A FIXED POINT THEOREM FOR WEAKLY COMPATIBLE MAPPINGS SATISFYING A GENERAL CONTRACTIVE CONDITION OF OPERATOR TYPE

ISHAK ALTUN AND DURAN TURKOGLU

ABSTRACT. In this paper, we prove a fixed point theorem for weakly compatible mappings satisfying a general contractive condition of operator type. In short, we are going to study mappings $A, B, S, T : X \to X$ for which there exists a right continuous function $\psi : \mathbb{R}^+ \to \mathbb{R}^+$, $\psi(0) = 0$ and $\psi(s) < s$ for $s > 0$ such that for each $x, y \in X$ one has $O(f; d(Sx, Ty)) \leq \psi(O(f; M(x, y)))$, where $O(f; \cdot)$ and f are defined in the first section. Also in the first section, we give some examples for $O(f; \cdot)$. The second section contains the main result. In the last section, we give some corollaries and remarks.

1. INTRODUCTION

Branciari [10] obtained a fixed point result for a single mapping satisfying an analogue of Banach's contraction principle for an integral type inequality. The authors in [4], [7], [8], [9], [13], [19], [25] and [27] proved some fixed point theorems involving more general contractive conditions. The authors in [5] have improved the concept of contractive condition of operator type and proved a fixed point theorem for single mapping using this type condition. In this paper, we establish a fixed point theorem for weakly compatible maps satisfying a general contractive inequality of operator type. This result substantially extends the theorems of the above work.

Sessa [21] generalized the concept of commuting mappings by calling self-mappings A and S of metric space (X, d) a weakly commuting pair if and only if $d(ASx, SAx) \leq d(Ax, Sx)$ for all $x \in X$, and he and others proved some common fixed point theorems of weakly commuting mappings [20]-[23]. Then, Jungck [14] introduced the concept of compatibility and he and others proved some common fixed point theorems using this concept [14]-[17], [26].

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Clearly, commuting mappings are weakly commuting and weakly commuting mappings are compatible. Examples in [21] and [14] show that neither converse is true.

Recently, Jungck and Rhoades [16] defined the concept of weak compatibility.

Definition 1 ([16], [24]). *Two maps $A, S : X \to X$ are said to be weakly compatible if they commute at their coincidence points.*

Again, it is obvious that compatible mappings are weakly compatible. Examples in [16] and [24] shows that neither converse is true. Many fixed point results have been obtained for weakly compatible mappings (see [3], [6], [8], [9], [11],[12],[16],[18] and [24]).

Let $F([0,\infty))$ be class of all function $f : [0,\infty) \to [0,\infty]$ and let $\mathcal{O}$ be class of all operators

$$\begin{array}{rccc} O(\bullet;\cdot): & F([0,\infty)) & \to & F([0,\infty)) \\ & f & \to & O(f;\cdot) \end{array}$$

satisfying the following conditions:

(i) $O(f;t) > 0$ for $t > 0$ and $O(f;0) = 0$,

(ii) $O(f;t) \leq O(f;s)$ for $t \leq s$,

(iii) $\lim_{n\to\infty} O(f;t_n) = O(f;\lim_{n\to\infty} t_n)$,

(iv) $O(f;\max\{t,s\}) = \max\{O(f;t), O(f;s)\}$
for some $f \in F([0,\infty))$.

Now we give some examples for $O(f;\cdot)$.

Example 1. *If $f : [0,\infty) \to [0,\infty]$ is a Lebesque integrable mapping which is finite integral on each compact subset of $[0,\infty)$, non-negative and such that for each $t > 0$, $\int_0^t f(s)ds > 0$, then the operator defined by*

$$O(f;t) = \int_0^t f(s)ds$$

satisfies the conditions (i)-(iv).

Example 2. *If $f : [0,\infty) \to [0,\infty)$ non-decreasing, continuous function such that $f(0) = 0$ and $f(t) > 0$ for $t > 0$, then the operator defined by*

$$O(f;t) = f(t)$$

satisfies the conditions (i)-(iv).

