The (s, t) Jacobsthal and (s, t) Jacobsthal-Lucas Matrix Sequences

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March 23, 2012

Abstract

In this study, we first define *new sequences* named (s,t)-*Jacobsthal and* (s,t) *Jacobsthal-Lucas sequences.* After that, by using these sequences, we establish (s,t)-*Jacobsthal and* (s,t) *Jacobsthal-Lucas matrix sequences.* Finally we present some important relationships between these matrix sequences.

Keywords: (s,t)-Jacobsthal sequence, (s,t)-Jacobsthal-Lucas sequence, (s,t)-Jacobsthal matrix sequence, (s,t)-Jacobsthal-Lucas matrix sequence.
AMS Classifications:15A24, 11B39

1 Introduction

In the last years, we have seen a great many studies on the different number sequences. From these sequences, Fibonacci F_n and Lucas L_n numbers are the terms of the sequences $\{0,\ 1,\ 1,\ 2,\ 3,\ 5,\ ...\}$ and $\{2,\ 1,\ 3,\ 4,\ 7,\ 11,\ ...\}$ wherein each term is the sum of the two previous terms, beginning with the values $F_0 = 0$, $F_1 = 1$ and $L_0 = 2$, $L_1 = 1$ respectively. Similarly Jacobsthal and Jacobsthal-Lucas numbers are also given by recurrence relations $S_{n+1} = S_n + 2S_{n-1}$, $S_0 = 0$, $S_1 = 1$ and $s_{n+1} = s_n + 2s_{n-1}$, $s_0 = 2$, $s_1 = 2$ for $n \geq 1$, respectively [1-5]. In the literature, in [6-8], there are the some generalizations of the Fibonacci, Jacobsthal and Pell families. For instance, in [6], Falcon and Plaza introduce $k-$Fibonacci sequence $\{F_{k,n}\}_{n=0}^{\infty}$ by using Fibonacci and Pell sequences. Many properties of these numbers were deduced directly from elementary matrix algebra. In [12], Authors defined generalized $k-$Horadam sequence and examined the properties of these sequence. Then, in

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[9], we defined a new generalization $\{G_{k,n}\}_{n\in\mathrm{N}}$ of $k-$Fibonacci family. Civciv, in [10-11], defined (s,t) Fibonacci and (s,t) Lucas matrix sequences by using (s,t) Fibonacci and (s,t) Lucas sequences. He also gave some properties related to these matrix sequences.

In this study, we firstly define (s,t)-Jacobsthal and (s,t)-Jacobsthal-Lucas sequences, then by using these sequences, we also define (s,t)-Jacobsthal and (s,t)-Jacobsthal Lucas matrix sequences. In the last of the study, we investigate the relationships between (s,t)-Jacobsthal and (s,t)-Jacobsthal Lucas matrix sequences.

2 Main Results

Let us first consider the following definitions of (s,t) *Jacobsthal and* (s,t) *Jacobsthal Lucas sequences* which will be needed for the definitions of (s,t) *Jacobsthal and* (s,t) *Jacobsthal Lucas matrix sequences* and relationships between them.

Definition 1 *For any real numbers* $s,\ t$; *the* $(s,\ t)$ *Jacobsthal* $\{j_n(s,\ t)\}_{n\in\mathrm{N}}$ *and the* $(s,\ t)$ *Jacobsthal-Lucas* $\{c_n(s,\ t)\}_{n\in\mathrm{N}}$ *sequences are defined recurrently by*

$$j_n(s,t) = sj_{n-1}(s,t) + 2tj_{n-2}(s,t), \quad j_0(s,t) = 0,\ j_1(s,t) = 1,\ n \geq 2, \tag{1}$$

and

$$c_n(s,t) = sc_{n-1}(s,t) + 2tc_{n-2}(s,t), \quad c_0(s,t) = 2,\ c_1(s,t) = s,\ n \geq 2, \tag{2}$$

respectively, where $t \neq 0$ *and* $s^2 + 8t \neq 0$.

