Matching Preclusion Number in Cartesian Product of Graphs and its Application to Interconnection Networks

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Abstract

The matching preclusion number of a graph G, $mp(G)$, is the minimum number of edges whose deletion leaves a resulting graph that has neither perfect matchings nor almost perfect matchings. Besides its theoretical linkage with conditional connectivity and extremal graph theory, the matching preclusion number is a measure of robustness in interconnection networks. In this paper we develop general properties related to matchings in the Cartesian product of graphs which allow us, in a simple manner, to establish the matching preclusion number for some interconnection (product) networks, namely: hyper Petersen, folded Petersen, folded Petersen cube, hyperstar, star-cube and hypercube. We also conclude that the Cartesian product of graphs operation inherits the matching preclusion number optimality from factor graphs of even order, which reinforces the Cartesian product as a good network-synthesizing operator.

Keywords: Cartesian Product, Perfect Matching, Matching Preclusion, Interconnection Network, Fault Tolerance

1 Introduction and Preliminaries

In this work we are concerned with the following theoretical framework. Let $\mathcal{G}$ stand for the family of all graphs, and let $\otimes : \mathcal{G} \times \mathcal{G} \rightarrow \mathcal{G}$ be a binary operation on graphs. Also, let γ be a graph parameter with the following property: if two graphs G_1 and G_2 are *optimal* with respect to parameter γ (in the sense that both values $\gamma(G_1)$ and $\gamma(G_2)$ reach an intrinsic bound for γ), then so will the graph $G_1 \otimes G_2$. One of the features of this general framework is the possibility of directly calculating the value of parameter γ for several networks created via the use of $\otimes$. In this paper, we exemplify for the *Cartesian product* operation and the *matching preclusion number* parameter.

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We consider only finite, undirected graphs with no loops and no parallel edges. Let G be a graph of order n. A *matching* M of G is a set of pairwise nonadjacent edges. A matching M of G is called a *perfect matching* (resp. an *almost perfect matching*) if its size $|M|$ is equal to $n/2$ (resp. $(n-1)/2$). If G has a perfect matching (almost-perfect matching), then we say G is *matchable* (*almost-matchable*). A set F of edges in G is called a *matching preclusion set* if $G - F$ is neither matchable nor almost-matchable. The *matching preclusion number* of G, denoted by $mp(G)$, is the cardinality of a minimum matching preclusion set in G. If G has neither a perfect matching nor an almost perfect matching, then $mp(G) = 0$.

The concept of matching preclusion was introduced by Brigham et al. [2]. They presented it as a measure of robustness in interconnection networks — if $mp(G)$ is large, networks for which it is essential to have each node possessing at any time a special partner will be robust in the event of edge failures —, as well as a theoretical linkage to conditional connectivity, behavior of graph invariants, and extremal graph theory. Also in their work, the matching preclusion number was calculated for the Petersen graph, complete graphs, complete bipartite graphs and hypercubes.

The matching preclusion number has a natural and trivial upper bound as described in [2] and reproduced in Proposition 1. It comes from the observation that we can always prevent a graph from having a perfect matching by removing all the edges incident to a single vertex. Because the matching preclusion number is to be minimized, we choose this vertex to be the one with minimum degree. Thus, for a graph G of even order, the natural upper bound for $mp(G)$ is $\delta(G)$. In case of an odd graph H, we must consider removing the edges incident to two vertices. Then the upper bound for $mp(H)$ is $d_1 + d_2$, where d_1 and d_2 are the first two elements of a non-decreasing degree sequence of H.

Proposition 1. [2] *Let the degree sequence of an n-vertex graph G be $d_1, d_2, \ldots, d_n$ where $\delta(G) = d_1 \leq d_2 \leq \ldots \leq d_n$. Then $mp(G) \leq \delta(G)$ if n is even, and $mp(G) \leq \delta(G) + d_2$ if n is odd.*

The matching preclusion number is considered to be optimal when equality holds in Proposition 1, as there is absolutely no possibility for it to be higher than the natural upper bound.

