Pairs of graphs having n-2 cards in common*

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Abstract

For a vertex v of a graph G, the unlabeled subgraph $G-v$ is called a *card* of G. We prove that connectedness of an n vertex graph G and presence of isolated vertices in G can be determined from any collection of $n-2$ of its cards. It is also proved that if two graphs on $n \geq 6$ vertices and minimum degree at least two have $n-2$ cards in common, then the numbers of edges in them differ by at most one.

Key words : vertex-deleted subgraph (card), common cards, number of edges.

1. Introduction

All graphs considered are finite simple and undirected. We use the terminology in Harary [2]. The degree of a vertex v of a graph G is denoted by $deg\ v$ (or $deg_G v$). The minimum degree among the vertices of a graph is denoted by δ. The number of edges of a graph G is denoted by $e(G)$. For a vertex v of G, the unlabeled subgraph $G-v$ is called a *card* of G. The collection of cards of G is called the *deck* of G.

The famous Ulam's Reconstruction Conjecture for graphs [1] claims that all graphs on at least three vertices are determined uniquely up to isomorphism by their decks. Myrvold studied [4] adversery reconstruction number of G which is the smallest k such that no subcollection of the deck of G of size k is contained in the deck of any other graph H, $H \ncong G$. It is well known [1] that the number of edges of a graph on n vertices can be determined from the deck of G. Manvel [3] proved that, if G and H are graphs on n vertices with $n-1$ cards in common, then $|e(G) - e(H)| \leq 1$. Myrvold [5] verified that for $n = 6$, all pairs of graphs G and H with five cards in common have $e(G) = e(H)$ except for the pair in Figure 1 and proved that " for $n \geq 7$, $e(G) = e(H)$ whenever G and H have $n-1$ cards in common". Here we study pairs of graphs with $n-2$ cards in common.

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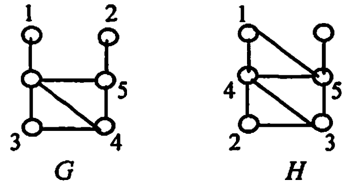

$G-i \cong H-i$, $i = 1$ to 5 and $e(H) - e(G) = 1$

Figure 1: A pair of 6-point graphs with 5 cards common

We prove that connectedness of G and presence of isolated vertices in G can be determined from any collection of $n-2$ of its cards. It is also proved that if two graphs on $n \geq 6$ vertices and $\delta \geq 2$ have $n-2$ cards in common, then the numbers of edges in them differ by at most one. However, it is not yet known whether $\delta \geq 2$ can be determined from $n-2$ cards or not.

2. Recognizability from $n-2$ cards

A family (property) of graphs is called recognizable from a specified collection of their cards if all graphs having that subcollection within their deck belongs to that family (have that property).

Here we prove that connected graphs and graphs having isolated vertices are recognizable from any collection of $n-2$ of their cards.

Lemma 1. Let G be a connected graph on n vertices. Then at most $\left\lfloor \frac{n}{2} \right\rfloor$ cards of G have isolated vertices.

Proof. Suppose k cards of G each have isolated vertices. Then G has at least k endvertices such that no two of them are adjacent with the same vertex and hence $n \geq 2k$. This proves the lemma. ■

Lemma 2. Connected graphs and disconnected graphs on n (≥ 7) vertices are recognizable from any collection of $n-2$ of their cards.

Proof. Let $\Im$ be a given collection of $n-2$ cards of a graph G. If two of the cards in $\Im$ are connected, then G is connected.

Now *let $\Im$ have at most one connected card.*

Case 1. There exists a connected card A in $\Im$.

Now G is either (i) connected, or

(ii) disconnected with exactly two components, one of them being K_1.

If all the disconnected cards in $\Im - \{A\}$ ($n-3$ in number) each have isolated vertices, then G is disconnected by Lemma 1 (as $n-3 > \left\lfloor \frac{n}{2} \right\rfloor$ for $n \geq 7$).

Otherwise, G has at least two cards having no isolated vertex. Hence (ii) does not hold and G is connected.

