i \le \frac{n}{2}\}$; $V_3 = \{v'_{2i} : 1 \le i \le \frac{n}{2}\} \cup \{u_{2i} : 1 \le i \le \frac{n}{2}\}$; $V_4 = \{u'_i : 1 \le i \le n\}$. Clearly V_1, V_2, V_3 and V_4 are independent sets of $M(S_n)$. Also $|V_1| = |V_2| = |V_3| = |V_4| = n$, it holds the inequality $||V_i| - |V_j|| \le 1$ for every pair (i, j). $\chi_=(M(S_n)) \le 4$. Since $M(S_n)$ contains a clique of order 4, $\chi(M(S_n)) \ge 4$. $4 \le \chi(M(S_n)) \le \chi_=(M(S_n))$, $\chi_=(M(S_n)) \ge 4$. Therefore $\chi_=(M(S_n)) = 4$.

Case(ii): If n is odd.

Case(ii)a:
If $n = 6k-3$ for some positive integer k, then set the partition of V as below. $V_1 = \{v_{3i-2} : 1 \le i \le 2k-1\} \cup \{u_{3i-2} : 1 \le i \le 2k-1\} \cup \{v'_{3i-1} : 1 \le i \le 2k-1\}$; $V_2 = \{v_{3i-1} : 1 \le i \le 2k-1\} \cup \{u_{3i-1} : 1 \le i \le 2k-1\} \cup \{v'_{3i} : 1 \le i \le 2k-1\}$; $V_3 = \{v_{3i} : 1 \le i \le 2k-1\} \cup \{u_{3i} : 1 \le i \le 2k-1\} \cup \{v'_{3i-2} : 1 \le i \le 2k-1\}$; $V_4 = \{u'_i : 1 \le i \le 6k-3\}$. Clearly V_1, V_2, V_3 and V_4 are independent sets of $M(S_n)$. Also $|V_1| = |V_2| = |V_3| = |V_4| = 6k-3$.

Case(ii)b:
If $n = 6k-1$ for some positive integer k, then set the partition of V as below. $V_1 = \{v_{3i} : 1 \le i \le 2k-1\} \cup \{u_{3i-2} : 1 \le i \le 2k\} \cup \{v'_{3i-2} : 1 \le i \le 2k\}$; $V_2 = \{v_{3i+1} : 1 \le i \le 2k-1\} \cup \{u_{3i-1} : 1 \le i \le 2k\} \cup \{v'_{3i-1} : 1 \le i \le 2k\}$; $V_3 = \{v_{3i-1} : 1 \le i \le 2k\} \cup \{v_1\} \cup \{u_{3i} : 1 \le i \le 2k-1\} \cup \{v'_{3i} : 1 \le i \le 2k-1\}$; $V_4 = \{u'_i : 1 \le i \le 6k-1\}$. Clearly V_1, V_2, V_3 and V_4 are independent sets of $M(S_n)$. Also $|V_1| = |V_2| = |V_3| = |V_4| = 6k-1$.

Case(ii)c:
If $n = 6k+1$ for some positive integer k, then set the partition of V as below. $V_1 = \{v_{3i} : 1 \le i \le 2k\} \cup \{u_{3i} : 1 \le i \le 2k\} \cup \{v_{6k+1}\} \cup \{u_{3i} : 1 \le i \le 2k\}$; $V_2 = \{v_{3i+1} : 1 \le i \le 2k-1\} \cup \{u_{3i-2} : 1 \le i \le 2k+1\} \cup \{v'_{3i-1} : 1 \le i \le 2k\} \cup \{v'_{6k+1}\}$; $V_3 = \{v_{3i-1} : 1 \le i \le 2k\} \cup \{u_{3i-1} : 1 \le i \le 2k\} \cup \{v_1\} \cup \{v'_{3i} : 1 \le i \le 2k\}$; $V_4 = \{u'_i : 1 \le i \le 6k+1\}$. Clearly V_1, V_2, V_3 and V_4 are independent sets of $M(S_n)$. Also $|V_1| = |V_2| = |V_3| = |V_4| = 6k+1$.

In all the three subcases of case (ii), V can be partitioned into four independent sets satisfying the relation $||V_i| - |V_j|| \le 1$ for every pair (i, j). $\chi_=(M(S_n)) \le 4$. Since $M(S_n)$ contains a clique of order 4, $\chi(M(S_n)) \ge 4$,

$4 \leq \chi(M(S_n)) \leq \chi_{=}(M(S_n))$, $\chi_{=}(M(S_n)) \geq 4$. Therefore $\chi_{=}(M(S_n)) = 4$. □

Theorem 4.2. *If $n \geq 3$ the equitable chromatic number on total graph of sun let graph $T(S_n)$, $\chi_{=}(T(S_n)) = 4$.*

Proof. Let $V(S_n) = \{v_1, v_2, \cdots, v_n\} \cup \{u_1, u_2, \cdots, u_n\}$ and $E(S_n) = \{e'_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n-1\} \cup \{e_n\}$ where e_i is the edge $v_i v_{i+1}$ $(1 \leq i \leq n-1)$, e_n is the edge $v_n v_1$ and e'_i is the edge $v_i u_i$ $(1 \leq i \leq n)$. By the definition of total graph $V(T(S_n)) = V(S_n) \cup E(S_n) = \{v_i : 1 \leq i \leq n\} \cup \{v'_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\} \cup \{u'_i : 1 \leq i \leq n\}$ where v'_i and u'_i represents the edge e_i and e'_i $(1 \leq i \leq n)$ respectively.

Case(i): If n is even.