Particular cases of the previous definition are:

- If $s = 1,\ t = 1/2$ and $j_0(1,1/2) = 0,\ j_1(1,1/2) = 1,$ the classic Fibonacci sequence is obtained,
- If $s = 1,\ t = 1/2$ and $c_0(1,1/2) = 2,\ c_1(1,1/2) = 1,$ the classic Lucas sequence is obtained,
- If $s = t = 1$ and $j_0(1,1) = 0,\ j_1(1,1) = 1,$ the classic Jacobsthal sequence is obtained,
- If $s = t = 1$ and $c_0(1,1) = 2,\ c_1(1,1) = 2,$ the classic Jacobsthal-Lucas sequence is obtained,

Furthermore, in the following proposition, (s,t)-Jacobsthal matrix sequence $\{J_n(s,t)\}_{n\in\mathrm{N}}$ is defined by carrying to matrix theory (s,t)-Jacobsthal sequence.

Definition 2 *For any integer numbers* $s,\ t$; *the* $(s,\ t)$ *th Jacobsthal matrix sequence* $\{J_n(s,\ t)\}_{n\in\mathrm{N}}$ *is defined recurrently by*

$$\begin{aligned} J_n(s,t) &= sJ_{n-1}(s,t) + 2tJ_{n-2}(s,t), \\ J_0(s,t) &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ J_1(s,t) = \begin{pmatrix} s & 2 \\ t & 0 \end{pmatrix},\ n \geq 2, \end{aligned} \tag{3}$$

respectively, where $t \neq 0$ *and* $s^2 + 8t \neq 0$.

Theorem 3 *For any integer* $n \geq 1$, *we have*

$$J_n(s,t) = \begin{pmatrix} j_{n+1}(s,t) & 2j_n(s,t) \\ tj_n(s,t) & 2tj_{n-1}(s,t) \end{pmatrix} \tag{4}$$

with initial values $J_0(s,t) = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $J_1(s,t) = \begin{pmatrix} s & 2 \\ t & 0 \end{pmatrix}$.

Proof. If we take $n = 2$ in (3), then we get

$$J_2(s,t) = \begin{pmatrix} j_3(s,t) & j_2(s,t) \\ tj_2(s,t) & 2tj_1(s,t) \end{pmatrix} = \begin{pmatrix} s^2 + 2t & 2s \\ st & 2t \end{pmatrix}.$$

By considering induction steps, let us suppose that the equality in (4) holds for all $n = k \in \mathbb{Z}^+$. To end up the proof, we have to show that the case also holds for $n = k + 1$. Therefore we can write

$$\begin{aligned} J_{k+1}(s,t) &= sJ_k(s,t) + 2tJ_{k-1}(s,t) \\ &= s\begin{pmatrix} j_{k+1}(s,t) & 2j_k(s,t) \\ tj_k(s,t) & 2tj_{k-1}(s,t) \end{pmatrix} \\ &\quad +2t\begin{pmatrix} j_k(s,t) & 2j_{k-1}(s,t) \\ tj_{k-1}(s,t) & 2tj_{k-2}(s,t) \end{pmatrix} \\ &= \begin{pmatrix} sj_{k+1}(s,t) + 2tj_k(s,t) & 2sj_k(s,t) + 4tj_{k-1}(s,t) \\ stj_k(s,t) + 2t^2j_{k-1}(s,t) & 2stj_{k-1}(s,t) + 4t^2j_{k-2}(s,t) \end{pmatrix} \\ &= \begin{pmatrix} j_{k+2}(s,t) & 2j_{k+1}(s,t) \\ tj_{k+1}(s,t) & 2tj_k(s,t) \end{pmatrix}. \end{aligned}$$

Hence the result. ∎

Theorem 4 *For any integer* $m, n \geq 0$, *we get*

$$J_{m+n}(s,t) = J_m(s,t)J_n(s,t). \tag{5}$$

Proof. It's proven by induction. Consider $n = 0$, it's true. Let us suppose that the equality in (5) holds for all $n = k \in \mathbb{Z}^+$. Consequently, we have to show that the case also holds for $n = k + 1$. Therefore we can write

$$\begin{aligned} J_{m+n+1}(s,t) &= sJ_{m+n}(s,t) + 2tJ_{m+n-1}(s,t) \\ &= sJ_m(s,t)J_n(s,t) + 2tJ_m(s,t)J_{n-1}(s,t) \\ &= J_m(s,t)\left[sJ_n(s,t) + 2tJ_{n-1}(s,t)\right] \\ &= J_m(s,t)J_{n+1}(s,t). \end{aligned}$$

Furthermore, in the following proposition, (s,t)-Jacobsthal-Lucas matrix sequence $\{C_n(s,t)\}_{n\in\mathbb{N}}$ is defined by carrying to matrix theory (s,t)-Jacobsthal-Lucas sequence. ∎