Since the results from graph theory can be applied to networks analysis and evaluation[3], further studies naturally arose in order to investigate the matching preclusion for interconnection networks, namely: the (n,k)-star and Cayley graphs generated by transpositions [6]; arrangement graphs [3];

[3]Topologies for interconnection networks are represented by graphs. In multiprocessor systems, vertices represent processors, and an edge p_1p_2 means that processors p_1 and p_2 are able to communicate directly with one another.

restricted HL-graphs and recursive circulant $G(2^m, 4)$ [15]; k-ary hypercubes [17]; tori and related Cartesian products [7]; regular interconnection networks [5]; crossed cubes [8]. Some other studies developed variations of the matching preclusion problem, such as the *conditional matching preclusion* [4] and the *strong preclusion* [16], which in turn brought other works related to the respective problems for different interconnection networks.

For the next definition we follow the notation in [12]. The *Cartesian product* of two graphs G and H, denoted by $G \square H$, is a graph with vertex set $V(G) \times V(H)$, that is, the set $\{(g,h) \mid g \in G, h \in H\}$. The edge set of $G \square H$ consists of all pairs $[(g_1,h_1),(g_2,h_2)]$ of vertices with $h_1 = h_2$ and $[g_1,g_2] \in E(G)$, or $g_1 = g_2$ and $[h_1,h_2] \in E(H)$. For ease of notation, use $(g_1,[h_1,h_2])$ to represent the edge $[(g_1,h_1),(g_1,h_2)]$ if $[h_1,h_2] \in E(H)$ and use $([g_1,g_2],h_1)$ to represent the edge $[(g_1,h_1),(g_2,h_1)]$ if $[g_1,g_2] \in E(G)$. Then the edge set of $G \square H$ can be represented by $(E(G) \times V(H)) \cup (V(G) \times E(H))$. We will call *$G$-edges* those from $E(G) \times V(H)$, and *H-edges* those from $V(G) \times E(H)$. That is, the pair $([x,y],z)$ is a G-edge if $z \in V(H)$ and $[x,y] \in E(G)$, and the pair $(x,[y,z])$ is an H-edge if $x \in V(G)$ and $[y,z] \in E(H)$. In order to have a uniform notation for edges by using square brackets, we write $[x,[y,z]]$ instead of $(x,[y,z])$.

A useful and intuitive manner for visually interpreting the Cartesian product of two graphs G_1 and G_2, where $|V(G_1)| = n_1$ and $|V(G_2)| = n_2$ is shown in Figure 1. Use the following steps:

Step 1. Draw G_1;

Step 2. Replace each vertex $v \in V(G_1)$ by a copy of G_2 (call it vG_2);

Step 3. Transform each edge $[u,v] \in E(G_1)$ into n_2 edges, linking uG_2 to vG_2 (these n_2 edges must link the corresponding vertices of the copies of G_2).

Some well-known properties of the Cartesian product that are relevant to this work follow below:

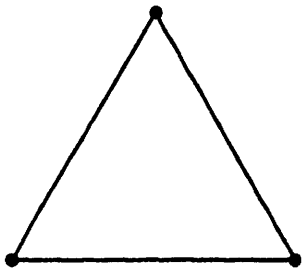
Property 2. $G_1 \square G_2 \cong G_2 \square G_1$.

Property 3. $d_{G_1 \square G_2}((x,y)) = d_{G_1}(x) + d_{G_2}(y)$, *where* $d(v)$ *is the degree of a vertex* v.

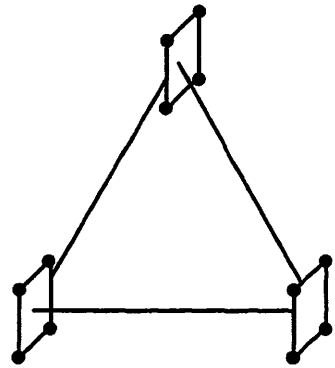
Property 4. $\delta(G_1 \square G_2) = \delta(G_1) + \delta(G_2)$, *where* $\delta(G)$ *is the minimum degree of a graph* G.

Property 5. $\ell_{G_1 \square G_2}((x_1,y_1),(x_2,y_2)) = \ell_{G_1}(x_1,x_2) + \ell_{G_2}(y_1,y_2)$, *where* $\ell_{G(u,v)}$ *is the distance between vertices* u *and* v *in a graph* G.

