Super Connectivity and Super Edge-connectivity of Transformation Graphs G^{+-+} *

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Abstract: For a graph G=(V(G), E(G)), the transformation graphs G^{+-+} is the graph with vertex set $V(G) \cup E(G)$ in which the vertices α and β are joined by an edge if and only if α and β are adjacent or incident in G while $\{\alpha, \beta\} \nsubseteq E(G)$, or α and β are not adjacent in G while $\{\alpha, \beta\} \subseteq E(G)$. In this note, we show that all but for a few exceptions, G^{+-+} is super connected and super edge-connected.

Key words: transformation graphs, super connectivity, super edge-connectivity

1 Introduction

With the rapid development of communication networks, many theoretical problems have come into focus, one of which is the reliability of the network. A network is often modeled as a graph. The classical measure of the reliability is the connectivity and the edge-connectivity. For further study, many variations have been introduced, which are known as

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higher connectedness, such as max-λ(max-κ),super-λ (super-κ) k-restricted edge-connectivity, etc. For classical connectivity, it is known that $\kappa(G) \leq \lambda(G) \leq \delta(G)$, where $\kappa(G)$, $\lambda(G)$ and $\delta(G)$ denote the connectivity, edge-connectivity and minimum degree of G, respectively. A graph G is said to have maximal connectivity (edge-connectivity) if $\kappa(G) = \delta(G)$ ($\lambda(G) = \delta(G)$). Furthermore, a graph G is said to have super connectivity (edge-connectivity) if every minimum vertex(edge) cut set is the neighbor vertex(edge) set of a vertex with the minimum degree.

In designing networks with high reiability, some graph operations, such as total graphs, line graphs and jump graphs, is an effiecient way. Inspired by the above operations, Wu and Meng defined the following transformation graphs[1].

Definition 1.1 Let $G = (V(G), E(G))$ be a graph, and x, y, z be three variables taking values + or -. The transformation graph G^{xyz} is the graph having $V(G) \cup E(G)$ as the vertex set, and for $\alpha, \beta \in V(G) \cup E(G)$, α and β are adjacent in G^{xyz} if and only if one of the following holds:

(1)$\alpha, \beta \in V(G)$, α and β are adjacent in G if x=+; and are not adjacent in G if x=-.

(2)$\alpha, \beta \in E(G)$, α and β are adjacent in G if y=+; and are not adjacent in G if y=-.

(3)$\alpha \in V(G), \beta \in E(G)$, α and β are incident in G if z=+; and are not incident in G if z=-.

Thus, as defined above, there are eight kinds of transformation graphs, among which G^{+++} is usually known as the total graph of G. In [1], the authors have proved that:

theorem 1.2: For a given graph G, G^{+-+} is connected if and only if G contains no isolated vertices.

In [6], Zhang and Huang charactered the maximally connected transformation graph G^{+-+}.

Theorem 1.3: For a given graph G, $\kappa(G^{+-+}) = \delta(G^{+-+})$ if and only if none of the following three conditions apply:

(1)G has at least two components, one of which is K_2, and $\delta(G) \geq 1$.

(2)G has at least two components, one of which is K_3, and $\delta(G) \geq 2$.

(3)$G \cong K_{1,n}$.

Corollary 1.4: If $G \ncong 2K_2$, then $\lambda(G^{+-+}) = \delta(G^{+-+})$.

In this paper, we study the super connectivity and super edge-connectivity of G^{+-+}.

2 Some Basics

Throughout the paper we only consider finite and simple graphs. Undefined symbols and concepts can be found in [5].

Let G=(V(G), E(G)) be a graph. $|V(G)|$ and $|E(G)|$ are called the order and the size of G, respectively. For a vertex v of G, $d_G(v)$ denotes the degree of v; $I_G(v)$ denotes the set of all edges incident with v. The neighborhood $N_G(v)$ of v is the set of all vertices of G adjacent to v. The symbols $\Delta(G), \delta(G), \lambda(G), \kappa(G), \omega(G), \alpha(G)$ denote the maximum degree, the minimum degree, the edge-connectivity, the connectivity, the number of components, and the independence number of G, respectively. For any $X \subseteq V(G)$, G[X] denotes the subgraph of G induced by X. For any $X, Y \subseteq V(G)$ such that $X \cap Y = \emptyset$, let $[X, Y] = \{e : e = uv \in E(G), u \in X, v \in Y\}$.