Case 2. No card in $\Im$ is connected.

Now G is either disconnected or P_n (since P_n is the only connected graph with $n-2$ cutvertices). If $\Im$ coincides with the collection of disconnected cards of P_n, then it can be proved that G is P_n. Otherwise, G is disconnected. ■

Theorem 3. A graph with an isolated vertex can be recognized from any collection of $n-2$ of its vertex-deleted subgraphs for $n \geq 7$.

Proof. Let $\Im$ be the given collection of $n-2$ cards of G. By Lemma 2, we know if G is connected or not. If G is disconnected and there is a connected card in $\Im$, then $\delta = 0$.

Now let G be disconnected and no card in $\Im$ *is connected.*

If at least two cards in $\Im$ have no isolated vertices, then $\delta \neq 0$. Hence we can take that *at most one card in $\Im$ has no isolated vertex.*

We consider two subcases.

Case 1. There exists a card without isolated vertices in $\Im$.

Now G has at most one isolated vertex. Let us characterize G when it has no isolated vertex.

Since all but at most three cards of G have isolated vertices, G must have at least $n-3$ vertices adjacent to a vertex of degree one and hence, *at least $n-3$ vertices of degree one.* Also G can have *at most one component on three or more vertices* (since each such component of G gives rise to at least 2 cards of G without isolated vertex). Other components of G are K_2's.

If G *has no components on three or more vertices*, then it is just a union of K_2's, leading to a contradiction (Since in $\Im$, there is a card with no isolated vertex).

If G *has a component which is a tree, and that tree has four or more endvertices* then there would be at least four cards with no isolated vertices (which is not the case here). So any components which are trees have at most three endvertices. If *the tree has three endvertices*, then it has one vertex v of degree three. If v is not adjacent to some endvertex, then the graph again has at least four cards with no isolated vertices (those obtained from deleting v and the three endvertices). Thus the degree three vertex v is adjacent to an endvertex. If there is a path of length three or more from v to one of the other endvertices which has first edge vu, the four cards from deleting the three endvertices or u have no isolated vertices. Thus, if G has a component which is a tree having three endvertices, this component must be one of F_1, F_2 and F_3 of Figure 2.

If G *has a component which is a tree having only two endvertices* (the only remaining case for trees) then it is a path. Any path on six or more vertices has at least four cards which have no isolated vertices. So such a component is one of F_4, F_5 and F_6 of Figure 2.

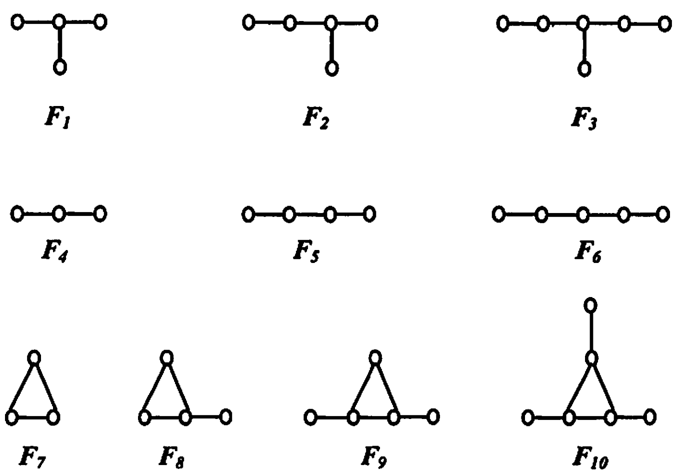

Figure 2 : Possibilities for the sole non K_2 component of G (Theorem 3)

The *remaining cases are when G has a component on three or more vertices which has a cycle.* Such a cycle can have size at most three. The reason is that either a cycle vertex v is adjacent to an endvertex or it is not. If the vertex v is adjacent to an endvertex u, then $G-u$ has no isolated vertices. If it is not, then $G-v$ has no isolated vertices. Similar reasoning can be used to argue that the component only has one cycle. Let the cycle be (a, b, c). These vertices may or may not be adjacent to an endvertex. But either way, there can be no other vertices besides a, b, c, and at most one endvertex adjacent to each which are in this component or else there will be four or more cards with no isolated vertices. The possible cases are F_7, F_8, F_9 or F_{10} of Figure 2.

