On super (a, d)-edge-antimagic total labeling of subdivided stars

M. Javaid*, A. A Bhatti*

Department of Mathematics
National University of Computer and Emerging
Sciences Lahore Campus, Pakistan.
javaidmath@{*gmail*, *yahoo*}.com,
akhlaq.ahmad@nu.edu.pk

Abstract. Let $G = (V, E)$ be a graph with $v = |V(G)|$ vertices and $e = |E(G)|$ edges. An (a, d)-*edge-antimagic total labeling* of the graph G is a one-to-one map λ from $V(G) \cup E(G)$ onto the integers $\{1, 2, \cdots, v + e\}$ such that the set of edge weights of the graph G, $W = \{w(xy) : xy \epsilon E(G)\}$ form an arithmetic progression with the initial term a and common difference d, where $w(xy) = \lambda(x) + \lambda(y) + \lambda(xy)$ for any $xy \in E(G)$. If $\lambda(V(G)) = \{1, 2, \cdots, v\}$ then G is *super* (a, d)-*edge-antimagic total* i.e ((a, d)-EAT). In this paper, for different value of d, we formulate super (a, d)-edge-antimagic total labeling on subdivision of stars $K_{1,p}$ for $p \geq 5$.

Keywords : Super (a, d)*-edge-antimagic total labeling, subdivision of star.*

1 Introduction

All graphs in this paper are finite, simple, planar and undirected. The graph G has the vertex-set $V(G)$ and edge-set $E(G)$. A general reference for graph-theoretic ideas can be seen in [4]. A *labeling* (or *valuation*) of a graph is a map that carries graph elements to numbers (usually to positive or non-negative integers). In this paper the domain will be the set of all vertices and edges and such labeling is called *total labeling*. Some labelings use the vertex-set only, or the edge-set only, and we shall call them *vertex-labelings* and *edge-labelings* respectively. The most complete recent survey of graph labelings can be seen in [8]. In this paper, we formulate super (a, d)-edge-antimagic total labeling on subdivided stars.

* The research contents of this paper is partially supported by the Higher Education Commission (HEC) of Pakistan and National University of Computer and Emerging Sciences (NUCES) Lahore Campus, Pakistan.

A graph G is called *(a, d)-edge-antimagic total* if there exist integers $a > 0$, $d \geq 0$ and a bijection $\lambda : V \cup E \rightarrow \{1, 2, ..., v + e\}$ such that $W = \{w(xy) : xy \in E\}$ forms an arithmetic progression starting from a with the difference d, where $w(xy) = \lambda(x) + \lambda(y) + \lambda(xy)$ for any $xy \in E$. W is called the set of *edge-weights* of the graph G. Additionally, if $\lambda(V) = \{1, 2, ..., v\}$ then G is super (a, d)-EAT. A number of classification studies on edge-antimagic total graphs has been intensively investigated. For further detail see a recent survey of graph labelings [8]. The subject of edge-magic total labeling of graphs has its origin in the work of Kotzig and Rosa [1, 2], on what they called magic valuations of graphs. The notion of super edge-magic total labeling was introduced by Enomoto et al. [7] and they proposed following conjecture:

Conjecture 1. *Every tree admits a super edge-magic total labeling.*

In the effort of attacking this conjecture, many authors have considered super edge-magic total labeling for some particular classes of trees for example [3, 5, 6, 9–13]. Lee and Shah [15] verified this conjecture by a computer search for trees with at most 17 vertices. Earlier Kotzig and Rosa in [1] proved that every caterpillar is super edge-magic total. However, this conjecture is still open.

A star is a particular type of tree. Super edge-magic total labeling for subdivision of star $K_{1,3}$ was studied by Baskoro et al. [6]. In [10] Javaid et al. furnished super edge-magic total labeling on subdivision of $K_{1,4}$ and w-tree. In [11] Javaid et al. proved super edge-magic total labeling on subdivision of $K_{1,p}$ for $p \geq 5$. Some of their results are presented in the following theorems.

