On (α, β)-generalized derivations on lattices

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Abstract

In this paper, we introduce the notion of (α, β)-generalized d-derivations on lattices and investigate some related properties. Also using the notion of permuting (α, β)-triderivation we characterize distributive element of a lattice.

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1 Introduction

Lattices play an important role in information theory and cryptanalysis [5, 13]. The well-established notion of derivations of C^*-algebras and rings has been recently studied by various researchers in the context of lattices (see [19] and references therein).

Ozturk [17], Ozden and Ozturk [16] introduced the notion of permuting triderivations in prime and semiprime rings and proved some results. Later on Ozturk, Yazarli and Kim [18] developed this notion for lattices.

Zhan and Liu [22] introduced the notions of left, right and regular f-derivations on BCI algebras and investigated some properties of such derivations. Yilmaz and Ozturk [20] introduced the notion of f-derivation on a lattice and discussed some related properties. Later on Zabal and Firat [21] introduced the notion of symmetric f-bi-derivations on lattices. Recently Khan and Chaudhry [1] has used the notion of f-derivation on lattices for proving some results about permuting f-triderivations.

Derivations on various algebraic structures have been an active area of research since the last fifty years due to their usefulness in various areas of mathematics.

A more general concept of (α, β)-derivations have been extensively studied in prime and semiprime rings. They have played an important role in the solution of some functional equations (see, e.g., Bresar [7] and references therein). (α, β)-derivation on prime and semiprime rings have also been studied by Chaudhry and Thaheem [8, 9, 10, 11] and Ali and Chaudhry [2]. Recently Asci et al. [3] use the notion of (f, g)-derivations on lattices and proved some results by using this notion.

In this paper, the notion of (α, β)-generalized d-derivation, which is more general than the notion of generalized d-derivation [18], is introduced. We study some properties of this general notion and using permuting (α, β)-triderivations give characterization of distributive elements of lattices.

2 Preliminaries

In this section we describe some definitions and results which will be used in the sequel.

Definition 2.1 [6] *A nonempty set L together with the operations* $\wedge$ *and* $\vee$ *is called a lattice if it satisfies the following conditions for all* $x, y, z \in L$:
(1) $x \wedge x = x,\ x \vee x = x$.
(2) $x \wedge y = y \wedge x,\ x \vee y = y \vee x$.
(3) $(x \wedge y) \wedge z = x \wedge (y \wedge z),\ (x \vee y) \vee z = x \vee (y \vee z)$.
(4) $(x \wedge y) \vee x = x,\ (x \vee y) \wedge x = x$.
The lattice L is denoted by $(L, \wedge, \vee)$.

Definition 2.2 [6] *Let* $(L, \wedge, \vee)$ *be a lattice and a nonempty subset M of L is called a sublattice of L if*
$a, b \in M$ *implies* $a \vee b \in M$ *and* $a \wedge b \in M$.

Definition 2.3 [6] *A lattice* $(L, \wedge, \vee)$ *is called a distributive lattice if it satisfies*
(5) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ *for all* $x, y, z \in L$,
and
(6) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ *for all* $x, y, z \in L$.
In any lattice, the above conditions are equivalent.

Definition 2.4 [6] *Let* $(L, \wedge, \vee)$ *be a lattice. A binary relation* $\leq$ *on L is defined by* $x \leq y$ *if and only if* $x \wedge y = x$ *and* $x \vee y = y$.

Definition 2.5 [6] *A lattice* $(L, \wedge, \vee)$ *is called a modular lattice if it satisfies the following condition for all* $x, y, z \in L$
(7) If $x \leq z$, *then* $x \vee (y \wedge z) = (x \vee y) \wedge z$.

The following Lemma is already known [19]

Lemma 2.6 [19] *Let $(L, \wedge, \vee)$ be a lattice. Let $\leq$ be as defined in definition 2.4. Then $(L, \leq)$ is a poset for any $x, y \in L$, $x \wedge y$ is the g.l.b of $\{x, y\}$ and $x \vee y$ is the l.u.b of $\{x, y\}$.*

Definition 2.7 [19] *Let $(L, \wedge, \vee)$ be a lattice. A function $d : L \to L$ is called a derivation on L if it satisfies the following condition:*
$d(x \wedge y) = (d(x) \wedge y) \vee (x \wedge d(y))$ for all $x, y \in L$.