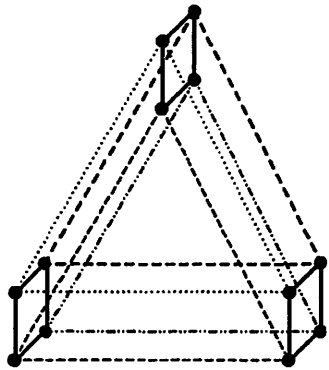
Property 6. $D(G_1 \square G_2) = D(G_1) + D(G_2)$, *where* $D(G)$ *is the diameter of a graph* G.



(a) Draw K_3



(b) Replace each vertex of K_3 by a C_4



(c) Transform each edge $[u, v] \in E(K_3)$ into $|V(C_4)|$ new edges, linking uC_4 to vC_4 by its corresponding vertices

Figure 1: The 3 steps for visualizing $K_3 \square C_4$.

The Cartesian product operation has many interesting and desirable properties for synthesizing interconnection networks: as the number of vertices grows geometrically, the vertex degrees, the diameter and the average distance grow arithmetically; network algorithms can be easily and systematically synthesized from the corresponding algorithms of the factor networks [18]. Not at random, some classical and highly implemented topologies — such as hypercubes, k-ary n-cubes, meshes, tori and generalized hypercubes — are the Cartesian product of graphs, and many other product topologies have been proposed/studied (some of them will be cited in Section 3). These properties of the Cartesian product are a part of the explanation why many works have been dedicated to studying this operation recently. Besides articles, works in this area include two books: one in 2008 [12] and another in 2011 [11].

In the following sections we develop general properties related to perfect matchings in the Cartesian product of graphs (Section 2) which allow us, in a simple manner, to establish the matching preclusion number for some interconnection product networks (Section 3), according to the general framework described in the beginning. We also develop properties related to almost-perfect matchings and matching preclusion number of graphs of odd order (Section 4).

2 Perfect Matchings in Cartesian Product of Graphs

In this section we develop properties related to perfect matchings in Cartesian product of graphs, including upper and lower bounds for the matching preclusion number under this operation. With these results, we formulate a theorem which shows that the Cartesian product inherits the optimality of the matching preclusion number of the factor graphs of even order.

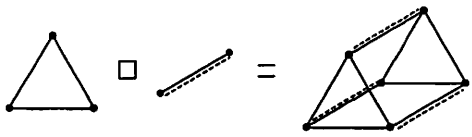
Lemma 7. *If G_1 or G_2 is matchable, then $G_1 \square G_2$ is matchable.*

Proof. Let G_1 be a matchable graph, and let $E_M(G_1)$ be a perfect matching in G_1. A perfect matching in $G_1 \square G_2$ is easily obtained from G_1-edges: $E_M(G_1) \times V(G_2)$. Since $E_M(G_1)$ covers all vertices in $V(G_1)$ — for it is a perfect matching in G_1 —, $E_M(G_1) \times V(G_2)$ will cover all vertices in $V(G_1) \times V(G_2)$, i.e., all vertices in $V(G_1 \square G_2)$. □

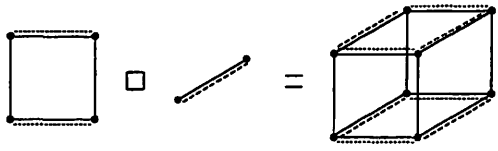
Figure 2 illustrates the idea of the proof.

Lemma 8. *$mp(G_1 \square G_2) \leq \delta(G_1) + \delta(G_2)$ if $n_1 \times n_2$ is even.*

Proof. From Proposition 1, we know that $mp(G) \leq \delta(G)$ if n is even. From Property 4, we know that $\delta(G_1 \square G_2) = \delta(G_1) + \delta(G_2)$. Therefore, $mp(G_1 \square G_2) \leq \delta(G_1 \square G_2) = \delta(G_1) + \delta(G_2)$. □



(a) $G_1 \cong C_3$ and $G_2 \cong K_2$. Perfect matching in $G_1 \,\square\, G_2$ from a perfect matching $E_M(G_2)$ in G_2: $V(G_1) \times E_M(G_2)$.



(b) $G_1 \cong C_4$ and $G_2 \cong K_2$. Two perfect matchings in $G_1 \,\square\, G_2$ from the perfect matchings $E_M(G_1)$ and $E_M(G_2)$ in factor graphs: one from G_1-*edges* ($E_M(G_1) \times V(G_2)$), and another from G_2-*edges* ($V(G_1) \times E_M(G_2)$).

Figure 2: Perfect matching in $G_1 \,\square\, G_2$, provided that G_1 and/or G_2 are matchable.