Theorem A. For any odd $n \geq 3$, $G \cong T(n, n, n-1, n, 2n-1)$ admits super edge-magic total labeling with magic constant $a = 15n$. □

Theorem B. For any odd $n \geq 3$, $G \cong T(n, n, n-1, n, 2n-1, 4n-3)$ admits super edge-magic total labeling with magic constant $a = 25n - 7$. □

Theorem C. For any odd $n \geq 3$ and $p \geq 5$, $G \cong T(n, n, n-1, n, n_5, ..., n_p)$ admits super edge-magic total labeling, where $n_p = n + \frac{(n-1)(p-3)(p-4)}{2}$. □

However, super edge-magic total labeling of $G \cong T(n_1, n_2, n_3, ..., n_p)$ for different n_p is still open. In this paper we fond super (a, d)-edge antimagic total labelings on subdivision of star $K_{1,p}$ for $p \geq 5$ with $n_p = 1 + (n + 1)2^{p-4}$.

2 Main Results

For $n_i \geq 1$ and $p \geq 5$, let $G \cong T(n_1, n_2, ..., n_p)$ be a graph obtained by inserting $n_i - 1$ vertices to each of the $i-$th edge of the star $K_{1,p}$, where $1 \leq i \leq p$. Thus, the graph $T\underbrace{(1, 1, ..., 1)}_{p-time}$ is a star $K_{1,p}$.

Before giving our main results, let us consider the following lemma found in [14] that gives a necessary and sufficient condition for a graph to be super edge-magic total.

Lemma 1. *A graph G with v vertices and e edges is super edge-magic total if and only if there exists a bijective function $\lambda : V(G) \rightarrow \{1, 2, \cdots, v\}$ such that the set of edge-sums $S = \{\lambda(x) + \lambda(y) | xy \in E(G)\}$ consists of e consecutive integers. In such a case, λ extends to a super edge-magic total labeling of G with magic constant $a = v + e + s$, where $s = min(S)$ and*

$$\begin{aligned} S &= \{\lambda(x) + \lambda(y) | xy \in E(G)\} \\ &= \{a - (v+1), a - (v+2), \cdots, a - (v+e)\}. \end{aligned}$$

Theorem 1. *For any odd $n \geq 3$, $G \cong T(n, n, n+2, n+2, 2n+3)$ admits super $(a, 0)$-edge-antimagic total labeling with $a = 2v + s - 1$ and super $(a, 2)$-edge-antimagic total labeling with $a = v + s + 1$, where $v = |V(G)|$ and $s = 3n + 8$.*

Proof. Let us denote the vertices and edges of G, as follows:
$V(G) = \{c\} \cup \{x_i^{l_i} \mid 1 \leq i \leq 5 \ ; \ 1 \leq l_i \leq n_i\}$,
$E(G) = \{c\, x_i^1 \mid 1 \leq i \leq 5\} \cup \{x_i^{l_i} x_i^{l_i+1} \mid 1 \leq i \leq 5 \ ; \ 1 \leq l_i \leq n_i - 1\}$.
If $v = |V(G)|$ and $e = |E(G)|$, then $v = 6n + 8$, and $e = 6n + 7$.
Now, we define the labeling $\lambda : V \rightarrow \{1, 2, ..., v\}$ as follows:

$$\lambda(c) = 4n + 6.$$

For $l_i = 1, 3, 5, ..., n_i$,

$$\lambda(u) = \begin{cases} \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ (n+1) - \frac{l_2-1}{2}, & \text{for } u = x_2^{l_2}, \\ (n+2) + \frac{l_3-1}{2}, & \text{for } u = x_3^{l_3}, \\ 2(n+2) - \frac{l_4-1}{2}, & \text{for } u = x_4^{l_4}, \\ 3(n+2) - \frac{l_5-1}{2}, & \text{for } u = x_5^{l_5}. \end{cases}$$

For $l_i = 2, 4, 6, ..., n_i - 1$,

$$\lambda(u) = \begin{cases} (3n+7) + \frac{l_1-2}{2}, & \text{for } u = x_1^{l_1}, \\ (4n+5) - \frac{l_2-2}{2}, & \text{for } u = x_2^{l_2}, \\ (4n+7) + \frac{l_3-2}{2}, & \text{for } u = x_3^{l_3}, \\ (5n+7) - \frac{l_4-2}{2}, & \text{for } u = x_4^{l_4}, \\ (6n+8) - \frac{l_5-2}{2}, & \text{for } u = x_5^{l_5}. \end{cases}$$