As usual, P_n, C_n, K_n are respectively, the path, cycle, and complete graph of order n. For two positive integers m and n, $K_{m,n}$ is the complete bipartite graph with two partite sets containing m and n vertices. In particular, $K_{1,n}$ is called a star. The graph $K_{1,n}^{+}$ is obtained by $K_{1,n}$ adding one edge. The graph $K_{1,n}^{*}$ is obtained by adding only one edge between $K_{1,n}$ and K_1. We say two graphs G and H are disjoint if they have no vertex in common, and denote their union by $G \cup H$.

A jump graph J(G) is a graph whose vertices are the edges of G, and two vertices of J(G) are adjacent if and only if the corresponding edges of G are independent. Clearly, G^{+-+} contains J(G) as a subgraph. Concerning the connectivity of J(G), in [2], Wu and Meng proved the following two theorems:

Theorem 2.1: Let G be any graph, J(G) is connected if and only if G contains no edges that is adjacent to every other edge of G and $G \ncong C_4$ or K_4.

Theorem 2.2: Let G be a graph having a connected jump graph, then $\kappa(J(G)) \geq q - \Delta(e) = \delta(J(G)) - 1$.

Let p and q denote the order and size of G, respectively. For any $v \in V(G)$, $d_{G^{+-+}}(v) = 2d_G(v)$, and for any $e = xy \in E(G)$, $d_{G^{+-+}}(e) = q+3-(d_G(x)+d_G(y))$. Thus, $\delta(G^{+-+}) = \min\{2\delta(G), q+3-\Delta(e)\}$, where $\Delta(e) = \max\{d_G(x) + d_G(y) : e = xy \in E(G)\}$.

3 Super Connectivity of G^{+-+}

In this section, we study the super connectivity of G^{+-+}. We will prove that:

Theorem 3.1: For a given graph G which contains no isolated vertices, G^{+-+} is super connected if and only if none of the following conditions holds:

(1)$\delta(G) = 1$, G has at least tow components and one of which is K_2 or $K_{1,2}$.

(2)$\delta(G)=2$, G has at least tow components and one of which is C_4 or K_3.
(3)$\delta(G)=2$, G has a component G_0 which has cut vertex v such that one component of G_0-v is K_2.
(4)$\delta(G)=3$, G has at least tow components and one of which is K_4.
(5)$G\cong K_{1,p-1}(p\geq 3)$, $K^{+}_{1,p-1}(p\geq 4)$, $K^{*}_{1,p-1}(p\geq 4)$.

Proof: Since G^{+-+} has the graph G as a subgraph, it is easy to see that if conditions (1),(2) or (4) apply, there exists a minimum vertex cut T such that one component of $G^{+-+}\backslash T$ is $K_2, K_{1,2}, C_4, K_3$ or K_4 which is also a component of graph G, respectively. Clearly, $\kappa(G^{+-+})\leq\delta(G^{+-+})$ and G^{+-+} is not super connected. If condition (3) applies, there exists a minimum vertex cut T such that one component of $G^{+-+}\backslash T$ is K_2 which is also a component of $G_0\setminus\{v\}$ and $\kappa(G^{+-+})=\delta(G^{+-+})$, G^{+-+} is not super connected. If condition (5) applies, $\kappa(G^{+-+})\leq\delta(G^{+-+})$, G^{+-+} is not super connected. Thus, the necessity is proved. Now we prove the sufficiency.

Suppose that T is a minimum vertex cut of G^{+-+}, then $G^{+-+}\setminus T$ is not connected. If one component of $G^{+-+}\setminus T$ has only one vertex, then the result holds. Hence, in the following, we assume that each component has at least two vertices. Clearly, It suffices to show that $|T|>\delta(G^{+-+})$.

Case 1: There exists a component G_0 in $G^{+-+}\backslash T$, such that $V(G_0)\subseteq V(G)$.

Subcase 1.1: $\alpha(G_0)\geq 3$.