Definition 2.8 [18] *Let $(L, \wedge, \vee)$ be a lattice. A mapping $D(.,.,.) : L \times L \times L \to L$ is called a permuting mapping if $D(x, y, z) = D(x, z, y) = D(y, x, z) = D(y, z, x) = D(z, x, y) = D(z, y, x)$ holds for all $x, y, z \in L$.*

Definition 2.9 [18] *Let $(L, \wedge, \vee)$ be a lattice and $D(.,.,.) : L \times L \times L \to L$ a permuting mapping. The mapping $d : L \to L$ defined by $d(x) = D(x, x, x)$ is called the trace of $D(.,.,.)$.*

Definition 2.10 [18] *Let $(L, \wedge, \vee)$ be a lattice and $D(.,.,.) : L \times L \times L \to L$ a permuting mapping. We call D a permutig triderivation on L, if it satisfies*
$D(x \wedge w, y, z) = (D(x, y, z) \wedge w) \vee (x \wedge D(w, y, z))$ for all $w, x, y, z \in L$.

Definition 2.11 [18] *Let $(L, \wedge, \vee)$ be a lattice and $D(.,.,.) : L \times L \times L \to L$ a permuting mapping. We call D a joinitive mapping, if it satisfies*
$D(x \vee w, y, z) = D(x, y, z) \vee D(w, y, z)$ for all $w, x, y, z \in L$.

Definition 2.12 [18] *Let $(L, \wedge, \vee)$ be a lattice and d be a trace of a permuting tri-derivation D. Let $G : L \to L$ be a mapping, then G is called a generalized d-derivation on L if it satisfies the following condition*
$G(x \wedge y) = (G(x) \wedge y) \vee (x \wedge d(y))$ for all $x, y \in L$.

Definition 2.13 [3] *Let $(L, \wedge, \vee)$ be a lattice and $\alpha : L \to L$, $\beta : L \to L$ are mappings. Let $D(.,.,.) : L \times L \times L \to L$ be a permuting mapping. We call D a permuting (α, β)-triderivation on L, if it satisfies the following condition*
$D(x \wedge w, y, z) = (D(x, y, z) \wedge \alpha(w)) \vee (\beta(x) \wedge D(w, y, z))$ for all $w, x, y, z \in L$.

Definition 2.14 [6] *Let $(L, \wedge, \vee)$ be a lattice. A mapping $f : L \to L$ is called a lattice homomorphism if*
(1) $f(x \wedge y) = f(x) \wedge f(y)$,
(2) $f(x \vee y) = f(x) \vee f(y)$ for all $x, y \in L$.

Definition 2.15 [3] *Let $(L, \wedge, \vee)$ be a lattice and $\alpha : L \to L$, $\beta : L \to L$ are mappings. Let D be a permuting (α, β)-triderivation of L with trace d. If $x \leq y$ implies $d(x) \leq d(y)$, then d is called an isotone mapping.*

In this paper we shall use the following results of [3].

Proposition 2.16 [3] *Let L be a lattice and d be the trace of permuting tri-(f,g)- derivation D on L. Then $d(x) \leq (f(x) \vee g(x))$ for all $x \in L$.*

Theorem 2.17 [3] *Let L be a distributive lattice and D be a permuting tri-(f,g)- derivation on L with the trace d. Then $d(x \wedge y) = (d(x) \wedge f(y)) \vee (g(x) \wedge d(y)) \vee \{(g(x) \wedge f(y)) \wedge [D(x,x,y) \vee D(x,y,y)]\}$ for all $x, y \in L$.*

Proposition 2.18 [3] *Let L be a lattice and d be the trace of permuting tri-(f,g)- derivation D on L. Then the following conditions are equivalent, (i) d is an isotone mapping, (ii) $dx \vee dy \leq d(x \vee y)$.*

Remark 2.19 *Imposing the additional condition $\alpha(x) \leq \beta(x)$ in the Proposition 2.16 mentioned above, the following result follows immediately.*

Let $(L, \wedge, \vee)$ be a distributive lattice and $\alpha : L \to L$, $\beta : L \to L$ are mappings satisfying $\alpha(x) \leq \beta(x)$. Let d be the trace of the permuting (α, β)-triderivation D, then $d(x) \leq \beta(x)$ for all $x \in L$.