Thus when G has no isolated vertices, it has to be one among the ten graphs

$F_1 \cup \left(\frac{n-4}{2}\right)K_2$, $F_2 \cup \left(\frac{n-5}{2}\right)K_2$, $F_3 \cup \left(\frac{n-6}{2}\right)K_2$, $F_4 \cup \left(\frac{n-3}{2}\right) K_2$, $F_5 \cup \left(\frac{n-4}{2}\right)K_2$, $F_6 \cup \left(\frac{n-5}{2}\right)K_2$, $F_7 \cup \left(\frac{n-3}{2}\right) K_2$, $F_8 \cup \left(\frac{n-4}{2}\right)K_2$, $F_9 \cup \left(\frac{n-5}{2}\right)K_2$ *and* $F_{10} \cup \left(\frac{n-6}{2}\right)K_2$.

Hence if $\mathfrak{I}$ does not coincide with some collection of $n-2$ cards of one these ten graphs, then G has $\delta = 0$. Otherwise, $\delta \neq 0$ as decided in each of the ten cases below.

Case 1.1. $\mathfrak{I}$ coincides with a subcollection of $n-2$ cards of $F_3 \cup \left(\frac{n-6}{2}\right)K_2$.

Now $\Im$ consists of $n-6$ cards $K_1 \cup F_3 \cup \left(\frac{n-8}{2}\right)K_2$, two cards $K_1 \cup P_4 \cup \left(\frac{n-6}{2}\right)K_2$, one card $K_1 \cup \left(\frac{n-2}{2}\right)K_2$ and one among $P_5 \cup \left(\frac{n-6}{2}\right)K_2$ and $F_2 \cup \left(\frac{n-6}{2}\right)K_2$.

The existence of a component F_3 in the card $K_1 \cup F_3 \cup \left(\frac{n-8}{2}\right)K_2$ implies that G has a component of order at least six. Since the card $P_5 \cup \left(\frac{n-6}{2}\right)K_2$ (or $F_2 \cup \left(\frac{n-6}{2}\right)K_2$) has no component of order at least six, the corresponding deleted vertex in G must be a vertex of that component of order at least six so that $\delta \neq 0$.

Case 1.2. $\Im$ coincides with a subcollection of $n-2$ cards of $F_5 \cup \left(\frac{n-4}{2}\right)K_2$.

Now $\Im$ consists of "$n-4$ cards $K_1 \cup F_5 \cup \left(\frac{n-6}{2}\right)K_2$, one card $K_1 \cup \left(\frac{n-2}{2}\right)K_2$ and one card $P_3 \cup \left(\frac{n-4}{2}\right)K_2$" or "$n-5$ cards $K_1 \cup F_5 \cup \left(\frac{n-6}{2}\right)K_2$, two cards $K_1 \cup \left(\frac{n-2}{2}\right)K_2$ and one card $P_3 \cup \left(\frac{n-4}{2}\right)K_2$". Hence G contains a component with $F_5 = P_4$ as an induced subgraph. Hence while augmenting the card $P_3 \cup \left(\frac{n-4}{2}\right)K_2$ to G, the annexed vertex cannot remain as an isolated vertex. Hence $\delta \neq 0$.

Similarly in the other eight cases also, it can be proved that $\delta \neq 0$.

Case 2. Each card in $\Im$ has an isolated vertex.

Now G has either an isolated vertex or at least $n-2$ endvertices, no two having the same neighbour. Hence either G has an isolated vertex or G is $P_4 \cup \left(\frac{n-4}{2}\right)K_2$ or $\left(\frac{n}{2}\right)K_2$ or $P_3 \cup \left(\frac{n-3}{2}\right)K_2$. As in Case 1, we can prove that if $\Im$ coincides with a collection of $n-2$ cards of one of these graphs, then $\delta(G) \neq 0$ and if not $\delta(G) = 0$. ■

3. Graphs with $n-2$ cards in common

Notation. Whenever G and H are taken as two graphs having $n-2$ cards in common, we assume that G and H are labeled with $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ respectively so that $G-v_i \cong H-u_i$ for $i = 1$ to $n-2$. A card $G-v_i$, $1 \leq i \leq n-2$ is called a common card of G and a card $H-u_i$, $1 \le i \le n-2$ is called a common card of H.