Example 3. *If $f : [0,\infty) \to [0,\infty)$ non-decreasing, continuous function such that $f(0) = 0$ and $f(t) > 0$ for $t > 0$, then the operator defined by*

$$O(f;t) = \frac{f(t)}{1+f(t)}$$

satisfies the conditions (i)-(iv).

Example 4. *If $f : [0,\infty) \to [0,\infty)$ non-decreasing, continuous function such that $f(0) = 0$ and $f(t) > 0$ for $t > 0$, then the operator defined by*

$$O(f;t) = \frac{f(t)}{1+\ln(1+f(t))}$$

satisfies the conditions (i)-(iv).

2. MAIN RESULT

Now we give our main theorem.

Theorem 1. *Let A, B, S and T be self-maps defined on a metric space (X, d) satisfying the following conditions:*

(a) $S(X) \subseteq B(X)$, $T(X) \subseteq A(X)$,

(b) for all $x, y \in X$, there exists a right continuous function $\psi : \mathbb{R}^+ \to \mathbb{R}^+$, $\psi(0) = 0$ and $\psi(s) < s$ for $s > 0$ such that

$$O(f; d(Sx, Ty)) \le \psi(O(f; M(x, y)))$$

where $O(\bullet;\cdot) \in \mathcal{O}$ and

$$M(x,y) = \max\{d(Ax,By), d(Sx,Ax), d(Ty,By), \frac{d(Sx,By)+d(Ty,Ax)}{2}\}. \tag{2.1}$$

If one of $A(X), B(X), S(X)$ or $T(X)$ is a complete subspace of X, then

(1) A and S have a coincidence point, or
(2) B and T have a coincidence point.

Further, if S and A as well as T and B are weakly compatible, then
(3) A, B, S and T have a unique common fixed point.

Proof. Let $x_0 \in X$ be an arbitrary point of X. From (a) we can construct a sequence $\{y_n\}$ in X as follows:

$$y_{2n+1} = Sx_{2n} = Bx_{2n+1} \text{ and } y_{2n+2} = Tx_{2n+1} = Ax_{2n+2}$$

for all $n = 0, 1, \ldots$. Define $d_n = d(y_n, y_{n+1})$. Suppose that $d_{2n} = 0$ for some n. Then $y_{2n} = y_{2n+1}$; i.e., $Tx_{2n-1} = Ax_{2n} = Sx_{2n} = Bx_{2n+1}$, and A and S have a coincidence point.

Similarly, if $d_{2n+1} = 0$, then B and T have a coincidence point. Assume that $d_n \neq 0$ for each n.

Then, by (b),

$$O(f; d(Sx_{2n}, Tx_{2n+1})) \leq \psi(O(f; M(x_{2n}, x_{2n+1}))) \tag{2.2}$$

where

$$\begin{aligned} M(x_{2n}, x_{2n+1}) &= \max\{d(Ax_{2n}, Bx_{2n+1}), \\ &\quad d(Sx_{2n}, Ax_{2n}), d(Tx_{2n+1}, Bx_{2n+1}), \\ &\quad \frac{d(Sx_{2n}, Bx_{2n+1}) + d(Tx_{2n+1}, Ax_{2n})}{2}\} \\ &= \max\{d_{2n}, d_{2n+1}\}. \end{aligned}$$

Thus from (2.2) we have

$$O(f; d_{2n+1}) \leq \psi(O(f; \max\{d_{2n}, d_{2n+1}\})). \tag{2.3}$$

Now, if $d_{2n+1} \geq d_{2n}$ for some n, then, from (2.3) we have

$$O(f; d_{2n+1}) \leq \psi(O(f; d_{2n+1})) < O(f; d_{2n+1})$$

which is a contradiction. Thus $d_{2n} > d_{2n+1}$ for all n, and so, from (2.3) we have