Lemma 9. *$mp(G_1 \,\square\, G_2) \geq mp(G_1) + mp(G_2)$ if n_1 and n_2 are even.*

Proof. A perfect matching in $G_1 \,\square\, G_2$ can be built from a perfect matching M in G_1: $M \times V(G_2)$ (Lemma 7). In order to ensure that $G_1 \,\square\, G_2$ will not have perfect matchings of this form, it is necessary that at least one of the copies of G_1 in $G_1 \,\square\, G_2$ shall not have perfect matchings. The minimum number of edges we need to remove from a copy of G_1, so that it is not matchable, is exactly $mp(G_1)$ by definition. Similarly, we consider perfect matchings from G_2-*edges*. Therefore, it is necessary to remove at least $mp(G_1) + mp(G_2)$ edges. □

Observation. As described in [12], if φ is a graph parameter and $\otimes$ is a graph product, then we say that φ is *supermultiplicative* (respectively, *submultiplicative*) on $\otimes$ if for every pair of graphs G_1 and G_2 it holds that $\varphi(G_1 \otimes G_2) \geq \varphi(G_1)\varphi(G_2)$ (resp., $\varphi(G_1 \otimes G_2) \leq \varphi(G_1)\varphi(G_2)$). By adapting this terminology, we can similarly define *superadditive* and *subadditive* parameters, replacing multiplication by addition on the right hand size of the previous inequalities. Therefore, from Lemma 9 we can say that the matching preclusion number *mp* is *superadditive* on $\square$ when both factor graphs are even.

We now state Theorem 10, which brings interesting corollaries. It shows that the Cartesian product of graphs operation inherits the matching preclusion number optimality from factor graphs of even order. This property reinforces the Cartesian product as a good network-synthesizing operator, since the topologies created by this operation will preserve the factors' optimality with respect to the matching preclusion number.

Theorem 10. *Let G_1 and G_2 be graphs of even order. If both G_1 and G_2 have optimal matching preclusion numbers, then so will $G_1 \square G_2$.*

Proof. Since G_1 and G_2 have optimal matching preclusion numbers, we have $mp(G_1) = \delta(G_1)$ and $mp(G_2) = \delta(G_2)$. We know from Lemma 8 that $mp(G_1 \square G_2) \leq \delta(G_1) + \delta(G_2)$. And from Lemma 9 we know that $mp(G_1 \square G_2) \geq mp(G_1) + mp(G_2)$. Since $mp(G_1) = \delta(G_1)$ and $mp(G_2) = \delta(G_2)$, we conclude that $mp(G_1 \square G_2) = \delta(G_1) + \delta(G_2) = \delta(G_1 \square G_2)$. □

3 Matching Preclusion Number for some Interconnection Networks

Theorem 10 allows us to calculate, in a simple manner, the matching preclusion number of Cartesian product of networks of even order with optimal matching preclusion number. In this section we show some examples.

- Hypercube. Denoted by H_n or Q_n, it is the Cartesian product of n copies of K_2.

  Corollary 11. [2, Theor. 12] $mp(Q_n) = \delta(Q_n) = n$.

  Proof. It is easy to verify that $mp(K_2) = \delta(K_2)$. Since $Q_n = (K_2)^n$, we conclude from Theorem 10 that $mp(Q_n) = \delta(Q_n)$. □

  As observed, the result in Corollary 11 was already known from [2, Theor. 12]. However, we have presented this alternative proof in order to exemplify the utility of Theorem 10, which allows us to calculate the matching preclusion number in a simpler manner.

- Hyper-Petersen [9]. Denoted by HP_n, $n \geq 3$, it is the Cartesian product of an $(n-3)$-dimensional hypercube and the *Petersen* graph.

  Corollary 12. $mp(HP_n) = \delta(HP_n) = n$.

  Proof. It is known that $mp(Q_{n-3}) = \delta(Q_{n-3})$ and $mp(P) = \delta(P)$ [2]. Since $HP_n = Q_{n-3} \square P$, we conclude from Theorem 10 that $mp(HP_n) = \delta(HP_n)$. □

- N-folded Petersen [13]. Denoted by FP_n, it is the Cartesian product of n *Petersen* graphs.

  Corollary 13. $mp(FP_n) = \delta(FP_n) = 3n$.