Definition 5 *For any integer numbers s, t; the $(s,\ t)$ th Jacobsthal-Lucas matrix sequence $\{C_n(s,\ t)\}_{n\in\mathbb{N}}$ is defined recurrently by*

$$\begin{aligned} C_n(s,t) &= sC_{n-1}(s,t) + 2tC_{n-2}(s,t),\ n \geq 2 \qquad (6)\\ C_0(s,t) &= \begin{pmatrix} s & 4 \\ 2t & -s \end{pmatrix},\ C_1(s,t) = \begin{pmatrix} s^2+4t & 2s \\ st & 4t \end{pmatrix}, \end{aligned}$$

respectively, where $t \neq 0$ and $s^2 + 8t \neq 0$.

Theorem 6 *For any integer $n \geq 1$, we have*

$$C_n(s,t) = \begin{pmatrix} c_{n+1}(s,t) & 2c_n(s,t) \\ tc_n(s,t) & 2tc_{n-1}(s,t) \end{pmatrix}, \qquad (7)$$

with initial values $C_0(s,t) = \begin{pmatrix} s & 4 \\ 2t & -s \end{pmatrix}$ and $C_1(s,t) = \begin{pmatrix} s^2+4t & 2s \\ st & 4t \end{pmatrix}$.

Proof. By using (2) and (7), for $n = 2$, we also have

$$C_2(s,t) = \begin{pmatrix} c_3(s,t) & 2c_2(s,t) \\ tc_2(s,t) & 2tc_1(s,t) \end{pmatrix} = \begin{pmatrix} s^3+6st & 2s^2+8t \\ s^2t+4t^2 & 2ts \end{pmatrix}.$$

By considering induction steps, let us suppose that the equality in (7) holds for all $n = k \in \mathbb{Z}^+$. To end up the proof, we have to show that the case also holds for $n = k+1$. Therefore we have

$$\begin{aligned} C_{k+1}(s,t) &= sC_k(s,t) + 2tC_{k-1}(s,t) \\ &= s\begin{pmatrix} c_{k+1}(s,t) & 2c_k(s,t) \\ tc_k(s,t) & 2tc_{k-1}(s,t) \end{pmatrix} \\ &\quad +2t\begin{pmatrix} c_k(s,t) & 2c_{k-1}(s,t) \\ tc_{k-1}(s,t) & 2tc_{k-2}(s,t) \end{pmatrix} \\ &= \begin{pmatrix} sc_{k+1}(s,t) + 2tc_k(s,t) & 2sc_k(s,t) + 4tc_{k-1}(s,t) \\ stc_k(s,t) + 2t^2c_{k-1}(s,t) & 2stc_{k-1}(s,t) + 4t^2c_{k-2}(s,t) \end{pmatrix} \\ &= \begin{pmatrix} c_{k+2}(s,t) & 2c_{k+1}(s,t) \\ tc_{k+1}(s,t) & 2tc_k(s,t) \end{pmatrix}. \end{aligned}$$

Hence the result.

Let us consider the following proposition which will be used for the results in this section. In fact, by this proposition, it will be given a relationship between the sequences $\{J_n(s,t)\}_{n\in\mathbb{N}}$ and $\{C_n(s,t)\}_{n\in\mathbb{N}}$. ∎

Proposition 7 *For $n \geq 0$, we get*

$$C_{n+1}(s,t) = C_1(s,t)J_n(s,t). \qquad (8)$$

Proof. For $n = 1$, it is obvious from $C_1(s,t) = \begin{pmatrix} s^2+4t & 2s \\ st & 4t \end{pmatrix}$ and $J_1(s,t) = \begin{pmatrix} s & 2 \\ t & 0 \end{pmatrix}$,

$$\begin{aligned} C_2(s,t) &= C_1(s,t)J_1(s,t) \\ &= \begin{pmatrix} s^2+4t & 2s \\ st & 4t \end{pmatrix}\begin{pmatrix} s & 2 \\ t & 0 \end{pmatrix} \\ &= \begin{pmatrix} s^3+6st & 2s^2+8t \\ s^2t+4t^2 & 2st \end{pmatrix} \\ &= \begin{pmatrix} c_3(s,t) & 2c_2(s,t) \\ tc_2(s,t) & 2tc_1(s,t) \end{pmatrix}. \end{aligned}$$