Now, we partition the vertex set $V(T(S_n))$ as $V_1 = \{v_{2i-1} : 1 \leq i \leq \frac{n}{2}\} \cup \{u'_{2i} : 1 \leq i \leq \frac{n}{2}\}$; $V_2 = \{v_{2i} : 1 \leq i \leq \frac{n}{2}\} \cup \{u'_{2i-1} : 1 \leq i \leq \frac{n}{2}\}$; $V_3 = \{u_{2i-1} : 1 \leq i \leq \frac{n}{2}\} \cup \{v'_{2i-1} : 1 \leq i \leq \frac{n}{2}\}$; $V_4 = \{u_{2i} : 1 \leq i \leq \frac{n}{2}\} \cup \{v'_{2i} : 1 \leq i \leq \frac{n}{2}\}$. Clearly V_1, V_2, V_3 and V_4 are the independent sets of $T(S_n)$. Also $|V_1| = |V_2| = |V_3| = |V_4| = n$, it holds the inequality $||V_i| - |V_j|| \leq 1$ for every pair (i, j). $\chi_{=}(T(S_n)) \leq 4$. Since $T(S_n)$ contains a clique of order 4, $\chi(T(S_n)) \geq 4$, $4 \leq \chi(T(S_n)) \leq \chi_{=}(T(S_n))$, $\chi_{=}(T(S_n)) \geq 4$. Therefore $\chi_{=}(T(S_n)) = 4$.

Case(ii): If n is odd.

Case(ii)a:
If $n = 6k-3$ for some positive integer k, then set the partition of V as below. $V_1 = \{v_{3i-2} : 1 \leq i \leq 2k-1\} \cup \{v'_{3i-2} : 1 \leq i \leq 2k-1\} \cup \{u_{3i-1} : 1 \leq i \leq 2k-1\}$; $V_2 = \{v_{3i} : 1 \leq i \leq 2k-1\} \cup \{v'_{3i-2} : 1 \leq i \leq 2k-1\} \cup \{u_{3i-2} : 1 \leq i \leq 2k-1\}$; $V_3 = \{v_{3i-1} : 1 \leq i \leq 2k-1\} \cup \{v'_{3i} : 1 \leq i \leq 2k-1\} \cup \{u_{3i} : 1 \leq i \leq 2k-1\}$; $V_4 = \{u'_i : 1 \leq i \leq 6k-3\}$. Clearly V_1, V_2, V_3 and V_4 are independent sets of $T(S_n)$. Also $|V_1| = |V_2| = |V_3| = |V_4| = 6k-3$.

Case(ii)b:
If $n = 6k-1$ for some positive integer k, then set the partition of V as below. $V_1 = \{v_{3i-1} : 1 \leq i \leq 2k-1\} \cup \{v'_{3i} : 1 \leq i \leq 2k-1\} \cup \{v'_{6k-1}\} \cup \{u_{3i-2} : 1 \leq i \leq 2k\}$; $V_2 = \{v_{3i} : 1 \leq i \leq 2k-1\} \cup \{u_{3i-1} : 1 \leq i \leq 2k\} \cup \{v'_{3i-2} : 1 \leq i \leq 2k\}$; $V_3 = \{v_{3i-2} : 1 \leq i \leq 2k\} \cup \{u_{3i} : 1 \leq i \leq 2k-1\} \cup \{v'_{3i-1} : 1 \leq i \leq 2k-1\} \cup \{u'_{6k-1}\}$; $V_4 = \{u'_i : 1 \leq i \leq 6k-2\} \cup \{v_{6k-1}\}$. Clearly V_1, V_2, V_3 and V_4 are independent sets of $T(S_n)$. Also $|V_1| = |V_2| = |V_3| = |V_4| = 6k-1$.

Case(ii)c:
If $n = 6k+1$ for some positive integer k, then set the partition of V as below. $V_1 = \{v_{4i-3} : 1 \leq i \leq 2k\} \cup \{u_{4i} : 1 \leq i \leq 2k-1\} \cup \{u'_{4i-1} : 1 \leq i \leq 2k\} \cup \{v'_{4i-3} : 1 \leq i \leq 2k\}$; $V_2 = \{v_{4i-3} : 1 \leq i \leq 2k\} \cup \{u_{4i-2} : 1 \leq i \leq 2k\} \cup \{u'_{4i} : 1 \leq i \leq 2k-1\} \cup \{v'_{4i-2} : 1 \leq i \leq 2k\}$; $V_3 = \{v_{4i-2} : 1 \leq i \leq$

$2k\} \cup \{u_{4i-1} : 1 \le i \le 2k\} \cup \{v'_{4i-1} : 1 \le i \le 2k\} \cup \{u'_{4i+1} : 1 \le i \le 2k-1\}$; $V_4 = \{v_{4i-1} : 1 \le i \le 2k\} \cup \{u_{4i} : 1 \le i \le 2k-1\} \cup \{v'_{4i} : 1 \le i \le 2k-1\} \cup \{u'_{4i-2} : 1 \le i \le 2k\} \cup \{u'_1\}$. Clearly V_1, V_2, V_3 and V_4 are independent sets of $T(S_n)$. Also $|V_1| = |V_2| = |V_3| = |V_4| = 8k-1$.

In all three subcases of case (ii), V can be partitioned into four independent set satisfying the relation $||V_i| - |V_j|| \le 1$ for every pair (i,j). $\chi_=(T(S_n)) \le 4$. Since $T(S_n)$ contains a clique of order 4, $\chi(T(S_n)) \ge 4$, $4 \le \chi(T(S_n)) \le \chi_=(T(S_n))$, $\chi_=(T(S_n)) \ge 4$. Therefore $\chi_=(T(S_n)) = 4$. □

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