  Proof. It is known that $mp(P) = \delta(P)$ [2]. Since $FP_n = P^n$, we conclude from Theorem 10 that $mp(FP_n) = \delta(FP_n)$. □

- Folded Petersen cube [14]. Denoted by $FPQ_{n,k}$, it is the Cartesian product of an n-dimensional hypercube and k *Petersen* graphs.

  Corollary 14. $mp(FPQ_{n,k}) = \delta(FPQ_{n,k}) = n + 3k$.

  Proof. It is known that $mp(Q_n) = \delta(Q_n)$ and $mp(P) = \delta(P)$ [2]. Since $FPQ_{n,k} = Q_n \square P^k$, we conclude from Theorem 10 that $mp(FPQ_{n,k}) = \delta(FPQ_{n,k})$. □

- Hyperstar [1]. Denoted by $S_{n_1,n_2,\ldots,n_k}$, it is the Cartesian product of k star graphs.

  Corollary 15. $mp(S_{n_1,n_2,\ldots,n_k}) = \delta(S_{n_1,n_2,\ldots,n_k}) = \sum_{i=1}^{k}(n_i - 1)$.

  Proof. It is known that $mp(S_n) = \delta(S_n)$ [6]. Since $S_{n_1,n_2,\ldots,n_k} = S_{n_1} \square S_{n_2} \square \cdots \square S_{n_k}$, we conclude from Theorem 10 that $mp(S_{n_1,n_2,\ldots,n_k}) = \delta(S_{n_1,n_2,\ldots,n_k})$. □

- Star-cube [10]. Denoted by $SQ_{m,n}$, it is the Cartesian product of an m-dimensional star graph and a n-dimensional hypercube.

  Corollary 16. $mp(SQ_{m,n}) = \delta(SQ_{m,n}) = m + n - 1$.

  Proof. It is known that $mp(S_m) = \delta(S_m)$ [6] and $mp(Q_n) = \delta(Q_n)$ [2]. Since $SQ_{m,n} = S_m \square Q_n$, we conclude from Theorem 10 that $mp(SQ_{m,n}) = \delta(SQ_{m,n})$. □

The above corollaries are examples of the applicability of Theorem 10 to obtain new information on the matching preclusion number of classes of graphs defined via the Cartesian product operation.

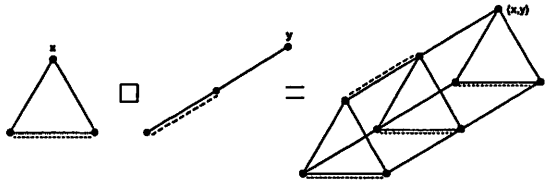
4 Almost-Perfect Matchings in Cartesian Product of Graphs

In this section we develop properties related to almost-perfect matchings in Cartesian product of graphs, including upper and lower bounds for the matching preclusion number under this operation. This means that we now investigate the problem in graphs of odd order.

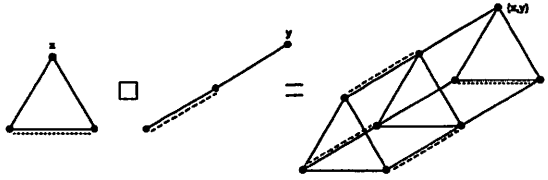
Proposition 17. *If G_1 and G_2 are both almost-matchable, then $G_1 \square G_2$ is almost-matchable.*

Proof. Let $E_M(G_1)$ and $E_M(G_2)$ be almost-perfect matchings in G_1 and G_2, respectively, and let $x \in G_1$ and $y \in G_2$ be the vertices not covered by these matchings. An almost-perfect matching in $G_1 \square G_2$ can be easily built in two different ways: (1) $(E_M(G_1) \times V(G_2)) \cup (\{x\} \times E_M(G_2))$ or (2) $(V(G_1) \times E_M(G_2)) \cup (E_M(G_1) \times \{y\})$. In both cases, (x, y) will be the vertex not covered by the almost-perfect matching in $G_1 \square G_2$. □

Figure 3 illustrates the idea of the proof.



(a) $(E_M(G_1) \times V(G_2)) \cup (\{x\} \times E_M(G_2))$



(b) $E_M(G_1) \times (V(G_2)) \cup (\{x\} \times E_M(G_2))$

Figure 3: Two ways of obtaining an almost-perfect matching in $G_1 \square G_2$, provided that G_1 and G_2 are both almost-matchable ($G_1 \cong C_3$ and $G_2 \cong P_3$).