Suppose that $C_1(s,t)J_N(s,t) = C_{N+1}(s,t)$, for $n = N$. We have to show that the case also holds for $n = N+1$.

$$\begin{aligned} C_1(s,t)J_{N+1}(s,t) &= C_1(s,t)J_N(s,t)J_1(s,t) \\ &= C_{N+1}(s,t)J_1(s,t) \\ &= \begin{pmatrix} c_{N+2}(s,t) & 2c_{N+1}(s,t) \\ tc_{N+1}(s,t) & 2tc_N(s,t) \end{pmatrix}\cdot\begin{pmatrix} s & 2 \\ t & 0 \end{pmatrix} \\ &= \begin{pmatrix} c_{N+3}(s,t) & 2c_{N+2}(s,t) \\ tc_{N+2}(s,t) & 2tc_{N+1}(s,t) \end{pmatrix} \\ &= C_{N+1}(s,t). \end{aligned}$$

■

Proposition 8 *For $m, n \geq 0$, we have*

$$J_m(s,t)C_{n+1}(s,t) = C_{n+1}(s,t)J_m(s,t). \tag{9}$$

Proof. From proposition 7 and Theorem 4, we can write

$$\begin{aligned} J_m(s,t)C_{n+1}(s,t) &= J_m(s,t)C_1(s,t)J_n(s,t) \\ &= J_m(s,t)\left[sJ_1(s,t)+4tJ_0(s,t)\right]J_n(s,t) \\ &= sJ_{n+m+1}(s,t)+4tJ_{n+m}(s,t) \\ &= \left[sJ_1(s,t)+4tJ_0(s,t)\right]J_{n+m}(s,t) \\ &= C_1(s,t)J_n(s,t)J_m(s,t) \\ &= C_{n+1}(s,t)J_m(s,t). \end{aligned}$$

■

Proposition 9 *For $n \geq 1$, we have*

$$C_n(s,t) = sJ_n(s,t) + 4tJ_{n-1}(s,t). \tag{10}$$

Proof. For $n = 1$, it is obvious

$$\begin{aligned} C_1(s,t) &= sJ_1(s,t) + 4tJ_0(s,t) \\ &= \begin{pmatrix} s^2+4t & 2s \\ ts & 4t \end{pmatrix} = s\begin{pmatrix} s & 2 \\ t & 0 \end{pmatrix} + 4t\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned}$$

For $n = M$, assume that $C_M(s,t) = sJ_M(s,t) + 4tJ_{M-1}(s,t)$. We will show that the case also holds for $n = N+1$.

$$\begin{aligned} C_{N+1}(s,t) &= C_1(s,t)J_N(s,t) \\ &= [sJ_1(s,t) + 4tJ_0(s,t)]\, J_N(s,t) \\ &= sJ_{N+1}(s,t) + 4tJ_N(s,t). \end{aligned}$$

■

Proposition 10 *For $n \geq 0$, we have*

a) $C^2_{n+1}(s,t) = C^2_1(s,t)J_{2n}(s,t)$,

b) $C^2_{n+1}(s,t) = C_1(s,t)C_{2n+1}(s,t)$,

c) $C_{2n+1}(s,t) = J_n(s,t)C_{n+1}(s,t)$.

Proof. From proposition 7 and the proof of a) is obvious

$$\begin{aligned} C^2_{n+1}(s,t) &= C_{n+1}(s,t)C_{n+1}(s,t) \\ &= C_1(s,t)J_n(s,t)C_1(s,t)J_n(s,t) \\ &= C^2_1(s,t)J_{2n}(s,t). \end{aligned}$$

From a) and proposition 7, we can write

$$\begin{aligned} C^2_{n+1}(s,t) &= C^2_1(s,t)J_{2n}(s,t) \\ &= C_1(s,t)C_1(s,t)J_{2n}(s,t) \\ &= C_1(s,t)C_{2n+1}(s,t). \end{aligned}$$

Using proposition 7, we have

$$\begin{aligned} C_{2n+1}(s,t) &= C_1(s,t)J_{2n}(s,t) \\ &= J_n(s,t)C_{n+1}(s,t). \end{aligned}$$

■

Corollary 11 *For $n \geq 0$, we have*

a) $c^2_{n+2}(s,t) + 2tc^2_{n+1}(s,t) = \left(s^2+8t\right) j_{2n+3}(s,t)$,

b) $c^2_{n+2}(s,t) + 2tc^2_{n+1}(s,t) = c_{2n+4}(s,t) + 2tc_{2n+2}(s,t)$,

c) $c_{2n}(s,t) = j_n(s,t)c_{n+1}(s,t) + 2tj_{n-1}(s,t)c_n(s,t)$.