Remark 2.20 [6] *Every distributive lattice is a modular lattice but the converse is not true, in general.*

3 Results

Proposition 3.1 *Let $(L, \wedge, \vee)$ be a distributive lattice and $\alpha : L \to L$, $\beta : L \to L$ are mappings satisfying $\alpha(x) \leq \beta(x)$. Let d be the trace of the permuting (α, β)-triderivation D, then $D(x,y,z) \leq \beta(x)$, $D(x,y,z) \leq \beta(y)$ and $D(x,y,z) \leq \beta(z)$ for all $x, y, z \in L$.*

Proof. Since $D(x,y,z) = D(x \wedge x, y, z) = (D(x,y,z) \wedge \alpha(x)) \vee (\beta(x) \wedge D(x,y,z))$. Since L is distributive, therefore $D(x,y,z) = D(x,y,z) \wedge (\alpha(x) \vee \beta(x))$, which alongwith $\alpha(x) \leq \beta(x)$ implies $D(x,y,z) = D(x,y,z) \wedge \beta(x)$. Thus $D(x,y,z) \leq \beta(x)$ for all $x, y, z \in L$. Similarly we can show that $D(x,y,z) \leq \beta(y)$ and $D(x,y,z) \leq \beta(z)$ for all $x, y, z \in L$.

Remark 3.2 *$(L, \wedge, \vee)$ be a lattice and $\alpha : L \to L$, $\beta : L \to L$ are mappings satisfying $\alpha(x) \leq \beta(x)$. Let D be a permuting (α, β)-triderivation. It is obvious from Proposition 3.1 that $D(x,y,z) \leq \beta(x) \wedge \beta(y)$, $D(x,y,z) \leq \beta(x) \wedge \beta(z)$ and $D(x,y,z) \leq \beta(y) \wedge \beta(z)$ for all $x, y, z \in L$. By a similar argument $D(x,y,z) \leq (\beta(x) \wedge \beta(y)) \wedge \beta(z)$ for all $x, y, z \in L$.*

Theorem 3.3 *Let $(L, \wedge, \vee)$ be a distributive lattice and $\alpha : L \to L$, $\beta : L \to L$ are mappings satisfying $\alpha(x) \leq \beta(x)$ and d a trace of a permuting (α, β)-triderivation D of L. Then d is an isotone if and only if $d(x \wedge y) = d(x) \wedge d(y)$.*

Proof. Let d be isotone. Since $x \wedge y \leq x$ and $x \wedge y \leq y$, therefore $d(x \wedge y) \leq d(x)$ and $d(x \wedge y) \leq d(y)$. Thus $d(x \wedge y) \leq d(x) \wedge d(y)$. Since L is a distributive lattice, therefore by Theorem 2.17
$d(x \wedge y) = (d(x) \wedge \alpha(y)) \vee (\beta(x) \wedge d(y)) \vee \{(\alpha(y) \wedge \beta(x)) \wedge [D(x, x, y) \vee D(x, y, y)]\}$, which implies $\beta(x) \wedge d(y) \leq d(x \wedge y)$. Using Remark 2.19, we get $d(x) \leq \beta(x)$. Therefore $d(x) \wedge d(y) \leq \beta(x) \wedge d(y) \leq d(x \wedge y)$ implies $d(x) \wedge d(y) \leq d(x \wedge y)$. This alongwith $d(x \wedge y) \leq d(x) \wedge d(y)$ gives $d(x \wedge y) = d(x) \wedge d(y)$ for all $x, y \in L$.
Conversely suppose that $d(x \wedge y) = d(x) \wedge d(y)$ and $x \leq y$. Since $x \wedge y = x$, we get $d(x) = d(x \wedge y) = d(x) \wedge d(y) \leq d(y)$. Hence $d(x) \leq d(y)$ for all $x, y \in L$. Hence d is an isotone.

