# INFINITE SYMMETRIC GROUPS

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ABSTRACT. In this work, infinite similarities of permutation groups are investigated by means of new methods. For this purpose, we handle distinct groups on the set of natural numbers and we give the separation of the subgroups of them. Afterwards, we give the matrix representation of this groups.

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## 1. Introduction

Infinite-degree permutations and their representations are some of the subjects that have mostly studied recently. Naturally, this subject has arisen as a result of the improvement in permutation concept which has been generated at the classical period of group theory ([1], [2], [3], [4]). We can indicate [8] and [9] as examples of the subject. In studies of V.A. Kasimov, new methods for constructing infinite analogs of infinite degree permutations have been discussed and their separation into subgroups has been given ([5], [6], [7]). Matrix representations of some infinite degree permutation groups are researched in this study.

Let $\mathbb{N}$ denote the set of natural numbers and let $m$ be an arbitrary natural number. Each natural number $m$ cuts off $\mathbb{N}$ into two parts. Denote by $\mathbb{N}_m$ the set of natural numbers from 1 to $m$, i.e., let $\mathbb{N}_m = \{1, 2, ..., m\}$. $\mathbb{N}_m^\infty$ will denote the complement of $\mathbb{N}_m$, i.e., $\mathbb{N}_m^\infty = \mathbb{N} \setminus \mathbb{N}_m$. Let $S(m)$

stand for the group of all permutations with $m$ degree. Each element of $S(m)$ is a bijective mapping of $\mathbb{N}_m$. Let us mark by ${}^{(m)}\varphi$ the elements of $S(m)$, that is ${}^{(m)}\varphi : \mathbb{N}_m \to \mathbb{N}_m$ is a bijective mapping. Let $\varphi_i$ denote the value of the bijection $\varphi$ at the point $i \in \mathbb{N}$. The group operation on $S(m)$ is the composition of bijections, in other words, if ${}^{(m)}\varphi$ and ${}^{(m)}\psi$ are two $m$ degree permutations, then their product ${}^{(m)}\varphi.{}^{(m)}\psi$ can be determined by ${}^{(m)}\varphi.{}^{(m)}\psi = {}^{(m)}\psi \circ {}^{(m)}\varphi$. ${}^{(m)}\varepsilon$ will denote the unique $m$ degree permutation. Clearly, in the above equation the product of distinct degree permutations ${}^{(m)}\varphi$ and ${}^{(n)}\varphi$ is not determined. We give a method which will dissipate this indefiniteness.

## 2. On Infinite Degree Permutations

Let $S(\mathbb{N})$ denote the group of all bijections on $\mathbb{N}$. We name the elements of $S(\mathbb{N})$ infinite degree permutations and denote them ${}^{(\infty)}\varphi$.We construct some subgroups of $S(\mathbb{N})$. For this purpose we determine some kinds of infinite degree permutations.

2.1. **Stabilized Permutations.** An infinite degree permutation ${}^{(\infty)}\varphi$ for a certain number $m \in \mathbb{N}$ satisfies the following condition:

$$\text{(2.1)} \qquad \text{If} j \leq m \text{then,} {}^{(\infty)}\varphi(j) \in \mathbb{N}_m \text{and if} j > m, \text{then} {}^{\infty}\varphi(j) = j.$$

Permutations which satisfy the condition (2.1) are said to be stabilized permutations after $m$. If an infinite degree permutation ${}^{(\infty)}\varphi$ is stabilized after $m$ then we denote it by ${}^{(\infty,m)}\varphi$:

$$ {}^{(\infty,m)}\varphi(j) = \begin{cases} {}^{(\infty)}\varphi(j) \in \mathbb{N}_m & \text{if } j \leq m \\ j & \text{if } j > m. \end{cases} $$

We can exhibit this permutation as follows:

$$ {}^{(\infty,m)}\varphi = \begin{pmatrix} 1 & 2 & \dots & m & m+1 & m+2 & \dots \\ \varphi_1 & \varphi_2 & \dots & \varphi_m & m+1 & m+2 & \dots \end{pmatrix}. $$