Proof. From a) of proposition 10, the proof of a) of this corollary is obvious:

$$\begin{aligned} C^2_{n+1}(s,t) &= C^2_1(s,t)J_{2n}(s,t) \\ \begin{pmatrix} c_{n+2}(s,t) & 2c_{n+1}(s,t) \\ tc_{n+1}(s,t) & 2tc_n(s,t) \end{pmatrix}^2 &= \begin{pmatrix} s^2+4t & 2s \\ st & 4t \end{pmatrix}^2 \begin{pmatrix} j_{2n+1}(s,t) & 2j_{2n}(s,t) \\ tj_{2n}(s,t) & 2tj_{2n-1}(s,t) \end{pmatrix}. \end{aligned}$$

From the equality of the first entries of matrices in both of sides of the equation, we write

$$\begin{aligned} c^2_{n+2}(s,t)+2tc^2_{n+1}(s,t) &= (s^4+10s^2t+16t^2)\,j_{2n+1}(s,t)+t(2s^3+16st)j_{2n}(s,t) \\ &= s^3(sj_{2n+1}(s,t)+2tj_{2n}(s,t))+ \\ &\quad +8st(sj_{2n+1}(s,t)+2tj_{2n}(s,t))+ \\ &\quad +2s^2tj_{2n+1}(s,t)+16t^2j_{2n+1}(s,t) \\ &= s^3j_{2n+2}(s,t)+8stj_{2n+2}(s,t)+ \\ &\quad +2s^2tj_{2n+1}(s,t)+16t^2j_{2n+1}(s,t) \\ &= s^2(sj_{2n+2}(s,t)+2tj_{2n+1}(s,t))+8t(sj_{2n+2}(s,t)+ \\ &\quad +2tj_{2n+1}(s,t)) \\ &= (s^2+8t)j_{2n+3}(s,t). \end{aligned}$$

From b) of proposition 10, the proof of b) of this corollary can be clearly seen:

$$\begin{aligned} C^2_{n+1}(s,t) &= C_1(s,t)C_{2n+1}(s,t) \\ \begin{pmatrix} c_{n+2}(s,t) & 2c_{n+1}(s,t) \\ tc_{n+1}(s,t) & 2tc_n(s,t) \end{pmatrix}^2 &= \begin{pmatrix} s^2+4t & 2s \\ st & 4t \end{pmatrix} \begin{pmatrix} c_{2n+2}(s,t) & 2c_{2n+1}(s,t) \\ tc_{2n+1}(s,t) & 2tc_{2n}(s,t) \end{pmatrix}. \end{aligned}$$

From the equality of the first entries of matrices in both of sides of the equation, we have

$$\begin{aligned} c^2_{n+2}(s,t)+2tc^2_{n+1}(s,t) &= s(sc_{2n+2}(s,t)+2tc_{2n+1}(s,t))+4tc_{2n+2}(s,t) \\ &= (sc_{2n+3}(s,t)+2tc_{2n+2}(s,t))+2tc_{2n+2}(s,t) \\ &= c_{2n+4}(s,t)+2tc_{2n+2}(s,t). \end{aligned}$$

From c) of proposition 10, the proof of c) of this corollary is obvious:

$$\begin{aligned} C_{2n+1}(s,t) &= J_n(s,t)C_{n+1}(s,t) \\ \begin{pmatrix} c_{2n+2}(s,t) & 2c_{2n+1}(s,t) \\ tc_{2n+1}(s,t) & 2tc_{2n}(s,t) \end{pmatrix} &= \begin{pmatrix} j_{n+1}(s,t) & 2j_n(s,t) \\ tj_n(s,t) & 2tj_{n-1}(s,t) \end{pmatrix} \\ &\quad \begin{pmatrix} c_{n+2}(s,t) & 2c_{n+1}(s,t) \\ tc_{n+1}(s,t) & 2tc_n(s,t) \end{pmatrix}. \end{aligned}$$

Equalizing the entries in the second coloumn and row in both of sides of the matrix equality gives

$$\begin{aligned} 2tc_{2n}(s,t) &= 2t(j_n(s,t)c_{n+1}(s,t)+2tj_{n-1}(s,t)c_n(s,t)) \\ c_{2n}(s,t) &= j_n(s,t)c_{n+1}(s,t)+2tj_{n-1}(s,t)c_n(s,t). \end{aligned}$$