The following two lemmas are obvious.

Lemma 4. Let G and H be graphs with $e(G)$ and $e(G)+k$, $k \ge 0$ edges respectively. If $G-v \cong H-u$, then $deg\, u = deg\, v + k$. ■

Lemma 5. Let G and H be graphs on n verities with $\delta \ge 2$, and with $e(G)$ and $e(G)+k$, $k \ge 0$ edges respectively. If G and H have $n-2$ cards in common, then $deg\, v_{n-1} + deg\, v_n - (deg\, u_{n-1} + deg\, u_n) = k(n-4)$. ■

We now prove our main theorem.

Theorem 6. Let G and H each be a graph on $n \ge 6$ vertices with $\delta \ge 2$ such that they have $n-2$ cards in common. Then $|e(G) - e(H)| \le 1$.

Proof. For $n = 6$, we have verified the theorem by hand using the list of graphs in [2]. Now let $n \ge 7$.

Without loss of generality, let us take that $e(G) \le e(H)$.

If possible, let $e(H) - e(G) \ge 3$. ----(I)

Now by Lemma 5, $deg\, v_{n-1} + deg\, v_n - (deg\, u_{n-1} + deg\, u_n) \ge 3(n-4)$ ----(1)

Since $2 \le deg\, v_i \le n-1$ and $2 \le deg\, u_i \le n-1$, $deg\, v_i - deg\, u_j \le n-3$ and hence

$$deg\, v_{n-1} + deg\, v_n - (deg\, u_{n-1} + deg\, u_n) \le 2(n-3) \quad \text{----(2)}$$

From (1) and (2), $3(n-4) \le 2(n-3)$. This is impossible as $n \ge 7$.

Now, if possible let $e(H) - e(G) = 2$. ----(II)

Notation : For simplicity, we will denote $\overline{G}$ and $\overline{H}$ by E and F respectively.

Then $e(E) - e(F) = 2$ and $E-v_i \cong F-u_i$, $1 \le i \le n-2$ (by hypothesis).

So Lemma 4 applied to E and F gives $deg_E\, v_i = deg_F\, u_i + 2$ and hence

$deg_E\, v_i \ge 2$ for $i = 1$ to $n-2$. ----(3)

Now by Lemma 5, $deg_G\, v_{n-1} + deg_G\, v_n - (deg_H\, u_{n-1} + deg_H\, u_n) = 2n-8$.
Consequently, $deg_F\, u_{n-1} + deg_F\, u_n - (deg_E\, v_{n-1} + deg_E\, v_n) = 2n-8$. ----(4)

Since $2 \le deg_G\, v_i \le n-1$ and $2 \le deg_H\, u_i \le n-1$ for $i = 1$ to n,

$0 \le deg_E\, v_i \le n-3$ and $0 \le deg_F\, u_i \le n-3$ for $i = 1$ to n.

Hence $deg_F\, u_{n-1} + deg_F\, u_n \le 2n-6$ and $deg_E\, v_{n-1} + deg_E\, v_n \ge 0$ and (4) gives rise to the three cases discussed below, each leading to a contradiction.

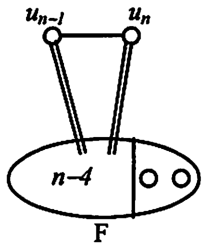

Figure 3: Structure of F when it has two isolated vertices in Case 1, Theorem 6.

Case 1. $deg_F\, u_{n-1} + deg_F\, u_n = 2n - 6$ and $deg_E\, v_{n-1} + deg_E\, v_n = 2$.