$$O(f; d_{2n+1}) \leq \psi(O(f; d_{2n})).$$

Similarly,

$$O(f; d_{2n}) \leq \psi(O(f; d_{2n-1})).$$

In general, we have for all $n = 1, 2, \ldots$,

$$O(f; d_n) \leq \psi(O(f; d_{n-1})). \tag{2.4}$$

From (2.4), we have

$$\begin{aligned} O(f; d_n) &\leq \psi(O(f; d_{n-1})) \\ &\leq \psi^2(O(f; d_{n-2})) \\ &\;\;\vdots \\ &\leq \psi^n(O(f; d_0)), \end{aligned}$$

and, taking the limit as $n \to \infty$

$$\lim_{n\to\infty} O(f; d_n) \leq \lim_{n\to\infty} \psi^n(O(f; d_0)) = 0,$$

which, from (i) and (iii), implies that

$$\lim_{n\to\infty} d_n = \lim_{n\to\infty} d(y_n, y_{n+1}) = 0. \tag{2.5}$$

We now show that $\{y_n\}$ is a Cauchy sequence. For this it is sufficient to show that $\{y_{2n}\}$ is a Cauchy sequence. Suppose that $\{y_{2n}\}$ is not a Cauchy sequence. Then there exists an $\varepsilon > 0$ such that for each even integer $2k$ there exist even integers $2m(k) > 2n(k) > 2k$ such that

$$d(y_{2n(k)}, y_{2m(k)}) \geq \varepsilon. \tag{2.6}$$

For every even integer $2k$, let $2m(k)$ be the least positive integer exceeding $2n(k)$ satisfying (2.6) such that

$$d(y_{2n(k)}, y_{2m(k)-2}) < \varepsilon. \tag{2.7}$$

Now

$$\begin{aligned} 0 &< \delta := O(f;\varepsilon) \\ &\leq O(f; d(y_{2n(k)}, y_{2m(k)})) \\ &\leq O(f; d(y_{2n(k)}, y_{2m(k)-2}) + d_{2m(k)-2} + d_{2m(k)-1}). \end{aligned}$$

Then by (2.5), (2.6) and (2.7) it follows that

$$\lim_{k\to\infty} O(f; d(y_{2n(k)}, y_{2m(k)})) = \delta. \tag{2.8}$$

Also, by the triangular inequality,

$$\left|d(y_{2n(k)}, y_{2m(k)-1}) - d(y_{2n(k)}, y_{2m(k)})\right| \leq d_{2m(k)-1}$$

and

$$\left|d(y_{2n(k)+1}, y_{2m(k)-1}) - d(y_{2n(k)}, y_{2m(k)})\right| \leq d_{2m(k)-1} + d_{2n(k)}$$

and so

$$O(f; \left|d(y_{2n(k)}, y_{2m(k)-1}) - d(y_{2n(k)}, y_{2m(k)})\right|) \leq O(f; d_{2m(k)-1}),$$

and

$$O(f; \left|d(y_{2n(k)+1}, y_{2m(k)-1}) - d(y_{2n(k)}, y_{2m(k)})\right|) \leq O(f; d_{2m(k)-1} + d_{2n(k)}).$$

Using (2.8), we get

$$O(f; d(y_{2n(k)}, y_{2m(k)-1})) \to \delta \tag{2.9}$$

and

$$O(f; d(y_{2n(k)+1}, y_{2m(k)-1})) \to \delta \tag{2.10}$$

as $k \to \infty$. Thus

$$\begin{aligned} d(y_{2n(k)}, y_{2m(k)}) &\leq d_{2n(k)} + d(y_{2n(k)+1}, y_{2m(k)}) \\ &\leq d_{2n(k)} + d(Sx_{2n(k)}, Tx_{2m(k)-1}), \end{aligned}$$

and so

$$O(f; d(y_{2n(k)}, y_{2m(k)})) \leq O(f; d_{2n(k)} + d(Sx_{2n(k)}, Tx_{2m(k)-1})).$$