The set of all edge-sums generated by the above formula forms a consecutive integer sequence $s = (3n+7)+1, (3n+7)+2, \cdots, (3n+7)+e$. Therefore, by Lemma 1, λ can be extended to a super (a,0)-edge-antimagic total labeling and we obtain the magic constant $a = v+e+s = 15n+23$. Similarly, λ can be extended to a super (a,2)-edge-antimagic total labeling and we obtain the magic constant $a = v+1+s = 9n+17$. □

Theorem 2. *For any odd $n \geq 3$, $G \cong T(n, n, n+2, n+2, 2n+3)$ admits super $(a,1)$-edge-antimagic total labeling with $a = s + \frac{3(v)}{2}$, where $v = |V(G)|$ and $s = 3n+8$.*

Proof. Let us denote the vertices and edges of G, as follows:
$V(G) = \{c\} \cup \{x_i^{l_i} \mid 1 \leq i \leq 5 \ ; \ 1 \leq l_i \leq n_i\}$,
$E(G) = \{c\, x_i^1 \mid 1 \leq i \leq 5\} \cup \{x_i^{l_i} x_i^{l_i+1} \mid 1 \leq i \leq 5 \ ; \ 1 \leq l_i \leq n_i - 1\}$.
If $v = |V(G)|$ and $e = |E(G)|$ then $v = 6n+8$ and $e = 6n+7$. Now, we define the labeling $\lambda : V \to \{1, 2, ..., v\}$ as in Theorem 1. It follows that the edge-sums of all edges of G constitute an arithmetic sequence $(3n+7)+1, (3n+7)+2, \cdots, (3n+7)+e$, with common difference 1. We denote it by $A = \{a_i; 1 \leq i \leq e\}$. Now for G we complete the edge labeling λ for super $(a,1)$-edge antimagic total labeling with values in the arithmetic sequence $v+1$, $v+2$, $\cdots$,$v+e$ with common difference 1. Let us denote it by $B = \{b_j \ ; \ 1 \leq j \leq e\}$. Define $C = \{a_{2i-1} + b_{e-i+1} \ ; \ 1 \leq i \leq \frac{e+1}{2}\} \cup \{a_{2j} + b_{\frac{e-1}{2}-j+1} \ ; \ 1 \leq j \leq \frac{e+1}{2} - 1\}$. It is easy to see that C constitute an arithmetic sequence with $d = 1$ and $a = s + \frac{3(v)}{2} = 12n+20$.

Since all vertices receive the smallest labels so λ is a super $(12n+20, 1)$-edge-antimagic total labeling. □

Theorem 3. *For any odd $n \geq 3$, $G \cong T(n, n, n+2, n+2, 2n+3, 4n+5)$ admits super $(a,0)$-edge-antimagic total labeling with $a = 2v + s - 1$ and super $(a,2)$-edge-antimagic total labeling with $a = v + s + 1$, where $v = |V(G)|$ and $s = 5n+11$.*

Proof. Let us denote the vertices and edges of G, as follows:

$V(G) = \{c\} \cup \{x_i^{l_i} \mid 1 \le i \le 6 \; ; \; 1 \le l_i \le n_i\}$,

$E(G) = \{c\, x_i^1 \mid 1 \le i \le 6\} \cup \{x_i^{l_i} x_i^{l_i+1} \mid 1 \le i \le 6 \; ; \; 1 \le l_i \le n_i - 1\}$.