Now $deg_F\, u_{n-1} = deg_F\, u_n = n-3$. Hence in F, there can be at most two vertices which are adjacent to neither u_n nor u_{n-1} and these alone can be isolated vertices of F (Figure 3). On the other hand, F has at least $n-6$ vertices which are adjacent to both u_n and u_{n-1} (Figure 4). Hence, apart from u_n and u_{n-1}, there can be at most four vertices in F which are nonadjacent to at least one among u_{n-1} and u_n and these alone can be endvertices of F.

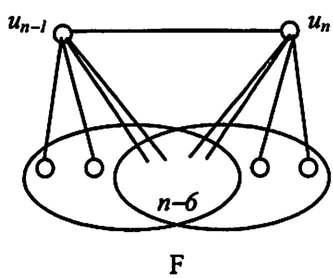

Figure 4: Structure of F when it has four endvertices in Case 1, Theorem 6.

Thus, F has at most two isolated vertices and at most four endvertices. ----(5)

Also $deg_E\, v_{n-1} + deg_E\, v_n = 2$, implies $\{deg_E\, v_{n-1}, deg_E\, v_n\} = \{2, 0\}$ or $\{1\}$.

Subcase 1.1. $\{deg_E\, v_{n-1}, deg_E\, v_n\} = \{2, 0\}$.

Without loss of generality, we can take that $deg_E\, v_{n-1} = 2$ and $deg_E\, v_n = 0$.

As $deg_E\, v_i \geq 2$ for $i = 1$ to $n-2$, this implies v_n is the only isolated vertex of E. Thus $E-v_i$ and hence $F-u_i$, $1 \leq i \leq n-2$ has at least one isolated vertex. ----(6)

If possible, let F have no isolated vertex. Then by (6), all the vertices of F other than u_{n-1} and u_n must be adjacent to endvertices. Therefore there must be at least $n-2$ (≥ 5) endvertices in F. This contradicts (5).

If possible, let F have exactly one isolated vertex. Then it is different from u_n and u_{n-1}, and the common card corresponding to deletion of that isolated vertex of F has no isolated vertex, contradicting (6).

So by (5), we can take that F has exactly two isolated vertices, say u_s and u_t. Then in F, the vertices u_{n-1} and u_n (each of degree $n-3$) are adjacent and each is adjacent to all the vertices other than u_s and u_t.

Now $F-u_i$, $i \in \{1, 2, \ldots, n-2\} - \{s, t\}$ has at least two isolated vertices (and there are $n-4$ such cards). Consequently, $n-4$ of the cards $E-v_1$, $E-v_2$, ..., $E-v_{n-2}$ each must have two isolated vertices. Hence E has at least $n-4$ endvertices (since E has exactly one isolated vertex), and this contradicts (3).

Subcase 1.2. $\{deg_E v_{n-1}, deg_E v_n\} = \{1\}$.

We consider two subcases as below.

Subcase 1.2.1. v_{n-1} and v_n are adjacent.

Now in E, the subgraph induced by $\{v_{n-1}, v_n\}$ is a component ($\cong K_2$). Thus $E-v_i$ and hence $F-u_i$, $1 \leq i \leq n-2$ are disconnected with a component K_2. ---- (7)

Since $deg_F u_n = n-3$ (≥ 4), all but two vertices of F are adjacent to u_n and thus u_n and its neighbours (totalling $n-2$) together can not occur in a component K_2 in $F-u_i$, $1 \leq i \leq n-2$. Hence neither of the two cards obtained by deleting one among the other two vertices can have a component K_2. However, at least one of the above two cards must be a common card and this *contradicts* (7).

Subcase 1.2.2. v_{n-1} and v_n are not adjacent.

Let v_s and v_t be the neighbours of v_{n-1} and v_n respectively. Two subcases arise depending on $v_s = v_t$ or not.

Subcase 1.2.2.1. $v_s = v_t$

Now the card $E-v_s$ (and hence $F-u_s$) has two isolated vertices (namely v_{n-1} and v_n).
Each card $E-v_i$ (and hence $F-u_i$), $i \in \{1, 2, \ldots, n-2\} - \{s\}$ has the following two properties.