Suppose x,y and z are any three independent vertices in G_0, then $N_{G^{+-+}}(x)\cap E(G), N_{G^{+-+}}(y)\cap E(G)$ and $N_{G^{+-+}}(z)\cap E(G)$ are pair-wise disjoint. Thus

$$\begin{aligned}|T|&\geq|N_{G^{+-+}}(x)\cap E(G)|+|N_{G^{+-+}}(y)\cap E(G)|+|N_{G^{+-+}}(z)\cap E(G)|\\&=d_G(x)+d_G(y)+d_G(z)\geq 3\delta(G)>\delta(G^{+-+}).\end{aligned}$$

Subcase 1.2: $\alpha(G_0) = 2$.

Suppose x and y are any two independent vertices in G_0. If there exists a vertex z, such that $z \in (N_{G^{+-+}}(x) \cap V(G)) \cup (N_{G^{+-+}}(y) \cap V(G))$ and $z \notin V(G_0)$, then

$$|T| \geq |N_{G^{+-+}}(x) \cap E(G)| + |N_{G^{+-+}}(y) \cap E(G)| + 1$$
$$= d_G(x) + d_G(y) + 1 \geq 2\delta(G) + 1 > \delta(G^{+-+}).$$

Otherwise, we have $V(G_0) = \{x, y\} \cup (N_{G^{+-+}}(x) \cap V(G)) \cup (N_{G^{+-+}}(y) \cap V(G))$.

Claim 1.2.1: For any $u, v \in V(G_0) \backslash \{x, y\}$, u and v are not adjacent in G.

If $e = uv \in E(G)$, then $e \in T$ and $e \notin (N_{G^{+-+}}(x) \cap E(G)) \cup (N_{G^{+-+}}(y) \cap E(G))$, thus
$|T| \geq |N_{G^{+-+}}(x) \cap E(G)| + |N_{G^{+-+}}(y) \cap E(G)| + 1 > \delta(G^{+-+})$.

Claim 1.2.2: $|V(G_0) \setminus \{x, y\}|$ =1 or 2.

Since G_0 is connected, $|V(G_0) \setminus \{x, y\}| \geq 1$. If $|V(G_0) \setminus \{x, y\}| \geq 3$, then there are at least three independent vertices in G_0, a contradiction.

By the above two claims, we see that G_0 is isomorphic to one of $\{C_4, P_3, K_{1,2}\}$. It is easy to see that: If $G_0 \cong C_4$ and $\delta(G) = 2$, then $|T| = 4 = \delta(G^{+-+})$, thus conditions (2) applies, a contradiction. If $G_0 \cong P_3$ and $\delta(G) = 1$, then $|T| = 3 > 2\delta(G)$. If $G_0 \cong K_{1,2}$ and $\delta(G) = 1$, then $|T| = 2 = \delta(G^{+-+})$, thus conditions (1) applies, a contradiction.

Subcase 1.3: $\alpha(G_0) = 1$.

That is, G_0 is a complete subgraph. Let $|V(G_0)| = m$, then

$$|T| \geq \frac{m(m-1)}{2} \geq \frac{(\delta(G)+1)\delta(G)}{2}.$$

Since $\frac{(\delta(G)+1)\delta(G)}{2} - 2\delta(G) = \frac{\delta(G)(\delta(G)-3)}{2}$, we have $|T| > 2\delta(G) \geq$

$\delta(G^{+-+})$ when $\delta(G) \geq 4 (m \geq 5)$. It is suffices to consider the cases in which m=2,3, or 4. Let $N(G_0) = \{u : e = uv \in E(G), v \in V(G_0), u \in V(G) \setminus V(G_0)\}$.

Subcase 1.3.1: m=2.

Then $G_0 \cong K_2$, and $|T| = d_G(x) + d_G(y) - 1 + |N(G_0)|$.

If $|N(G_0)| \geq 2$, then $|T| \geq 2\delta(G) + 1 > \delta(G^{+-+})$.

If $|N(G_0)| = \delta(G) = 1$, then $|T| \geq 3 \geq 2\delta(G) = 2$.

If $|N(G_0)| = 1$ and $\delta(G) = 2$, then $|T| = 4 = \delta(G^{+-+})$ only if conditions (3) applies, a contradiction.

If $|N(G_0)| = 0$, then $|T| = 1 < 2\delta(G) = 2$ only if conditions (3) applies, a contradiction.