It is clear that the product of stabilized permutations is also stabilized. Suppose that infinite degree permutations $\alpha$ and $\beta$ are stabilized after suitable $m$ and $n$. Then their product becomes stabilized after $max\{m, n\}$. Let us show by $S^m(\mathbb{N})$ the set of all infinite degree permutations stabilized after $m$. The set $S^m(\mathbb{N})$ is a group. Define the mapping $w : S(m) \to S^m(\mathbb{N})$ by

$$ w({}^{(m)}\varphi) = {}^{(\infty,m)}\varphi. \tag{2.2} $$

**Propositon 1** *The mapping $w : S(m) \to S^m(\mathbb{N})$ defined by equation* (2.2) *is a group isomorphism.*

**Proof** It is easy to see that $w$ preserves an algebraic operation. On the other hand, it is also clear that we have ${}^{(\infty,m)}\varphi \neq {}^{(\infty,m)}\psi$ if the condition ${}^{(m)}\varphi \neq {}^{(m)}\psi$ satisfies. Then $w$ is a monomorphism. At the same time, the mapping $\varphi$ is an epimorphism since there exist a number $m \in \mathbb{N}$ such that ${}^{(\infty)}\varphi = {}^{(\infty,m)}\varphi$ holds for all stabilized infinite degree permutations ${}^{(m)}\varphi$. In that case $\varphi$ is a group isomorphism.□

It is natural now to deal with stabilized infinite degree groups $S^m(\mathbb{N}) < S^{m+1}(\mathbb{N})$. Thus we consider successive groups

$$(2.3) \qquad S^1(\mathbb{N}) < S^2(\mathbb{N}) < S^3(\mathbb{N}) < \ldots < S^m(\mathbb{N}) < S^{m+1}(\mathbb{N}) < \ldots$$

As mentioned before, the groups $S(m)$ and $S^m(\mathbb{N})$ are isomorphic for each natural number $m$. Therefore, we can identify them. Let

$$(2.4) \qquad {}^{\infty}S = \bigcup_{\mathbf{N}} S^m(\mathbb{N}) \text{ and } S = \bigcup_{\mathbf{N}} S(m).$$

Then the following holds.

**Proposition 2** *The set* ${}^{\infty}S$ *is a subgroup of the group* $S(\mathbb{N})$ *of all in infinite degree permutations.*

**Proof** It suffices to show that ${}^{\infty}S = \bigcup_{\mathbf{N}} S^m(\mathbb{N})$ is a group. Assume that two stabilized permutations ${}^{(\infty,m)}\varphi \in S^m(\mathbb{N})$ and ${}^{(\infty,n)}\psi \in S^n(\mathbb{N})$ are given. As we already said, their product becomes stabilized after $max\{m, n\}$. That is to say that the product ${}^{(\infty,m)}\varphi.{}^{(\infty,n)}\psi$ belongs to ${}^{\infty}S = \bigcup_{\mathbf{N}} S^m(\mathbb{N})$. The identity element of this group is the identity mapping $\varepsilon : \mathbb{N} \to \mathbb{N}$ which is stabilized after 1. The inverse of the stabilized permutation ${}^{(\infty,m)}S \in S^m(\mathbb{N})$ is also stabilized after $m$, so it is belongs to $S^m(\mathbb{N})$, hence it is in ${}^{\infty}S$. Therefore, the set ${}^{\infty}S$ forms a group.□

The group $\varepsilon = \bigcap {}^{\infty}S = S^1(\mathbb{N})$ of the intersection of the subgroups $S^m(\mathbb{N})$ is also a subset of ${}^{\infty}S$. Thus, we can define successive groups in (2.3) as follows:

(2.5)

$$\varepsilon = S^1(\mathbb{N}) < S^2(\mathbb{N}) < \ldots < S^m(\mathbb{N}) < S^{m+1}(\mathbb{N}) < \ldots < {}^{\infty}S < S(\mathbb{N}).$$