(i) has no isolated vertex
(ii) has at least two endvertices. ---(8)

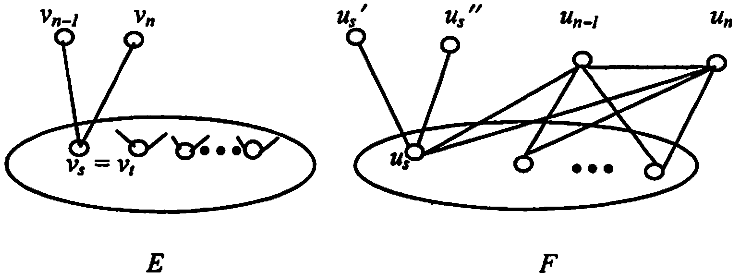

Figure 5 : Structure of E and F in Subcase 1.2.2.1, Theorem 6.

As a result, there will be two endvertices, say u_s' and u_s'' which are adjacent to u_s in F and hence for $i \notin \{s, n-1, n\}$, u_i is adjacent neither with u_s' nor with u_s''. Thus $deg_F\, u_s' = deg_F\, u_s'' = 1$, $deg_F\, u_s \geq 4$ (since u_s is adjacent to both u_{n-1} and u_n each of which is different from u_s' and u_s'') and $deg_F\, u_i \geq 2$ for all other vertices u_i of F (Figure 5). Hence the two cards $F-u_s'$ and $F-u_s''$ each has exactly one endvertex. Thus, at most $n-5$ of the cards $F-u_1$, $F-u_2$, ..., $F-u_{n-2}$ have more than one endvertex, giving a contradiction to (8).

Subcase 1.2.2.2. $v_s \neq v_t$

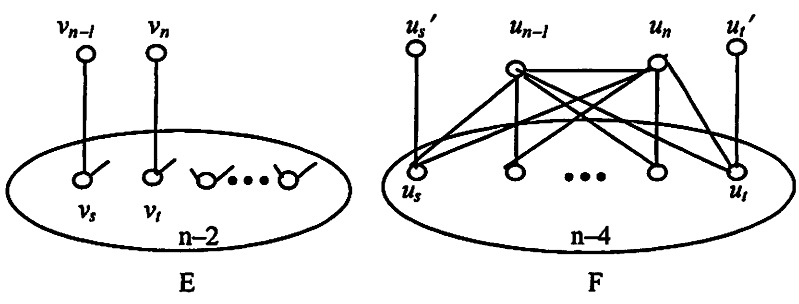

Figure 6 : Structure of E and F in Subcase 1.2.2.2 , Theorem 6.

Now the common cards $E-v_s$ and $E-v_t$ (and hence $F-u_s$ and $F-u_t$)each has exactly one isolated vertex and at least one endvertex. Each of the other $n-4$ common cards $E-v_i$ (and hence $F-u_i$), $i \in \{1, 2, ..., n-2\}-\{s, t\}$ has the following two properties.

(i) has two endvertices

(ii) has no isolated vertex. .....(9)

As a result, F will not have an isolated vertex and there will be two endvertices in F, say u_s' and u_t' adjacent with u_s and u_t respectively. Thus in F, the vertices u_n and u_{n-1} (each of degree $n-3$) are adjacent and each is adjacent to all the vertices other than u_s' and u_t' and hence $deg_F\, u_s \geq 3$, $deg_F\, u_t \geq 3$ and $deg_F\, u_i \geq 2$ for all $u_i \in \{u_1, u_2, \ldots, u_{n-2}\} - \{u_s, u_t, u_s', u_t'\}$ (Figure 6). Now the common card $F-u_s'$ (respectively $F-u_t'$) has exactly one endvertex namely u_t' (respectively u_s'). This *contradicts* (9).

Thus Case 1 can not arise.

Case 2. $deg_F\, u_{n-1} + deg_F\, u_n = 2n-7$ and $deg_E\, v_{n-1} + deg_E\, v_n = 1$.