Letting $k \to \infty$ on both sides of the last inequality, we have

$$\begin{aligned} \delta &\leq \lim_{k\to\infty} O(f; d(Sx_{2n(k)}, Tx_{2m(k)-1})) \\ &\leq \lim_{k\to\infty} \psi(O(f; M(x_{2n(k)}, x_{2m(k)-1}))), \end{aligned} \tag{2.11}$$

where

$$M(x_{2n(k)}, x_{2m(k)-1}) = \max\{d(y_{2n(k)}, y_{2m(k)-1}), d_{2n(k)}, d_{2m(k)-1}, \frac{d(y_{2n(k)+1}, y_{2m(k)-1}) + d(y_{2n(k)}, y_{2m(k)})}{2}\}.$$

Combining (2.5), (2.6), (2.7), (2.8), (2.9) and (2.10), yields the following contradiction from (2.11):

$$\delta \le \psi(\delta) < \delta.$$

Thus $\{y_{2n}\}$ is a Cauchy sequence and so $\{y_n\}$ is a Cauchy sequence.

Now, suppose that $A(X)$ is complete. Note that the sequence $\{y_{2n}\}$ is contained in $A(X)$ and has a limit in $A(X)$. Call it u. Let $v \in A^{-1}u$. Then $Av = u$. We shall use the fact that the sequence $\{y_{2n-1}\}$ also converges to u. To prove that $Sv = u$, let $r = d(Sv, u) > 0$. Then taking $x = v$ and $y = x_{2n-1}$ in (b),

$$\begin{aligned} O(f; d(Sv, y_{2n})) &= O(f; d(Sv, Tx_{2n-1})) \\ &\le \psi(O(f; M(v, x_{2n-1}))), \end{aligned}$$

where

$$M(v, x_{2n-1}) = \max\{d(u, y_{2n-1}), d(Sv, u), d(y_{2n}, y_{2n-1}), \frac{d(Sv, y_{2n-1}) + d(y_{2n}, u)}{2}\}.$$

Since $\lim_n d(Sv, y_{2n}) = r$, $\lim_n d(u, y_{2n-1}) = \lim_n d(y_{2n}, y_{2n-1}) = 0$ and $\lim_n[d(Sv, y_{2n-1}) + d(y_{2n}, u)] = r$, we may conclude that

$$O(f; r) \le \psi(O(f; r)) < O(f; r)$$

which is a contradiction. Hence from (i), $Sv = u$. This proves (1).

Since $S(X) \subseteq B(X)$, $Sv = u$ implies that $u \in B(X)$. Let $w \in B^{-1}u$. Then $Bw = u$. By using the argument of the previous section it can be easily verified that $Tw = u$. This proves (2).

The same result holds if we assume that $B(X)$ is complete instead of $A(X)$.

Now if $T(X)$ is complete, then by (a), $u \in T(X) \subseteq A(X)$. Similarly if $S(X)$ is complete, then $u \in S(X) \subseteq B(X)$. Thus (1) and (2) are completely established.

To prove (3), note that S, A and T, B are weakly compatible and

(2.12) $$u = Sv = Av = Tw = Bw$$

then

(2.13) $$Au = ASv = SAv = Su$$

and

$$(2.14) \qquad Bu = BTw = TBw = Tu.$$

If $Tu \neq u$ then, from (b), (2.12), (2.13) and (2.14)

$$\begin{aligned} O(f; d(u,Tu)) &= O(f; d(Sv,Tu)) \\ &\leq \psi(O(f; M(v,u))) \\ &= \psi(O(f; d(u,Tu))) \\ &< O(f; d(u,Tu)) \end{aligned}$$

which is a contradiction. So $Tu = u$. Similarly $Su = u$. Then, evidently from (2.13) and (2.14), u is a common fixed point of A, B, S and T.

The uniqueness of the common fixed point follows easily from condition (b).

□

3. FINAL REMARKS

Remark 1. *Theorem 1 is a generalization of Main Theorem of* [5].

If we combine Example 1 and Theorem 1, we have the following corollary, which is Theorem 2.1 of [9].