For each $(m, n)$ of positive integers, the product operation

$$({}^{(\infty,m)}\varphi, {}^{(\infty,n)}\psi) \mapsto {}^{(\infty,m)}\varphi . {}^{(\infty,n)}\psi$$

in the group ${}^{\infty}S$ gives rise to mapping

$$S^m(\mathbb{N}) \times S^n(\mathbb{N}) \to S^{max\{m,n\}}(\mathbb{N}). \tag{2.6}$$

Now turn back to the bijections (permutations) of $\mathbb{N}_m^{\infty} = \mathbb{N} \setminus \mathbb{N}_m$, and denote them by $\varphi_m^{\infty}$. Let $S(\mathbb{N}_m^{\infty})$ represent the set of all bijections of the set $\mathbb{N}_m^{\infty}$. We can display each permutation $\varphi_m^{\infty}$ as

$$\varphi_m^{\infty} = \begin{pmatrix} m+1 & m+2 & \dots \\ \varphi_{m+1} & \varphi_{m+2} & \dots \end{pmatrix}.$$

The identity mapping of $\mathbb{N}_m^{\infty}$ is written $\varepsilon_m^{\infty}$. The set $S(\mathbb{N}_m^{\infty})$ is also a group, and we call its elements permutations. Let ${}^{(m)}\varphi = \begin{pmatrix} 1 & 2 & \dots & m \\ \varphi_1 & \varphi_2 & \dots & \varphi_m \end{pmatrix}$ be an arbitrary $m$ degree permutation and let $\varepsilon_m^{\infty}$ be the identity permutation of the group $S(\mathbb{N}_m^{\infty})$, that is, $\varepsilon_m^{\infty} = \begin{pmatrix} m+1 & m+2 & \dots \\ m+1 & m+2 & \dots \end{pmatrix}$. Then from these two permutations we can obtain the following one:

$${}^{(\infty,m)}\varphi = \begin{pmatrix} 1 & 2 & \dots & m & m+1 & m+2 & \dots \\ \varphi_1 & \varphi_2 & \dots & \varphi_m & m+1 & m+2 & \dots \end{pmatrix}.$$

Conditionally, we will call those permutations the right composition of ${}^{(m)}\varphi$ with $\varepsilon_m^{\infty}$, and will write $({}^{(m)}\varphi \mid \varepsilon_{\infty}^{m})$. Thus, the infinite value permutation ${}^{(\infty,m)}\varphi$ stabilized after $m$ consists of the right composition of ${}^{(m)}\varphi$ with $\varepsilon_m^{\infty}$, i.e., ${}^{(\infty,m)}\varphi = ({}^{(m)}\varphi \mid \varepsilon_m^{\infty})$.

**2.2. Activated Permutations.** We have determined above the series of the subgroups ${}^{\infty}S$ and $S^m(\mathbb{N})$ of the group $S(\mathbb{N})$. Now we will meet other subgroups series of $S(\mathbb{N})$. The elements of the subgroup $S_m(\mathbb{N})$ are the permutations displayed as follows:

$$_{(m,\infty)}\varphi = \begin{pmatrix} 1 & 2 & \dots & m & m+1 & m+2 & \dots \\ 1 & 2 & \dots & m & \varphi_{m+1} & \varphi_{m+2} & \dots \end{pmatrix}.$$

We call them activated permutations after $m$. We can write the left hand side of this permutation as ${}^{(m)}\varepsilon$, and its right hand side hand side is of the form $\begin{pmatrix} m+1 & m+2 & \dots \\ \varphi_{m+1} & \varphi_{m+2} & \dots \end{pmatrix}$ and we denote it by $\varphi_m^{\infty}$. Accordingly, we can write $_{(m,\infty)}\varphi$ conditionally as $_{(m,\infty)}\varphi = ({}^{(m)}\varepsilon \mid \varphi_m^{\infty})$. Let $_{(m,\infty)}\varphi$ and $_{(m,\infty)}\psi$ be activated permutations in $S_m(\mathbb{N})$. Then their product belongs to $S_m(\mathbb{N})$. It can be easily shown that the identity permutation $\varepsilon$ for every $m$ and the inverse of a permutation activating after $m$ are also activated permutation after $m$. Thus, the set $S_m(\mathbb{N})$ of permutations activated after $m$ is a subgroup of $S(\mathbb{N})$ of permutations.