Subcase 1.3.2: m=3.

That is, $G_0 \cong K_3$. Let $V(G_0) = \{x, y, z\}$, then

$|T| = d_G(x) + d_G(y) + d_G(z) - 3 + |N(G_0)|$.

If $|N(G_0)| \geq 2$, then $|T| \geq 2\delta(G) + 1 > \delta(G^{+-+})$.

If $|N(G_0)| = 1$, then $\delta(G) \leq 3$. It is easy to verify that $|T| > \delta(G^{+-+})$.

If $|N(G_0)| = 0$, then $|T| = 3 < 2\delta(G) = 4$ only if conditions (2) applies, a contradiction.

Subcase 1.3.3: m=4.

Then $G_0 \cong K_4$, and $|T| = \sum d_G(v) - 6 + |N(G_0)| \geq 2\delta(G) + |N(G_0)|$.

If $|N(G_0)| \geq 1$, then $|T| \geq 2\delta(G) + 1 > \delta(G^{+-+})$.

If $|N(G_0)| = 0$, then $|T| = 6$ and $\delta(G) \leq 3$. If $\delta(G) \leq 2$, then $|T| > \delta(G^{+-+})$. If $\delta(G) = 3$, $|T| = 6 = \delta(G^{+-+})$ only if conditions (4) applies, a contradiction.

Case 2: Each component of $G^{+-+} \setminus T$ has at least one vertex of E(G) and J(G) is connected.

In this case, there exists a set $T_E \subseteq V(J(G)) \cap T$, such that $J(G) \setminus T_E$ is disconnected. By Theorem 2.2, $|T_E| \geq q - \Delta(e)$. Let e, f be any two vertices, with the restriction that they lie in different components of $G^{+-+} \setminus T$, since e and f are not adjacent in G^{+-+}, they must have a common end vertex, say u, in G. Clearly, $u \in T$, and $T \cap V(G) \neq \emptyset$. Let $T_V = T \cap V(G)$. Without loss generality, suppose $e = uv \in V(G_1)$, $f = uw \in V(G_2)$ where G_1 and G_2 are two different components of $G^{+-+} \setminus T$. If $|T| \leq \delta(G^{+-+})$, we have the following claims.

Claim 2.1: $1 \leq |T_V| \leq 3$.

If $|T_V| \geq 4$, then $|T| = |T_E| + |T_V| \geq q - \Delta(e) + 4 > d_{G^{+-+}}(e) \geq \delta(G^{+-+})$, a contradiction.

Claim 2.2: $|T_V| = 1$.

If $|T_V| = 2, 3$, then $|T| = |T_E| + |T_V| \geq 3$. $|T| \leq \delta(G^{+-+})$ only if $\delta(G) \geq 2$.

If $v, w \notin T_V$, then $N_{J(G)}(e) \subseteq T$. Otherwise, suppose $g \in N_{J(G)}(e)$ and $g \notin T$, then $g \in V(G_1)$, u is also the common end vertex of g and f in G. Thus $g \notin N_{J(G)}(e)$, a contradiction.

Since $d_G(v) \geq 2$, there exists one edge h such that $h \notin N_{J(G)}(e)$ and $h \in T$, then $|T| = |N_{J(G)}(e)| + |T_V| + 1 \geq \delta(J(G)) + 3 > \delta(G^{+-+})$.

If $v \in T_V, w \notin T_V$ or $w \in T_V, v \notin T_V$, without loss of generality, we only consider $v \in T_V, w \notin T_V$. Since G_1 is connected, there exists one edge h in G_1 such that h and e are adjacent in G^{+-+}, so u is not incident with h. On the other hand, h and f are not adjacent in G^{+-+}, they have common end vertex u, a contradiction.

If $v, w \in T_V$, then $T_V = \{u, v, w\}$. Since G_1 is connected, there exist one edge g in G_1 such that g and e are adjacent in G^{+-+}, so u and v are not incident with g. As g and f are not adjacent in G^{+-+}, they have common end vertex which must be w. On the other hand, since G_2 is connected, there exist one edge h in G_2 such that h and f are adjacent in G^{+-+}, so u and w are not end vertex of edge h. Because h and e are not adjacent in G^{+-+}, they have common end vertex which must be v. But g and h are not adjacent in G^{+-+}, they have common end vertex which must be in T, a contradiction.