Corollary 1. *Let A, B, S and T be self-maps defined on a metric space (X, d) satisfying the following conditions:*

(a) $S(X) \subseteq B(X)$, $T(X) \subseteq A(X)$,

(b) for all $x, y \in X$, there exists a right continuous function $\psi : \mathbb{R}^+ \to \mathbb{R}^+$, $\psi(0) = 0$ and $\psi(s) < s$ for $s > 0$ such that

$$\int_0^{d(Sx,Ty)} f(s)ds \leq \psi(\int_0^{M(x,y)} f(s)ds)$$

where $f : [0, \infty) \to [0, \infty]$ is a Lebesque integrable mapping which is finite integral on each compact subset of $[0, \infty)$, non-negative and such that for each $t > 0$, $\int_0^t f(s)ds > 0$ and

$$\begin{aligned} M(x,y) &= \max\{d(Ax,By), d(Sx,Ax), d(Ty,By), \\ &\quad \frac{d(Sx,By) + d(Ty,Ax)}{2}\}. \end{aligned}$$

If one of $A(X), B(X), S(X)$ or $T(X)$ is a complete subspace of X, then

(1) A and S have a coincidence point, or
(2) B and T have a coincidence point.

Further, if S and A as well as T and B are weakly compatible, then (3) A, B, S and T have a unique common fixed point.

Remark 2. *Corollary 1 is a generalization of Theorem 2.1 of* [10], *Theorem 2 of* [19] *and Theorem 2 of* [27].

Remark 3. *Theorem 1 is a generalization of Theorem 2.1 of* [24], *in fact letting $f = I$ (identity map) and $O(f;t) = f(t) = t$ in (b) (it is obvious that $O(f;\cdot) \in \mathcal{O}$) one has*

$$d(Sx,Ty) = O(f; d(Sx,Ty)) \leq \psi(O(f; M(x,y))) = \psi(M(x,y)),$$

thus the contraction of Theorem 2.1 of [24] *also satisfies (b).*

Now we give an example to illustrate Theorem 1.

Example 5. *Let $X = \{\frac{1}{n} : n \in N\} \cup \{0\}$ with Euclidean metric and S, T, A, B are self maps of X defined by*

$$S(\frac{1}{n}) = \begin{cases} \frac{1}{n+1} & \text{if } n \text{ is odd} \\ \frac{1}{n+2} & \text{if } n \text{ is even} \\ 0 & \text{if } n = \infty \end{cases}, \quad T(\frac{1}{n}) = \begin{cases} \frac{1}{n+1} & \text{if } n \text{ is even} \\ \frac{1}{n+2} & \text{if } n \text{ is odd} \\ 0 & \text{if } n = \infty \end{cases}$$

$$A(\frac{1}{n}) = B(\frac{1}{n}) = \frac{1}{n} \text{ for all } n \in N \cup \{\infty\}.$$

Clearly $S(X) \subseteq B(X)$, $T(X) \subseteq A(X)$, $A(X)$ is a complete subspace of X and A, S and B, T are weakly compatible.

Now we claim that the mappings S, T, A and B satisfy the condition (b) of Theorem 1 with $O(\bullet;\cdot) \in \mathcal{O}$ defined by $O(f;t) = \int_0^t f(s)ds$, $f \in F([0,\infty))$ defined by $f(t) = \max\{0, t^{\frac{1}{t-2}}[1 - \log t]\}$ for $t > 0$ and $f(0) = 0$ and $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ defined by $\psi(s) = \frac{s}{2}$. That is, we claim that the following inequality is satisfies:

$$(d(Sx,Ty))^{\frac{1}{d(Sx,Ty)}} \leq \frac{1}{2}\left((M(x,y))^{\frac{1}{M(x,y)}}\right) \tag{3.1}$$

for all $x, y \in X$, since $O(f;t) = \int_0^t f(s)ds = t^{\frac{1}{t}}$ for any $t \in (0,e)$. Since the function $t \to t^{\frac{1}{t}}$ is nondecreasing, we show sufficiently that

$$(d(Sx,Ty))^{\frac{1}{d(Sx,Ty)}} \leq \frac{1}{2}\left((d(x,y))^{\frac{1}{d(x,y)}}\right) \tag{3.2}$$

instead of (3.1). Now using Example 4 of [27], *we have (3.2), thus the condition (b) of Theorem 1 is satisfied.*