We can determine the mapping $\delta : S_m(\mathbb{N}) \to S(\mathbb{N}_m^{\infty})$ as follows:

$$\delta(_{(m,\infty)}\varphi) = \varphi_m^{\infty}. \tag{2.7}$$

Then the following holds.

**Proposition 3** The mapping $\delta : S_m(\mathbb{N}) \to S(\mathbb{N}_m^{\infty})$ in (2.7) is a group isomorphism.⊠

By the definition of the group of activated permutations, we have $S_m(\mathbb{N}) > S_{m+1}(\mathbb{N})$ for each $m \in \mathbb{N}$. Thus we obtain the successive groups

$$S_1(\mathbb{N}) > S_2(\mathbb{N}) > \dots > S_m(\mathbb{N}) > S_{m+1}(\mathbb{N}) > \dots \tag{2.8}$$

That $S(\mathbb{N}) > S_1(\mathbb{N})$ is obvious. Hence, we can define (2.8) under the following form:

$$(2.9) \qquad S(\mathbb{N}) > S_1(\mathbb{N}) > S_2(\mathbb{N}) > \ldots > S_m(\mathbb{N}) > S_{m+1}(\mathbb{N}) > \ldots$$

Given activated permutations ${}_{(m,\infty)}\varphi$ and ${}_{(n,\infty)}\psi$, their product becomes after $\min\{m,n\}$. For each $(m,n)$ of positive integers, the product operation

$$\left({}_{(m,\infty)}\varphi, {}_{(n,\infty)}\psi\right) \mapsto {}_{(m,\infty)}\varphi \cdot {}_{(n,\infty)}\psi$$

in the group $S(\mathbb{N})$ generates the mapping

$$(2.10) \qquad S_m(\mathbb{N}) \times S_n(\mathbb{N}) \to S^{min\{m,n\}}(\mathbb{N})$$

Thus, we have determined two classes of infinite degree permutations:

(*i*) $S_m(\mathbb{N})$ : stabilized after $m$; ${}^{(\infty,m)}\varphi = ({}^{(m)}\varphi \mid \varepsilon_m^\infty)$.

(*ii*) $S^m(\mathbb{N})$ : activated after $m$; ${}_{(m,\infty)}\varepsilon = ({}^{(m)}\varepsilon \mid \varphi_m^\infty)$.

2.3. **Detached Permutations.** Let's define class of infinite degree. If the condition

"There exists $m \in \mathbb{N}$ such that for all $j \leq m$ implies ${}^{\infty}\varphi(j) \leq m$" $\qquad (*)$

is satisfied, then we say that the permutation ${}^{(\infty)}\varphi$ is detached and the number $m$ detaches the permutation ${}^{(\infty)}\varphi$. We will denote the set of permutations detached by the number $m$ by $B^m(\mathbb{N})$.
The following holds.

**Proposition 4** The set $B^m$ of all permutations detached by the number $m$ is a subgroup of the group $S(\mathbb{N})$ of permutations of infinite degree.⊠

It is clear that the permutation groups $S_m(\mathbb{N})$ activated after $m$ and groups $S^m(\mathbb{N})$ stabilized after $m$ are subgroups of the group $B^m$:

$$S^m(\mathbb{N}) < B^m \text{ and } S_m(\mathbb{N}) < B^m.$$

Suppose that the permutations ${}^{(\infty,m)}\varphi = ({}^{(m)}\varphi \mid \varepsilon_m^\infty) \in S^m(\mathbb{N})$ and ${}_{(m,\infty)}\psi = ({}^{(m)}\varepsilon \mid \psi_m^\infty) \in S_m(\mathbb{N})$ are given.