If $v = |V(G)|$ and $e = |E(G)|$ then $v = 10n + 13$ and $e = 10n + 12$. Now, we define the labeling $\lambda : V \to \{1, 2, ..., v\}$ as follows:

$$\lambda(c) = 6n + 9.$$

For $l_i = 1, 3, 5, ..., n_i$,

$$\lambda(u) = \begin{cases} \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ (n+1) - \frac{l_2-1}{2}, & \text{for } u = x_2^{l_2}, \\ (n+2) + \frac{l_3-1}{2}, & \text{for } u = x_3^{l_3}, \\ 2(n+2) - \frac{l_4-1}{2}, & \text{for } u = x_4^{l_4}, \\ 3(n+2) - \frac{l_5-1}{2}, & \text{for } u = x_5^{l_5}, \\ (5n+9) - \frac{l_6-1}{2}, & \text{for } u = x_6^{l_6}. \end{cases}$$

For $l_i = 2, 4, 6, ..., n_i - 1$,

$$\lambda(u) = \begin{cases} 5(n+2) + \frac{l_1-2}{2}, & \text{for } u = x_1^{l_1}, \\ 2(3n+4) - \frac{l_2-2}{2}, & \text{for } u = x_2^{l_2}, \\ 2(3n+5) + \frac{l_3-2}{2}, & \text{for } u = x_3^{l_3}, \\ (7n+10) - \frac{l_4-2}{2}, & \text{for } u = x_4^{l_4}, \\ (8n+11) - \frac{l_5-2}{2}, & \text{for } u = x_5^{l_5}, \\ (10n+13) - \frac{l_6-2}{2}, & \text{for } u = x_6^{l_6}. \end{cases}$$

The set of all edge-sums generated by the above formula forms a consecutive integer sequence $s = (5n + 10) + 1, (5n + 10) + 2, \cdots, (5n + 10) + e$. Therefore, by Lemma 1, λ can be extended to a super (a,0)-edge-antimagic total labeling and we obtain the magic constant $a = v + e + s = 25n + 36$. Similarly, λ can be extended to a super (a,2)-edge-antimagic total labeling and we obtain the magic constant $a = v + 1 + s = 15n + 25$. □

Theorem 4. *For any odd* $n \geq 3$, $G \cong T(n, n, n+2, n+2, 2n+3, 4n+5, 8n+9)$ *admits super* $(a, 0)$*-edge-antimagic total labeling with* $a = 2v+s-1$ *and super* $(a, 2)$*-edge-antimagic total labeling with* $a = v+s+1$*, where* $v = |V(G)|$ *and* $s = 9n+16$.

Proof. Let us denote the vertices and edges of G, as follows:

$V(G) = \{c\} \cup \{x_i^{l_i} \mid 1 \leq i \leq 7\ ;\ 1 \leq l_i \leq n_i\}$,

$E(G) = \{c\, x_i^1 \mid 1 \leq i \leq 7\} \cup \{x_i^{l_i} x_i^{l_i+1} \mid 1 \leq i \leq 7\ ;\ 1 \leq l_i \leq n_i - 1\}$. If $v = |V(G)|$ and $e = |E(G)|$ then $v = 18n+22$ and $e = 18n+21$. Now, we define the labeling $\lambda : V \to \{1, 2, ..., v\}$ as follows:

$$\lambda(c) = 10n + 14.$$

For $l_i = 1, 3, 5, ..., n_i$,

$$\lambda(u) = \begin{cases} \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ (n+1) - \frac{l_2-1}{2}, & \text{for } u = x_2^{l_2}, \\ (n+2) + \frac{l_3-1}{2}, & \text{for } u = x_3^{l_3}, \\ 2(n+2) - \frac{l_4-1}{2}, & \text{for } u = x_4^{l_4}, \\ 3(n+2) - \frac{l_5-1}{2}, & \text{for } u = x_5^{l_5}, \\ (5n+9) - \frac{l_6-1}{2}, & \text{for } u = x_6^{l_6}, \\ (9n+14) - \frac{l_7-1}{2}, & \text{for } u = x_7^{l_7}. \end{cases}$$

For $l_i = 2, 4, 6, ..., n_i - 1$,

$$\lambda(u) = \begin{cases} (9n+15) + \frac{l_1-2}{2}, & \text{for } u = x_1^{l_1}, \\ (10n+13) - \frac{l_2-2}{2}, & \text{for } u = x_2^{l_2}, \\ (10n+15) + \frac{l_3-2}{2}, & \text{for } u = x_3^{l_3}, \\ (11n+15) - \frac{l_4-2}{2}, & \text{for } u = x_4^{l_4}, \\ (12n+16) - \frac{l_5-2}{2}, & \text{for } u = x_5^{l_5}, \\ (14n+18) - \frac{l_6-2}{2}, & \text{for } u = x_6^{l_6}, \\ (18n+22) - \frac{l_7-2}{2}, & \text{for } u = x_7^{l_7}. \end{cases}$$