Now suppose that the contractive condition of Corollary 3.1 of [11] is satisfied, that is, there exists $h \in [0,1)$ such that

$$d(Sx, Ty) \le hM(x, y) \tag{3.3}$$

for all $x, y \in X$. Therefore, for $x \neq y$, we have

$$\frac{d(Sx, Ty)}{M(x, y)} \le h < 1$$

but since $\sup_{x \neq y} \frac{d(Sx,Ty)}{M(x,y)} = 1$ one has a contradiction. Thus the condition (3.3) is not satisfied.

If we combine Example 2 and Theorem 1, we have the following corollary.

Corollary 2. *Let A, B, S and T be self-maps defined on a metric space (X, d) satisfying the following conditions:*

(a) $S(X) \subseteq B(X)$, $T(X) \subseteq A(X)$,

(b) for all $x, y \in X$, there exists a right continuous function $\psi : \mathbb{R}^+ \to \mathbb{R}^+$, $\psi(0) = 0$ and $\psi(s) < s$ for $s > 0$ such that

$$f(d(Sx, Ty)) \le \psi(f(M(x, y))),$$

where

$$\left\{ \begin{array}{l} f : [0, \infty) \to [0, \infty) \text{ non-decreasing,} \\ \quad \text{continuous function such that} \\ \quad f(0) = 0 \text{ and } f(t) > 0 \text{ for } t > 0, \end{array} \right. \tag{3.4}$$

and

$$\begin{aligned} M(x,y) \quad = \quad & \max\{d(Ax, By), d(Sx, Ax), d(Ty, By), \\ & \frac{d(Sx, By) + d(Ty, Ax)}{2}\}. \end{aligned}$$

If one of $A(X), B(X), S(X)$ or $T(X)$ is a complete subspace of X, then

(1) A and S have a coincidence point, or
(2) B and T have a coincidence point.

Further, if S and A as well as T and B are weakly compatible, then
(3) A, B, S and T have a unique common fixed point.

Remark 4. *Note that if f is absolutely continuous in Corollary 2, then we have Corollary 1. Indeed, if we consider Theorem 39.15 in [1], i.e. " A function $f : [a, b] \to \mathbb{R}$ is absolutely continuous if and only if $f' \in L_1([a, b])$ and*

$$f(x) - f(a) = \int_a^x f'(t)dt$$

holds for each $x \in [a,b]$ ", then we have

$$f(d(Sx,Ty)) = \int_0^{d(Sx,Ty)} f'(t)dt \leq \psi(\int_0^{M(x,y)} f'(t)dt) = \psi(f(M(x,y))).$$

Nevertheless, f has not to be absolutely continuous in Corollary 2. Thus Corollary 2 is a generalization of Corollary 1. The Exercise 8 in [1, Page 383] *and Problem 39.8 in* [2, Page 386] *shows that there exist some functions f which are not absolutely continuous but continuous and satisfying the other condition of (3.4).*

Remark 5. *Remark 4 shows that operator type contraction is more general then integral type contraction.*

Remark 6. *We can have new results, if we combine Theorem 1 and some examples of $O(f;\cdot)$.*

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Department of Mathematics, Faculty of Science and Arts, Kirikkale University, 71450 Yahsihan, Kirikkale / Turkey
E-mail address: ialtun@kku.edu.tr, ishakaltun@yahoo.com

Department of Mathematics, Faculty of Science and Arts, Gazi University, 06500-Teknikokullar, Ankara / Turkey
E-mail address: dturkoglu@gazi.edu.tr