Since $0 \leq deg_F\, u_i \leq n-3$, $\{deg_F\, u_{n-1},\ deg_F\, u_n\} = \{n-3,\ n-4\}$ and $\{deg_E\, v_{n-1},\ deg_E\, v_n\} = \{1, 0\}$. Without loss of generality, let us take that $deg_F\, u_{n-1} = n-3$, $deg_F\, u_n = n-4$, $deg_E\, v_{n-1} = 1$ and $deg_E\, v_n = 0$. Let v_s be the neighbour of v_{n-1} in E. Then, since $deg_E\, v_i \geq 2$ for $i \notin \{n-1, n\}$, the common card $E-v_s$ has exactly two isolated vertices. Each of the other $n-3$ common cards has the following two properties.

(i) has exactly one isolated vertex

(ii) has at least one endvertex. ---- (10)

Now, if F has two isolated vertices, say u_r and u_q, then the common cards other than $F-u_r$ and $F-u_s$ ($n-4$ in number), each has at least two isolated vertices, *contradicting* (10). If F has only one isolated vertex, say u_r then the common card $F-u_r$ has no isolated vertex, again *contradicting* (10).

Otherwise, F has no isolated vertex. Now since $F-u_s$ has exactly two isolated vertices, there will be two endvertices in F, say u_s' and u_s'', both of them adjacent with u_s. Then neither of the common cards $F-u_s'$ and $F-u_s''$ has an isolated vertex, and this *contradicts* (10).

Thus Case 2 can not arise.

Case 3. $deg_F\, u_{n-1} + deg_F\, u_n = 2n-8$ and $deg_E\, v_{n-1} + deg_E\, v_n = 0$.

Now $\{deg_F\, u_{n-1},\ deg_F\, u_n\} = \{n-3,\ n-5\}$ or $\{n-4\}$ and $deg_E\, v_{n-1} = deg_E\, v_n = 0$. Since $deg_E\, v_i \geq 2$ for $i \notin \{n-1, n\}$, each common card $E-v_i$ (and hence $F-u_i$), $1 \leq i \leq n-2$ has exactly two isolated vertices. ---(11)

Now, if F has at least three isolated vertices, then there will be a common card with at least three isolated vertices, *contradicting* (11). Hence F has at most two isolated vertices. Now since u_{n-1} and u_n are not isolated vertices, there will be a common card $F-u_s$, for some $s \in \{1,2,\ldots,n-2\}$ with at most one isolated vertex, again *contradicting* (11). Thus Case 3 can not arise.

Hence (II) can not occur. As (I) can not also occur, we have $|e(H)-e(G)| \leq 1$ proving the theorem. ■

4. Conclusion

There are graph pairs G and H on six vertices with $\delta \geq 2$ and having four cards in common such that $|e(G) - e(H)| = 1$. It seems that, for sufficiently large n, the number of such pairs with different number of edges will be zero.

It will be useful if an algorithm to determine the number (within a neighbourhood of 1) of edges of a graph G from an arbitrary collection of $n-2$ of its cards can be found. Some immediate open problems arising out of this paper are the following.

1. Proving that e(G) = e(H) for sufficiently large n.
2. Finding families of examples where $|e(G) - e(H)| = 1$, showing that the bound given by Theorem 6 is tight.
3. Removing the condition $\delta \geq 2$ from the hypothesis of Theorem 6, or providing counterexamples showing that $\delta \geq 2$ cannot be removed.

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Reference

[1]. J.A. Bondy and R.L. Hemminger, Graph reconstruction – a survey, J. Graph Theory 1 (1977), 227–268.

[2]. F. Harary, Graph Theory, Addison–Wesley, Mass 1969.

[3]. B. Manvel, On reconstruction of graphs, in : The Many facets of Graph Theory (Proceedings of the conference held at Western Michigan University, Kalamazoo, MI, 1968), Lecture Notes in Math. Vol. 110 (Springer, Berlin, 1969), 207–214.

[4]. W.J.Myrvold, The ally-reconstruction number of a disconnected graph, Ars Combinatoria 28 (1989) 123–127.

[5]. W.J. Myrvold, The degree sequence is reconstructible from n–1 cards, Discrete Math. 102 (1992) 187–196.

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