The set of all edge-sums generated by the above formula forms a consecutive integer sequence $s = (9n+15)+1, (9n+15)+2, \cdots, (9n+15)+e$. Therefore, by Lemma 1, λ can be extended to a super (a,0)-edge-antimagic total labeling and we obtain the magic constant $a = v+e+s = 45n+59$. Similarly, λ can be extended to a super (a,2)-edge-antimagic total labeling and we obtain the magic constant $a = v+1+s = 27n+39$. □

Theorem 5. *For any odd $n \geq 3$, $G \cong T(n, n, n+2, n+2, 2n+3, 4n+5, 8n+9)$ admits super $(a,1)$-edge-antimagic total labeling with $a = s + \frac{3(v)}{2}$, where $v = |V(G)|$ and $s = 9n+16$.*

Proof. Let us denote the vertices and edges of G, as follows:
$V(G) = \{c\} \cup \{x_i^{l_i} \mid 1 \leq i \leq 7 \; ; \; 1 \leq l_i \leq n_i\}$,
$E(G) = \{c\, x_i^1 \mid 1 \leq i \leq 7\} \cup \{x_i^{l_i} x_i^{l_i+1} \mid 1 \leq i \leq 7 \; ; \; 1 \leq l_i \leq n_i - 1\}$. If $v = |V(G)|$ and $e = |E(G)|$ then $v = 18n+22$ and $e = 18n+21$. Now, we define the labeling $\lambda : V \to \{1, 2, ..., v\}$ as in Theorem 4. It follows that the edge-weights of all edges of G constitute an arithmetic sequence $(9n+15)+1, (9n+15)+2, \cdots, (9n+15)+e$, with common difference 1. We denote it by $A = \{a_i; 1 \leq i \leq e\}$. Now for G we complete the edge labeling λ for super $(a,1)$-edge-antimagic total labeling with values in the arithmetic sequence $v+1, v+2, \cdots, v+e$ with common difference 1. Let us denote it by $B = \{b_j \; ; \; 1 \leq j \leq e\}$. Define $C = \{a_{2i-1} + b_{e-i+1} \; ; \; 1 \leq i \leq \frac{e+1}{2}\} \cup \{a_{2j} + b_{\frac{e-1}{2}-j+1} \; ; \; 1 \leq j \leq \frac{e+1}{2} - 1\}$. It is easy to see that C constitute an arithmetic sequence with $d = 1$ and $a = s + \frac{3(v)}{2} = 36n+49$.

Since all vertices receive the smallest labels so λ is a super $(36n+49, 1)$-edge-antimagic total labeling.

Theorem 6. *For any odd $n \geq 3$ and $p \geq 5$, $G \cong T(n, n, n+2, n+2, n_5, ..., n_p)$ admits super $(a,0)$-edge-antimagic total labeling with $a = 2v + s - 1$ and super $(a,2)$-edge-antimagic total labeling with $a = v+s+1$ where $v = |V(G)|$, $s = (2n+6) + \sum_{m=5}^{p} [(n+1)2^{m-5}+1]$ and $n_p = 1 + (n+1)2^{p-4}$.*