**Proposition 5** For each natural number $m$, the elements of the groups $S^m(\mathbb{N})$ and $S_m(\mathbb{N})$ are translocation.

**Theorem 1** The group $B^m$ is equal to the simple product of the subgroups $S^m(\mathbb{N})$ and $S_m(\mathbb{N})$:

$$B^m = S^m(\mathbb{N}).S_m(\mathbb{N}). \boxtimes$$

Each detached permutation $\beta \in B^m$ is shown as

$$\beta = \begin{pmatrix} 1 & 2 & \dots & m & m+1 & m+2 & \dots \\ \beta_1 & \beta_2 & \dots & \beta_m & \beta_{m+1} & \beta_{m+2} & \dots \end{pmatrix} = ({}^{(m)}\beta \mid \beta_m^\infty).$$

2.4. **Matrix Representation of Permutations.** We shall give a product group representation. To do this, associate each permutation $\varphi$ with the matrix $A_\varphi$ determined by the following formula:

$$A_\varphi(i,j) = \begin{cases} 1 & \text{if } \varphi_i = j \\ 0 & \text{if } \varphi_i \neq j \end{cases} \qquad (**)$$

In each row and column of the matrix specified by formula (**), 1 can occur only once, and the remaining entries are 0. Such matrices are called of type $0-1$ with dimension $m$ by $M_m(0-1)$. Thus, every $m$ degree permutation can be represented by an $m$ dimension matrix, and hence the product of permutations corresponds to the product of matrices. The correspondence determined by formula (**) will be denoted by $A : S(m) \to M_m(0-1)$, $A(\varphi) = A_\varphi$.On the other hand, we denote by $MS(m)$ the image of the group of permutations $S(m)$ in the mapping $A$. The group $MS(m)$ is a subgroup of the group of unimodular matrices (i.e., matrices with determinants $\pm 1$). We would like to emphasize that in this representation, even permutations correspond to matrices with positive determinants, while odd permutations correspond to matrices with negative determinants. Consequently, we have the isomorphism:

$$S(m) \approx MS(m).$$

Foregoing discussion can be expanded to the permutation group of infinite degree. If ${}^{(\infty,m)}\varphi = ({}^{(m)}\varphi \mid \varepsilon_m^\infty)$ is a stabilized permutation after $m$, then the matrix

$$A_{(\infty,m)_\varphi} = \begin{pmatrix} A_{(m)_\varphi} & 0 \\ 0 & E_\infty \end{pmatrix}$$

is given, where $A_{(m)_\varphi}$ an element of the group $MS(m)$. We will write $MS^m(\mathbb{N})$ for the group of all matrices of the form $A_{(\infty,m)_\varphi}$.

**Proposition 6** For each $m$ the group ${}^mS(\mathbb{N})$ stabilized after $m$ is isomorphic to the group $MS(\mathbb{N})$ of matrices.

$${}^mS(\mathbb{N}) \approx MS^m(\mathbb{N}) \approx MS(m).\boxtimes$$

If we consider the permutation ${}_{(m,\infty)_\varphi} = ({}^{(m)}\varepsilon \mid \varphi_m^\infty)$ activated after $m$, then the matrix

$$A_{(m,\infty)\varphi} = \begin{pmatrix} E_m & 0 \\ 0 & A_{\varphi_m^\infty} \end{pmatrix}$$

corresponds to it. Such matrices form a group, which is denoted by $MS_m(\mathbb{N})$.

**Proposition 7** For every $m$, the group $S_m(\mathbb{N})$ activated after $m$ is isomorphic to the group $MS_m(\mathbb{N})$ of matrices.$\boxtimes$

Now let's move on to the representation for the detached permutations matrix. As indicated in Theorem 1 above, each detached permutation $\beta \in B^m$ can be represented as $\beta = ({}^{(m)}\beta \mid \beta_m^\infty)$. Then the matrix

$$A_{(m,\infty)}\varphi = \begin{pmatrix} A_{(m)}\beta & 0 \\ 0 & A_{\beta_m^\infty} \end{pmatrix}$$

corresponds to it.