Definition 3.4 [14] *Let $(L, \wedge, \vee)$ be a lattice. An element a of L is said to be distributive whenever, for every $x, y \in L$,*
$a \wedge (x \vee y) = (a \wedge x) \vee (a \wedge y)$.

Example 3.5 *Let $\alpha : L \to L$, $\beta : L \to L$ be mappings satisfying $\alpha(x) \leq \beta(x)$ and $\beta(x \wedge y) = \beta(x) \wedge \beta(y)$. We define $D(x, y, z) = \alpha(a) \wedge (\beta(x) \wedge (\beta(y) \wedge \beta(z))$. We now verify that D is a permuting (α, β)-triderivation.*

We consider $D(x \wedge w, y, z) = \alpha(a) \wedge (\beta(x \wedge w) \wedge (\beta(y) \wedge \beta(z)) = \alpha(a) \wedge \{(\beta(x) \wedge \beta(w)) \wedge (\beta(y) \wedge \beta(z))\}$. (1)
Also $(D(x, y, z) \wedge \alpha(w)) \vee (\beta(x) \wedge D(w, y, z)) = \{\alpha(a) \wedge (\beta(x) \wedge (\beta(y) \wedge \beta(z))) \wedge \alpha(w)\} \vee \{\beta(x) \wedge (\alpha(a) \wedge (\beta(w) \wedge (\beta(y) \wedge \beta(z))))\} = \{\alpha(a) \wedge (\beta(x) \wedge (\beta(y) \wedge \beta(z))) \wedge \alpha(w)\} \vee \{\beta(w) \wedge (\alpha(a) \wedge (\beta(x) \wedge (\beta(y) \wedge \beta(z))))\}$. Let $M = (\alpha(a) \wedge (\beta(x) \wedge (\beta(y) \wedge \beta(z))))$, then the last equation gives $(M \wedge \alpha(w)) \vee (M \wedge \beta(w))$. Since $\alpha(w) \leq \beta(w)$, therefore $(D(x, y, z) \wedge \alpha(w)) \vee (\beta(x) \wedge D(w, y, z)) = (M \wedge \beta(w)) = \{\alpha(a) \wedge (\beta(x) \wedge (\beta(y) \wedge \beta(z))) \wedge \beta(w)\} = \alpha(a) \wedge ((\beta(x) \wedge \beta(w)) \wedge (\beta(y) \wedge \beta(z))$. (2)
From equation (1) and (2) we get
$D(x \wedge w, y, z) = (D(x, y, z) \wedge \alpha(w)) \vee (\beta(x) \wedge D(w, y, z))$.
Hence $D(x, y, z) = \alpha(a) \wedge (\beta(x) \wedge (\beta(y) \wedge \beta(z)))$ is permuting (α, β)-triderivation.

Theorem 3.6 *Let $(L, \wedge, \vee)$ be a lattice and $\alpha : L \to L$ $\beta : L \to L$ be lattice homomorphisms satisfying $\alpha(x) \leq \beta(x)$. Let $D : L \times L \times L \to L$ be a permuting (α, β)-triderivation on L defined by $D(x, y, z) = \alpha(a) \wedge \beta(x \wedge (y \wedge z)) = \alpha(a) \wedge (\beta(x) \wedge (\beta(y) \wedge \beta(z)))$. Then $\alpha(a)$ is distributive if and only if D is joinitive.*

Proof. Let D be a joinitive. By definition of D, we have $D(x \vee w, y, z) = \alpha(a) \wedge (\beta(x \vee w) \wedge (\beta(y) \wedge \beta(z)) = \alpha(a) \wedge ((\beta(x) \vee \beta(w)) \wedge (\beta(y) \wedge \beta(z)))$.