Proof. Let us denote the vertices and edges of G, as follows:
$V(G) = \{c\} \cup \{x_i^{l_i} \mid 1 \leq i \leq p \; ; \; 1 \leq l_i \leq n_i\}$,
$E(G) = \{c\, x_i^1 \mid 1 \leq i \leq p\} \cup \{x_i^{l_i} x_i^{l_i+1} \mid 1 \leq i \leq p \; ; \; 1 \leq l_i \leq n_i - 1\}$.
If $v = |V(G)|$ and $e = |E(G)|$ then

$$v = (4n+5) + \sum_{m=5}^{p} [(n+1)2^{m-4}+1]$$

and

$$e = 4(n+1) + \sum_{m=5}^{p} [(n+1)2^{m-4} + 1].$$

Now, we define the labeling $\lambda : V \to \{1, 2, ..., v\}$ as follows:

$$\lambda(c) = (3n+4) + \sum_{m=5}^{p} [(n+1)2^{m-5} + 1].$$

When $l_i = 1, 3, 5, ..., n_i$, where $i = 1, 2, 3, 4$ and $5 \le i \le p$, we define

$$\lambda(u) = \begin{cases} \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ (n+1) - \frac{l_2-1}{2}, & \text{for } u = x_2^{l_2}, \\ (n+2) + \frac{l_3-1}{2}, & \text{for } u = x_3^{l_3}, \\ 2(n+2) - \frac{l_4-1}{2}, & \text{for } u = x_4^{l_4}. \end{cases}$$

$$\lambda(x_i^{l_i}) = (2n+4) + \sum_{m=5}^{i} [(n+1)2^{m-5} + 1] - \frac{l_i - 1}{2} \text{ respectively.}$$

When $l_i = 2, 4, 6, ..., n_i - 1$ and $\alpha = (2n+4) + \sum_{m=5}^{p} [(n+1)2^{m-5} + 1]$.
For $i = 1, 2, 3, 4$ and $5 \le i \le p$, we define

$$\lambda(u) = \begin{cases} (\alpha+1) + \frac{l_1-2}{2}, & \text{for } u = x_1^{l_1}, \\ (\alpha+n-1) - \frac{l_2-2}{2}, & \text{for } u = x_2^{l_2}, \\ (\alpha+n) + \frac{l_3}{2}, & \text{for } u = x_3^{l_3}, \\ (\alpha+2n+1) - \frac{l_4-2}{2}, & \text{for } u = x_4^{l_4}. \end{cases}$$

and

$$\lambda(x_i^{l_i}) = (\alpha+2n+1) + \sum_{m=5}^{i} [(n+1)2^{m-5}] - \frac{l_i - 2}{2} \text{ respectively.}$$

The set of all edge-sums generated by the above formula forms a consecutive integer sequence $s = \alpha+2, \alpha+3, \cdots, \alpha+1+e$. Therefore, by Lemma 1, λ can be extended to a super (a, 0)-edge-antimagic total labeling and we obtain the magic constant $a = v+e+s = 2v+(2n+5)+\sum_{m=5}^{p}[(n+1)2^{m-5}+1]$. Similarly, λ can be extended to a super (a,2)-edge-antimagic total labeling and we obtain the magic constant $a = v+1+s = v+(2n+7)+\sum_{m=5}^{p}[(n+1)2^{m-5}+1]$. □

Theorem 7. *For any odd $n \geq 3$ and $p \geq 5$, $G \cong T(n,n,n+2,n+2,n_5,...,n_p)$ admits super $(a,1)$-edge-antimagic total labeling with $a = s+\frac{3(v)}{2}$ if v is even, where $v = |V(G)|$, $s = (2n+6)+\sum_{m=5}^{p}[(n+1)2^{m-5}+1]$ and $n_p = 1+(n+1)2^{p-4}$.*

Proof. Let us denote the vertices and edges of G, as follows:
$V(G) = \{c\} \cup \{x_i^{l_i} \mid 1 \leq i \leq p \,;\, 1 \leq l_i \leq n_i\}$,
$E(G) = \{c\, x_i^1 \mid 1 \leq i \leq p\} \cup \{x_i^{l_i} x_i^{l_i+1} \mid 1 \leq i \leq p \,;\, 1 \leq l_i \leq n_i - 1\}$.
If v=$|V(G)|$ and e=$|E(G)|$ then