Binet Formula enables us to state (s,t) *Jacobsthal and* (s,t) *Jacobsthal Lucas numbers.* It can be clearly obtained from the roots r_1 and r_2 of characteristic equations of (1) and (2) as the form $x^2 = sx + 2t$, where $r_1 = \frac{s+\sqrt{s^2+8t}}{2}$, $r_2 = \frac{s-\sqrt{s^2+8t}}{2}$. ■

Corollary 12 *For $n \geq 0$, the Binet Formulas for nth (s,t) Jacobsthal number and nth (s,t) Jacobsthal Lucas number are given by*

$$j_n(s,t) = \frac{r_1^n - r_2^n}{r_1 - r_2}, \tag{11}$$

and

$$c_n(s,t) = r_1^n + r_2^n \tag{12}$$

respectively.

Proof. The proof of first equality is obvious from the principle of induction n. Now, let us prove second equality. It follows from proposition 9 that we have

$$c_n(s,t) = sj_n(s,t) + 4tj_{n-1}(s,t).$$

From (11), we have

$$\begin{aligned} c_n(s,t) &= s\frac{r_1^n - r_2^n}{r_1 - r_2} + 4t\frac{r_1^{n-1} - r_2^{n-1}}{r_1 - r_2} \\ &= \frac{\left(s + \frac{4t}{r_1}\right) r_1^n - \left(s + \frac{4t}{r_2}\right) r_2^n}{r_1 - r_2} \\ &= \frac{\frac{r_1 s + 4t}{r_1} r_1^n - \frac{r_2 s + 4t}{r_2} r_2^n}{r_1 - r_2} \\ &= \frac{\frac{r_1^2 + 2t}{r_1} r_1^n - \frac{r_2^2 + 2t}{r_2} r_2^n}{r_1 - r_2} \\ &= \frac{\left(\frac{r_1^2 - r_1 r_2}{r_1}\right) r_1^n - \left(\frac{r_2^2 - r_1 r_2}{r_2}\right) r_2^n}{r_1 - r_2} \\ &= r_1^n + r_2^n. \end{aligned}$$

■

Proposition 13 *For $n \geq 0$, we get*

$$J_n(s,t) = \left(\frac{J_1(s,t) - r_2 J_0(s,t)}{r_1 - r_2}\right) r_1^n - \left(\frac{J_1(s,t) - r_1 J_0(s,t)}{r_1 - r_2}\right) r_2^n.$$

Proof.

$$\begin{aligned} J_n(s,t) &= \frac{r_1^n}{r_1-r_2}\begin{pmatrix} s-r_2 & 2 \\ t & -r_2 \end{pmatrix} - \frac{r_2^n}{r_1-r_2}\begin{pmatrix} s-r_1 & 2 \\ t & -r_1 \end{pmatrix} \\ &= \frac{1}{r_1-r_2}\begin{pmatrix} s\left(r_1^n-r_2^n\right)-r_1r_2\left(r_1^{n-1}-r_2^{n-1}\right) & 2\left(r_1^n-r_2^n\right) \\ t\left(r_1^n-r_2^n\right) & -r_1r_2\left(r_1^{n-1}-r_2^{n-1}\right) \end{pmatrix} \\ &= \left(\frac{J_1(s,t)-r_2J_0(s,t)}{r_1-r_2}\right)r_1^n - \left(\frac{J_1(s,t)-r_1J_0(s,t)}{r_1-r_2}\right)r_2^n. \end{aligned}$$

■

Proposition 14 *For $n \geq 0$, we have*

$$C_{n+1}(s,t) = \left(\frac{C_2(s,t)-r_2C_1(s,t)}{r_1-r_2}\right)r_1^n - \left(\frac{C_2(s,t)-r_1C_1(s,t)}{r_1-r_2}\right)r_2^n.$$

Proof. By using the above proposition and $C_{n+1}(s,t) = C_1(s,t)J_n(s,t)$, it can be clearly seen.

Conclusion 15 *We define new sequences named (s,t) Jacobsthal and (s,t) Jacobsthal Lucas sequences. By using these sequences, we establish (s,t) Jacobsthal and (s,t) Jacobsthal Lucas matrix sequences. Similarly, one can define matrix sequences releted to other number sequences and can examine their properties.*

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