## 3. Equivalence Relation

Suppose that $\alpha$ and $\beta$ are infinite degree permutations, that is, $\alpha, \beta \in S(\mathbb{N})$. If

$$\exists K \forall j > K \Rightarrow \alpha(j) = \beta(j) \tag{3.11}$$

holds then the infinite degree permutations $\alpha$ and $\beta$ are equivalent and we denote it by $\alpha \sim \beta$.

**Proposition 8** The relation defined by (3.11) is an equivalence relation on the set of infinite degree permutations $S\mathbb{N})$.

**Proof** Since $\alpha(j) = \alpha(j)$ holds for all $j \in \mathbb{N}$, the relation $\sim$ is reflective.

Let$\alpha \sim \beta$. Then we write $\exists K \forall j > K \Rightarrow \alpha(j) = \beta(j)$. Hence, we get $\exists K \forall j > K \Rightarrow \beta(j) = \alpha(j)$, that is, $\alpha \sim \beta$. Therefore, the relation $\sim$ is symmetric.

Let $\alpha \sim \beta$ and $\beta \sim \gamma$. Since $\alpha \sim \beta$, $\exists K \forall j > K \Rightarrow \alpha(j) = \beta(j)$ and since $\beta \sim \gamma$, $\exists L \forall i > L \Rightarrow \beta(i) = \gamma(i)$. Let $M$ be the maximum number of the numbers $K$ and $L$. For $M = \max\{K, L\}$, $\exists M \forall k > M \Rightarrow \gamma(k) = \alpha(k) = \beta(k)$ is obtained. In this case we get $\alpha \sim \gamma$ and so the relation is transitive.□

Let $S(I\!N)/\sim$ denote the factor set of the set of infinite degree permutations set $S(I\!N)$ with respect to equivalence relation $\sim$. Let us show the elements of factor set $S(I\!N)/\sim$, namely the equivalence class of permutations, denoted by $[\varphi]$.

We define group operation on factor set $S(I\!N)/\sim$. The product of $[\varphi]$ and $[\psi]$ is defined by

$$[\varphi].[\psi] = [\varphi.\psi]. \tag{3.12}$$

**Theorem 2** The product of infinite degree permutations satisfies the equivalence relation.

**Proof** We show that the condition (3.12) holds. In other words, if the equations

$$\varphi \sim \varphi' \Leftrightarrow \exists K \forall j > K \Rightarrow \varphi(j) = \varphi'(j) \tag{3.13}$$

and

$$\psi \sim \psi' \Leftrightarrow \exists L \forall i > L \Rightarrow \psi(i) = \psi'(i) \tag{3.14}$$

are given, it is clear that the relation $\varphi.\psi \sim \varphi'.\psi'$ is true.

There are two cases for each $i \in M$: Case I: $\psi(i) > L$ and Case II: $\psi(i) < L$. $K$ and $L$ are the natural numbers that satisfy the conditions (3.13) and (3.14).

**Case I:** Since $\forall i > L \Rightarrow \psi(i) = \psi'(i)$, then $\psi'(i) > K$. Then $\varphi.\psi(i) = \varphi'.\psi'(i)$ for all $i > L$.

**Case II:** $i > L$, only $\psi(i) < K$.The set of numbers which takes values than the infinite degree permutations $\psi$ is given by $\{x \in \mathbb{N} : \psi(x) < K\}$ and this set is finite. Then there exists $\exists k \in \mathbb{N}$ such that $\forall t > k \Rightarrow \psi(t) > K$. From (3.13) we get $\forall t > k \Rightarrow \varphi.\psi(t) = \varphi'.\psi'(t)$. Thus we get $\varphi.\psi \sim \varphi'.\psi'$.□

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