$$v = (4n+5)+\sum_{m=5}^{p}[(n+1)2^{m-4}+1]$$

and

$$e = 4(n+1)+\sum_{m=5}^{p}[(n+1)2^{m-4}+1].$$

Now, we define the labeling $\lambda : V \rightarrow \{1,2,...,v\}$ as in Theorem 6. It follows that the edge-weights of all edges of G constitute an arithmetic sequence $s = \alpha+2, \alpha+3, \cdots, \alpha+1+e$ with common difference 1, where $\alpha = (2n+4)+\sum_{m=5}^{p}[(n+1)2^{m-5}+1]$. We denote it by $A = \{a_i; 1 \leq i \leq e\}$. Now for G we complete the edge labeling λ for super $(a,1)$-edge-antimagic total labeling with values in the arithmetic sequence $v+1, v+2, \cdots, v+e$ with common difference 1. Let us denote it by $B = \{b_j \,;\, 1 \leq j \leq e\}$. Define $C = \{a_{2i-1}+b_{e-i+1} \,;\, 1 \leq i \leq \frac{e+1}{2}\} \cup \{a_{2j}+b_{\frac{e-1}{2}-j+1} \,;\, 1 \leq j \leq \frac{e+1}{2}-1\}$. It is easy to see that C constitute an arithmetic sequence with $d = 1$ and $a = s+\frac{3(v)}{2} = \frac{16n+27}{2}+\frac{\sum_{m=5}^{p}[(n+1)2^{m-2}+5]}{2}$. Since all vertices receive the smallest labels so λ is a super $(a,1)$-edge-antimagic total labeling. □

3 Conclusion

In this paper, we have shown that a subclass of trees, namely subdivided stars $G \cong T(n,n,n+2,n+2,n_5,...,n_p)$, admits super (a,d)-edge-antimagic total labeling when $d = 0, 1, 2$, odd $n \geq 3$, $n_p = 1 + (n+1)2^{p-4}$ and $p \geq 5$. For the remaining cases problem is still open.

References

1. A. Kotzig and A. Rosa, Magic valuations of finite graphs, *Canad. Math. Bull.* 13 (1970), 451-461.
2. A. Kotzig and A. Rosa, Magic valuation of complete graphs, *Centre de Recherches Mathematiques, Universite de Montreal*, (1972), CRM-175.
3. A. N. M. Salman, A. A. G. Ngurah and N. Izzati, On (Super) Edge-Magic Total Labelings of a Subdivision of a Star S_n, *Utilitas Math.* 81(2010), 275-284.
4. D. B. West, An Introduction to Graph Theory (*Prentice-Hall, 1996*).
5. E. T. Baskoro and A. A. G. Ngurah, On super edge-magic total labelings, *Bull. Inst. Combin. Appil.* 37 (2003), 82-87.
6. E. T. Baskoro, A. A. G. Ngurah and R. Simanjuntak, On (super) edge-magic total labeling of subdivision of $K_{1,3}$, *SUT J. Math.* 43 (2007), 127-136.
7. H. Enomoto, A. S. Llado, T. Nakamigawa and G. Ringle, Super edge-magic graphs *SUT J. Math.* 34 (1980), 105-109.
8. J. A. Gallian, A dynamic survey of graph labeling, *J. Combin.* January 2010.
9. M. Hussain, E. T. Baskoro and Slamin, On super edge-magic total labeling of banana trees, *Utilitas Math.* 79 (2009), 243-251.
10. M. Javaid, M. Hussain, K. Ali and H. Shaker, Super edge-magic total labeling on subdivision of trees, *Utilitas Math.* to appear.
11. M. Javaid, M. Hussain, K. Ali and H. Shaker, Super edge-magic total labeling on subdivision of stars, *Ars Combin.* to appear.
12. M. Javaid, M. Hussain, K. Ali and K. H. Dar, Super edge-magic total labeling on $w - trees$, *Utilitas Math.* to appear.
13. M. Javaid, A. A. Bhatti, M. Hussain and K. Ali, Super edge-magic total labeling on forest of extended $w - trees$, *Utilitas Math.* to appear.
14. R. M. Figueroa-Centeno, R. Ichishima, and F. A. Muntaner-Batle, The place of super edge-magic labeling among other classes of labeling, *Discrete Math.* 231 (2001), 153-168.
15. S. M. Lee and Q. X. Shah, All trees with at most 17 vertices are super edge-magic, *16th MCCCC Conference, Carbondale*, University Southern Illinois, November 2002.