Claim 2.3: $N_{J(G)}(e) \subseteq T$.

If $g \in N_{J(G)}(e)$ and $g \notin T$, then $g \in V(G_1)$. On the other hand, u is also the common end vertex of g and f in G. We have $g \notin N_{J(G)}(e)$, a contradiction.

Claim 2.4: $d_G(v) \leq 2$.

If $d_G(v) \geq 3$, there exists $g = vx, h = vy$ such that $\{g, h\} \in T$ and $\{g, h\} \notin N_{J(G)}(e)$. Thus $|T| \geq |N_{J(G)}(e)| + |\{u, g, h\}| > d_{G^{+-+}}(e) \geq \delta(G^{+-+})$, a contradiction.

By Claim 2.4, we have that $\delta(G) \leq 2$.

Subcase 2.1: $\delta(G) = 1$.

Then, $\delta(G^{+-+}) = 2$. Since J(G) is connected, $|T_E| \geq 1$. If $|T_E| \geq 2$, then $|T| \geq 3 > \delta(G^{+-+})$, a contradiction. If $|T_E| = 1$, then $|T| = 2 = \delta(G^{+-+})$ only if condition (6) applies, a contradiction.

Subcase 2.2: $\delta(G) = 2$.

Let $V_i = V(G_i) \cap V(G)$, i=1,2. By Claim 2.4, $d_G(v) = 2$. Thus, there exists a vertex x_1 in $V(G_1)$ such that v and x_1 are incident with an edge, say g_1, in T. Thus, $E(G[V_i]) \subseteq T$ for i=1,2 and $|T| \leq \delta(G^{+-+})$ only if $|E(G[V_1])| + |E(G[V_2])| \leq 3$, so $G[V_1]$ or $G[V_2]$ is K_2, condition (3) applies, a contradiction.

Case 3: J(G) is disconnected.

By Theorem 1.1, $G \cong C_4$ or K_4, or there exists an edge adjacent to every other edges in G. It is easy to verify that among these graphs, G^{+-+} is not super edge-connected only if condition (5) applies.

4 Super Edge-connectivity of G^{+-+}

In this section, we study the super edge-connectivity of G^{+-+}. Our main result of this section is the following:

Theorem 4.1: Let G be a graph not containing no isolated vertices. Then G^{+-+} is super edge-connected if and only if G has no isolated edges and $G \ncong K_{1,n}$.

Proof: The necessity is self-evident, we now prove the sufficiency.

Case 1: J(G) is disconnected.

By Theorem 1.1, $G \cong C_4$ or K_4, or there exists an edge adjacent to every other edges in G. It is easy to verify that among these graphs, G^{+-+} is not super edge-connected only if G is $K_{1,p-1}$.

Case 2: J(G) is connected.

Let S be a minimum edge cut set of G^{+-+}. Then $G^{+-+} - S$ has exactly two components, say G_1 and G_2. If one component has one vertex, then the result holds. In the following, we show that if each component has at least two vertices, then that $|S| > \delta(G^{+-+})$.

Subcase 2.1: One component contains E(G).

Without loss of generality, we assume that $E(G) \subseteq V(G_1)$. Let $|V(G_2)| = p_2$. Since for any $v \in V(G_2)$, v is incident with $d_G(v)$ edges in G. We have $|S| \geq \sum_{v \in V(G_2)} d_G(v) \geq p_2 \cdot \delta(G)$.

If $p_2 \geq 3$, $|S| > 2\delta(G) \geq \delta(G^{+-+})$.

For $p_2 = 2$, if $p = 2$, then $G \cong K_2$, $G^{+-+} \cong K_3$ is super edge-connected. If $p \geq 3$, then $|S| = 2\delta(G)$ only if $G \cong K_2 \cup H$ where graph H contains no isolated vertices.

Subcase 2.2: One component contains V(G).

