Fuzzy parameterized fuzzy soft lattice implication algebras

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Abstract: Cagman et al. introduced the concept of a fuzzy parameterized fuzzy soft set(briefly, FPFS) which is an extension of a fuzzy set and a soft set. In this paper, we introduce the concepts of FPFS filters and FPFS implicative filters of lattice implication algebras and obtain some related results. Finally, we define the concept of FPFS-aggregation operator of lattice implication algebras.

Keywords: Lattice implication algebra, fuzzy soft set, fuzzy parameterized fuzzy soft set, FPFS filter, FPFS implicative filter.

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1 Introduction

The logical system whose propositional value is given in a lattice was first studied by Xu in [14] from the semantic viewpoint. He then proposed the concept of lattice implication algebras and discussed some of their properties. Later on, Xu and Qin [15] discussed the properties of implicative filters in a lattice implicative algebra. Since then this logical algebras have been extensively investigated by several researchers, see [8–10, 16, 18, 19, 21]. For more details, the reader is refereed to the book [17].

There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. However, all of these theories have their own difficulties which have been pointed out by Molodtsov [13]. Maji et al. [12] and Molodtsov [13] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. At present, research on the soft set theory is progressing rapidly. Maji et al. [11] described the application of soft set theory to a decision making problem. They also studied several operations on the theory of soft sets. Chen et al. [2] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attribute reduction in rough set theory. Feng [4] investigated some characterizations of soft semirings. Zhan [20, 22] discussed the properties of soft BL-algebras and lattice implication algebras based on fuzzy sets, respectively.

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In the same time, this theory has been proven useful in many different fields such as decision making [3, 5–7].

In this paper, we discuss some important properties of FPFS filters and FPFS implicative filters of lattice implication algebras. The methods can be successfully applied to many problems that contain uncertainties.

2 Preliminaries

To start with, we first recall that an algebra $(L, \vee, \wedge, ', \rightarrow, 0, 1)$ is a lattice implication algebra [14] if $(L, \vee, \wedge, 0, 1)$ is a bounded lattice with an order-reversing involution " $'$ " and is a binary operation " $\rightarrow$ " such that the following axioms are satisfied:

(I1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$,

(I2) $x \rightarrow x = 1$,

(I3) $x \rightarrow y = y' \rightarrow x'$,

(I4) $x \rightarrow y = y \rightarrow x = 1 \Rightarrow x = y$,

(I5) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$,

(L1) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$,

(L2) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$, for all $x, y, z \in L$.

A non-empty subset A of L is called a filter of L if it satisfies the following conditions: (A1) $1 \in A$; (A2) $\forall x \in A, y \in L, x \rightarrow y \in A \Rightarrow y \in A$. Now, we call a non-empty subset A of L an implicative filter if it satisfies (A1) and (A3) $x \rightarrow (y \rightarrow z) \in A, x \rightarrow y \in A \Rightarrow x \rightarrow z \in A$. Equivalently, a non-empty subset A of L is an implicative filter of L if and only if it satisfies (A1) and (A4) $x \rightarrow ((y \rightarrow z) \rightarrow y) \in A, x \in A \Rightarrow y \in A$, for all $x, y, z \in L$.

Definition 2.1 [15] A fuzzy set μ of L is called a fuzzy filter of L if it satisfies:

(F1) $\forall x \in L, \mu(1) \geq \mu(x)$;

(F2) $\forall x, y \in L, \mu(y) \geq \min\{\mu(x \rightarrow y), \mu(x)\}$.

Definition 2.2 [9] A fuzzy set μ of L is called a fuzzy implicative filter of L if it satisfies (F1) and

(F3) $\mu(x \rightarrow z) \geq \min\{\mu(x \rightarrow (y \rightarrow z)), \mu(x \rightarrow y)\}$, for all $x, y, z \in L$.

Example 2.3 Let $L = \{0, a, b, c, d, 1\}$ be a set with an involution "$'$" and a binary operation " $\rightarrow$ " with the following Cayley table:

x	x'
0	1
a	c
b	d
c	a
d	b
1	0

$\rightarrow$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	c	1	b	c	b	1
b	d	a	1	b	a	1
c	a	a	1	1	a	1
d	b	1	1	b	1	1
1	0	a	b	c	d	1

Define $\vee-$ and $\wedge-$ operations on L as follows $x \vee y = (x \to y) \to y$, $\quad x \wedge y = ((x' \to y') \to y')'$. Then L is clearly a lattice implication algebra.

Define a fuzzy subset μ of L by $F(1) = F(b) = F(c) = 0.7, F(0) = F(a) = F(d) = 0.6$. One can easily check that μ is a fuzzy implicative filter of L.

Molodtsov [13] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the non-empty power set of U and $A \subseteq E$.

Definition 2.4 [13] A pair (F, A) is called a soft set over U if F is a mapping of A into the set of all subsets of the set U.

Definition 2.5 [4] The bi-intersection of two soft sets (F, A) and (G, B) over U is the soft set (H, C), where $C = A \cap B$, and $\forall e \in C$, $H(e) = F(e) \cap G(e)$. We write $(F, A)\tilde{\cap}(G, B) = (H, C)$.