Proposition 18. *$mp(G_1 \square G_2) \leq d_1(G_1) + d_1(G_2) + \min(d_1(G_1) + d_2(G_2); d_2(G_1) + d_1(G_2))$ if both n_1 and n_2 are odd, where $d_1(G_i)$ and $d_2(G_i)$ are the first two elements of a non-decreasing degree sequence of G_i.*

Proof. Directly from Proposition 1, it follows that $mp(G) \leq d_1(G) + d_2(G)$ if n is odd, and from Property 3 it follows that $d_{G_1 \,\square\, G_2}((x,y)) = d_{G_1}(x) + d_{G_2}(y)$. □

The following definition will be useful. Let F be a subset of $E(G_1 \,\square\, G_2)$ such that F contains only G_1-edges (for G_2-edges the definition is analogous). For a vertex $v \in V(G_2)$, let vF denote the *projection of F onto the copy vG_1*, that is, the subset of edges

$$vF = \{\, [\,[x,y],v] : \text{there is an edge } [\,[x,y],w] \text{ in } F \text{ for some } w \in V(G_2)\}.$$

Informally, for each edge $e = [[x,y],w] \in F$, vF contains a corresponding edge e' obtained by replacing w by v, i.e., $e' = [[x,y],v]$. (If $w = v$ then $e = e'$.) The idea is to consider the edges of F as if they were all in copy vG_1. Clearly, $|F| = |vF|$. Now we are ready to state the next proposition.

Proposition 19. *$mp(G_1 \,\square\, G_2) \geq \min(mp(G_1); mp(G_2))$ if both n_1 and n_2 are odd.*

Proof. Let F be a subset of $E(G_1 \,\square\, G_2)$ such that $|F| < \min(mp(G_1); mp(G_2))$. We will prove that $G_1 \,\square\, G_2 - F$ admits an almost-perfect matching. Let F_2 be the subset formed by the G_2-edges of F. For $v \in V(G_1)$, consider the projection vF_2. Note that

$$|vF_2| = |F_2| \leq |F| < mp(G_2) \leq \min(mp(G_1); mp(G_2)).$$

Thus, by removing vF_2 from vG_2, the copy vG_2 admits an almost-perfect matching M_2. Consider the projections wM_2 of M_2, for every $w \in V(G_1)$ (note that $M_2 = vM_2$). Then, for each $w \in V(G_1)$, by removing wF_2 from wG_2, the copy wG_2 admits the almost-perfect matching wM_2. Let (w,x) be the wM_2-unsaturated vertex, for each $w \in V(G_1)$. Consider the copy xG_1 and let $F_1 = F \setminus F_2$. Since $|F_1| \leq |F| < mp(G_1)$, by removing F_1 from $G_1 \,\square\, G_2$, the copy xG_1 admits an almost-perfect matching M_1 (note that this happens even if all the edges in F_1 lie in copy xG_1). By putting all together, we conclude that the graph

$$G_1 \,\square\, G_2 - (F_1 \cup (\cup_{w \in V(G_1)} wF_2))$$

admits an almost-perfect matching, namely $M_1 \cup (\cup_{w \in V(G_1)} wM_2)$. Since $F \subseteq (F_1 \cup (\cup_{w \in V(G_1)} wF_2))$, it is clear that $G_1 \,\square\, G_2 - F$ admits an almost-perfect matching as well. □

5 Concluding Remarks

We have developed properties related to the existence of perfect and almost-perfect matchings in Cartesian product of graphs, associated to properties

of the factor graphs. We then developed upper and lower bounds for the matching preclusion number of this operation. Under these conditions, we established the matching preclusion number for some interconnection networks, namely: hyper Petersen, folded Petersen, folded Petersen cube, hyperstar, star-cube and hypercubes (it was already known for this latter; however, we have presented an alternative simpler proof). We also concluded that the Cartesian product of graphs operation inherits the matching preclusion number optimality from factor graphs of even order, which reinforces the Cartesian product as a good network-synthesizing operator. It is an application of a general framework described in Section 1 for a graph operation and a parameter related to matching. As future research directions, other binary operations can be chosen and in combination with many other parameters related to matching, connectivity, planarity, hamiltonicity, coloring, an so on.

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