Without loss of generality, we assume that $V(G) \subseteq V(G_1)$. Since G_2

is connected and $|V(G_2)| \geq 2$, there exists $e_i = u_i v_i \in V(G_2)$, i=1,2, such that e_1 and e_2 are adjacent in G^{+-+}. If $V(G) = V(G_1)$, then $|S| \geq 2q = \sum_{v \in V(G)} d_G(v) \geq 4\delta(G) > \delta(G^{+-+})$. So we have $E_1 = V(G_1) \cap E(G) \neq \emptyset$. Let $I_G(v_i) \cap E_1 = m_i$, i=1,2. Then $d_G(v_i) - 1 \geq m_i \geq 0$ and e_i is adjacent with $|E_1| - m_i$ vertices of E_1 in G^{+-+} for $1 \leq i \geq 2$. Thus

$$
\begin{aligned}
|S| &\geq 2|V(G_2)| + (|E_1| - m_1) + (|E_1| - m_2) = 2q - (m_1 + m_2) \\
&\geq \sum_{v \in V(G)} d_G(v) - ((d_G(v_1) - 1) + (d_G(v_2) - 1)) \\
&\geq \delta(G) + 2 > \delta(G^{+-+}).
\end{aligned}
$$

Subcase 2.3: $V_i = V(G) \cap V(G_i) \neq \emptyset$ and $E_i = E(G) \cap V(G_i) \neq \emptyset$, $i = 1, 2$.

Since J(G) is connected, there exists a subset $S_J \subseteq E(J(G)) \cap S$, such that $J(G) - S_J$ is disconnected. By Theorem 3.2, $|S_J| \geq q - \Delta(e)$.

Subcase 2.3.1: $|[V_1, V_2]| \geq 2$.

That is, there are at least two edges between V_1 and V_2 in G^{+-+}, which contribute at least four edges to S. Thus

$$|S| \geq |S_J| + 4 > d_{G^{+-+}}(e) \geq \delta(G^{+-+}).$$

Subcase 2.3.2: $|[V_1, V_2]| = 0$.

That is, $E_i \subseteq G[V_i]$, i=1,2, and for any $e \in E_1$, $f \in E_2$, e and f is adjacent in G^{+-+}. Thus

$$|S| = |E_1| \cdot |E_2| = |E_1| \cdot (q - |E_1|) \geq q - 1 = q + 3 - \Delta(e) + (\Delta(e) - 4).$$

If $\Delta(e) \geq 5$, then $|S| > \delta(G^{+-+})$.

If $2 \leq \Delta(e) \leq 4$ then $\delta(G) = 1$ or 2. For $\delta(G) = 1$, $\delta(G^{+-+}) = 2$, and $|S| = \delta(G^{+-+}) = 2$ only if $G \cong K_2 \cup H$ where H has no isolated vertices. For $\delta(G) = 2$, we have $\Delta(e) = 4$. Thus G is the union of cycles and $q \geq 6$, and $\delta(G^{+-+}) = \min\{4, q - 1\} = 4$. Hence $|S| \geq 5 > \delta(G^{+-+})$.

Subcase 2.3.3: $|[V_1, V_2]| = 1$.

Let $e = uv$ be the unique one edge between V_1 and V_2, and without loss generality, we assume $e \in E_1$. If $|E_1| = 1$, then $|S| \geq \delta(J(G)) + 2 =$

$q+3-\Delta(e) \geq \delta(G^{+-+})$ and the equality holds only if G is the star, contradicting the assumption that J(G) is connected. So we have $|E_1| \geq 2$, and for any $g \in E_1 \setminus \{e\}$, $f \in E_2$, g and f are adjacent in G^{+-+}. Thus

$$|S| \geq (|E_1|-1)\cdot|E_2|+2 = (|E_1|-1)\cdot(q-|E_1|)+2$$
$$\geq (q-2)+2 = q+3-\Delta(e)+(\Delta(e)-3).$$

If $\Delta(e) \geq 4$, $|S| > \delta(G^{+-+})$.

If $2 \leq \Delta(e) \leq 3$, then $\delta(G)=1$, $\delta(G^{+-+})=2$, and $|S| > \delta(G^{+-+})$ for $q \geq 3$. It is easy to see that for $q \leq 2$, G^{+-+} is not super edge-connected only if G is $2K_2, K_{1,2}$.

Thus, we complete the proof of the Theorem 4.1. Clearly, we can contain the Corollary 1.4.

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