Since D is joinitive, therefore $D(x \vee w, y, z) = D(x, y, z) \vee D(w, y, z) = (\alpha(a) \wedge (\beta(x) \wedge (\beta(y) \wedge \beta(z)))) \vee (\alpha(a) \wedge (\beta(w) \wedge (\beta(y) \wedge \beta(z))))$. Hence $\alpha(a) \wedge ((\beta(x) \vee \beta(w)) \wedge (\beta(y) \wedge \beta(z))) = (\alpha(a) \wedge (\beta(x) \wedge (\beta(y) \wedge \beta(z)))) \vee (\alpha(a) \wedge (\beta(w) \wedge (\beta(y) \wedge \beta(z)))$. Thus $\alpha(a)$ is a distributive.
Conversely let $\alpha(a)$ be distributive. Then $\alpha(a) \wedge ((\beta(x) \vee \beta(w)) \wedge (\beta(y) \wedge \beta(z))) = (\alpha(a) \wedge (\beta(x) \wedge (\beta(y) \wedge \beta(z)))) \vee (\alpha(a) \wedge (\beta(w) \wedge (\beta(y) \wedge \beta(z)))$, which alongwith definition of D implies
$D(x \vee w, y, z) = D(x, y, z) \vee D(w, y, z)$. Hence D is joinitive.

Taking $\alpha = \beta = 1$, the identity on L, we get the following result as a corollary.

Corollary 3.7 *Let $(L, \wedge, \vee)$ be a lattice and $D : L \times L \times L \to L$ be a permuting triderivation on L defined by $D(x, y, z) = a \wedge (x \wedge (y \wedge z))$. Then a is distributive if and only if D is joinitive.*

4 (α, β)-generalized d-derivations

In this section, we describe the concept of an (α, β)-generalized d-derivation on lattices and prove our results regarding this notion.

Definition 4.1 *Let $(L, \wedge, \vee)$ be a lattice and $\alpha : L \to L$, $\beta : L \to L$ are mappings. Let d be a trace of a permuting (α, β)-triderivation D. Let $G : L \to L$ be a mapping, then G is called an (α, β)-generalized d-derivation on L if it satisfies the following condition*
$G(x \wedge y) = (G(x) \wedge \alpha(y)) \vee (\beta(x) \wedge d(y))$ for all $x, y \in L$.

Proposition 4.2 *Let $(L, \wedge, \vee)$ be a lattice and $\alpha : L \to L$, $\beta : L \to L$ are mappings satisfying $\alpha(x) \leq \beta(x)$. Let D be a permuting (α, β)-triderivation of L with trace d. If $G : L \to L$ is an (α, β)-generalized d-derivation on L, then*
(i) $d(x) \leq G(x)$,
(ii) $G(x \wedge y) \leq G(x) \vee G(y)$.

Proof. (i) Since $x \wedge x = x$ for all $x \in L$, we get
$G(x) = G(x \wedge x) = (G(x) \wedge \alpha(x)) \vee (\beta(x) \wedge d(x))$. By Remark 2.19, we get $G(x) = (G(x) \wedge \alpha(x)) \vee d(x)$.
Hence $d(x) \leq G(x)$ for all $x \in L$.
(ii) Since $G(x) \wedge \alpha(y) \leq G(x)$ and $\beta(x) \wedge d(y) \leq d(y)$, which alongwith $d(y) \leq G(y)$ gives
$G(x \wedge y) = (G(x) \wedge \alpha(y)) \vee (\beta(x) \wedge d(y)) \leq G(x) \vee G(y)$.
Hence $G(x \wedge y) \leq G(x) \vee G(y)$ for all $x, y \in L$.

Proposition 4.3 *Let $(L, \wedge, \vee)$ be a lattice, $\alpha : L \to L$, $\beta : L \to L$ are mappings satisfying $\alpha(x) \leq \beta(x)$ with β is an increasing function and D*

a permuting (α,β)-triderivation of L with trace d. Let $G: L \to L$ be an (α,β)-generalized d-derivation on L. If 1 is the greatest element of L, then
(i) $G(1) \leq d(x) \Rightarrow d(x) = G(x)$,
(ii) $d(x) \leq G(1) \Rightarrow G(x) \leq G(1)$.

Proof. (i) Since $G(1) \leq d(x)$ and $x \wedge 1 = x$ for all $x \in L$, we have $G(x) = G(1 \wedge x) = (G(1) \wedge \alpha(x)) \vee (\beta(1) \wedge d(x)) \leq (d(x) \wedge \alpha(x)) \vee d(x) \leq d(x)$. Which implies $G(x) \leq d(x)$ for all $x \in L$. By Proposition 4.2, we have $d(x) \leq G(x)$ for all $x \in L$, this alongwith $G(x) \leq d(x)$ gives $G(x) = d(x)$ for all $x \in L$.
(ii) Let $d(x) \leq G(1)$ for all $x \in L$. Since $x \wedge 1 = x$ for all $x \in L$, therefore we have $G(x) = G(1 \wedge x) = (G(1) \wedge \alpha(x)) \vee (\beta(1) \wedge d(x)) \leq (G(1) \wedge \alpha(x)) \vee G(1) = G(1)$. Thus $G(x) \leq G(1)$ for all $x \in L$.

Definition 4.4 *Let $(L, \wedge, \vee)$ be a lattice, $\alpha: L \to L$, $\beta: L \to L$ are mappings and D a permuting (α,β)-triderivation of L with trace d. Let $G: L \to L$ be an (α,β)-generalized d-derivation on L. We define the set by : $F = \{x \in L : G(x) = d(x)\}$.*

Theorem 4.5 *Let $(L, \wedge, \vee)$ be a lattice, $\alpha: L \to L$, $\beta: L \to L$ are mappings satisfying $\alpha(x) \leq \beta(x)$ and D a permuting (α,β)-triderivation of L with trace d. Let $G: L \to L$ be an (α,β)-generalized d-derivation on L. If d is decreasing function on L, then $y \leq x$ and $x \in F$ imply $y \in F$.*

Proof. Let $y \leq x$, $x \in F$, $G(y) = G(x \wedge y) = (G(x) \wedge \alpha(y)) \vee (\beta(x) \wedge d(y)) \leq G(x) \vee d(y) = d(y)$. Hence $G(y) \leq d(y)$ for all $y \in L$. By Proposition 4.2, $d(y) \leq G(y)$ for all $y \in L$, which along with $G(y) \leq d(y)$ implies $G(y) = d(y)$ for all $y \in L$. Hence $y \in F$.

Theorem 4.6 *Let $(L, \wedge, \vee)$ be a lattice, $\alpha: L \to L$, $\beta: L \to L$ are homomorphism satisfying $\alpha(x) \leq \beta(x)$ and D a permuting (α,β)-triderivation of L with trace d. Let $G: L \to L$ be an (α,β)-generalized d-derivation on L. If G is a decreasing function on L, then $x \vee y \in F$ for all $x, y \in F$.*

Proof. Since $x \leq x \vee y$ and $y \leq x \vee y$ for all $x, y \in F$ and G is a decreasing function on L, therefore $G(x \vee y) \leq G(x)$ and $G(x \vee y) \leq G(y)$ for all $x, y \in F$. So $G(x \vee y) \leq G(x) \vee G(y) = d(x) \vee d(y)$. By Proposition 2.18, we get $d(x) \vee d(y) \leq d(x \vee y)$. Thus $G(x \vee y) \leq d(x) \vee d(y) \leq d(x \vee y)$. So $G(x \vee y) \leq d(x \vee y)$ for all $x, y \in F$. By Proposition 4.2, we have $d(x \vee y) \leq G(x \vee y)$, which alongwith $G(x \vee y) \leq d(x \vee y)$ implies $G(x \vee y) = d(x \vee y)$ for all $x, y \in F$. Hence $x \vee y \in F$ for all $x, y \in F$.

Theorem 4.7 *Let $(L, \wedge, \vee)$ be a lattice, $\alpha: L \to L$, $\beta: L \to L$ are homomorphism satisfying $\alpha(x) \leq \beta(x)$ and D a permuting (α,β)-triderivation of L with trace d. Let $G: L \to L$ be an (α,β)-generalized* *d-derivation on L. If G and d are decreasing functions on L, then the set* $F = \{x \in L : G(x) = d(x)\}$ *is an ideal of L.*

Proof. Proof follows from Theorem 4.5 and Theorem 4.6.

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