Definition 2.6 [12] The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if} \quad e \in A - B, \\ G(e), & \text{if} \quad e \in B - A, \\ F(e) \cup G(e), & \text{if} \quad e \in A \cap B. \end{cases}$$

We write $(F, A)\tilde{\cup}(G, B) = (H, C)$.

3 Fuzzy parameterized fuzzy soft filters

In [3], Feng et al. presented an adjustable approach to fuzzy soft set based decision making and gave some illustrative examples. By a different way, Cagman introduced the concept of FPFS-sets as follows.

Definition 3.1 [1] Let U be an initial universe, E a set of parameters and μ be a fuzzy set over E with the membership function $\mu : E \to [0, 1]$ and $\gamma(e)$ be a fuzzy set over U for all $e \in E$. Then, an FPFS-set Γ over U is a set defined by a function representing a mapping

$\gamma : E \to F(U)$ such that $\gamma(e) = \emptyset$ if $\mu(e) = 0$.

Here, γ is called fuzzy approximate function of the FPFS-set Γ, and the value $\gamma(e)$ is a fuzzy set called e-element of the FPFS-set for all $e \in E$. Thus, an FPFS-set Γ over U can be represented by the set of ordered pairs

$\Gamma = \{(\frac{\mu(e)}{e} | e \in E, \gamma(e) \in F(U), \mu(e) \in [0, 1]\}$.

Definition 3.2 Let L be an lattice implication algebra, E be a parameter and μ be a fuzzy set over U and $\gamma(e)$ be a fuzzy set over L for all $e \in E$. Then an

FPFS-set $\Gamma = (\frac{\mu(e)}{e}, \gamma(e))$ over L is called an FPFS filter of L if for all $e \in E$ and $x, y \in L$, it satisfies:

(FP1) $\gamma(e)(1) \geq \gamma(e)(x)$;

(FP2) $\gamma(e)(y) \geq \gamma(e)(x \to y) \wedge \gamma(e)(x)$.

Example 3.3 Let $L = \{0, a, b, 1\}$ be a set with Caylay tables as follows:

x	x'
0	1
a	b
b	a
1	0

$\to$	0	a	b	1
0	1	1	1	1
a	b	1	1	1
b	a	b	1	1
1	0	a	b	1

Define $\vee-$ and $\wedge-$ operations on L as follows $x \vee y = (x \to y) \to y$, $x \wedge y = ((x' \to y') \to y')'$. Then it can be verified that L is a lattice implication algebra.

Let $E = \{e_1, e_2\}$ and $\mu = \frac{0.5}{e_1} + \frac{0.3}{e_2}$. Define a fuzzy set γ over L by

$\gamma(e_1) = \frac{1}{1} + \frac{0.6}{a} + \frac{0.6}{b} + \frac{0.2}{c} + \frac{0.2}{0}$ and $\gamma(e_2) = \frac{1}{1} + \frac{0.4}{a} + \frac{0.4}{b} + \frac{0.4}{c} + \frac{0.4}{0}$.

One can easily check that $\Gamma = (\frac{\mu(e)}{e}, \gamma(e))$ over L is an FPFS filter of L.

Now, we define $\Gamma_t = \{x \in L | \gamma(e)(x) \geq t\}$ for all $e \in E$.

Theorem 3.4 Let μ be a fuzzy set over U and $\gamma(e)$ be a fuzzy set over L for all $e \in E$. Then an FPFS-set $\Gamma = (\frac{\mu(e)}{e}, \gamma(e))$ over L is an FPFS filter of L if and only the level subset Γ_t is a filter of L for all $t \in [0, 1]$.

Proof. Let $\Gamma = (\frac{\mu(e)}{e}, \gamma(e))$ be an FPFS filter of L and $t \in [0, 1]$. If $x \in \Gamma_t$ for all $e \in E$ and $x \in L$, then $\gamma(e)(x) \geq t$, and so $\gamma(e)(1) \geq \gamma(e)(x) \geq t$, which implies, $1 \in \Gamma_t$. Let $x, y \in L$ be such that $x, x \to y \in \Gamma_t$, then $\gamma(e)(x) \geq t$ and $\gamma(e)(x \to y) \geq t$ Thus, $\gamma(e)(y) \geq \gamma(e)(x \to y) \wedge \gamma(e)(x) \geq t$ and so $y \in \Gamma_t$. Γ_t is a filter of L for all $t \in [0, 1]$.

Conversely, assume that Γ_t is a filter of L for all $t \in [0, 1]$. If $\gamma(e)(1) < t < \gamma(e)(x)$, then $1 \notin \Gamma_t$, but $x \in \Gamma_t$, contradiction. If there exist $x \to y, x \in \Gamma_t$ such that $\gamma(e)(y) < t = \gamma(e)(x \to y) \wedge \gamma(e)(x)$, then $y \notin \Gamma_t$, but $x \to y \in \Gamma_t$ and $x \in \Gamma_t$, and so $y \in \Gamma_t$, contradiction. Thus Γ is an FPFS implicative filter of L. □

Proposition 3.5 Let (F, A) and (G, B) be two FPFS filters of L. Then the set $(F, A) \tilde{\cap} (G, B)$ is an FPFS filter of L, when it is non-null.

Proposition 3.6 Let (F, A) be an FPFS filter of L. If B is a subset of A, then $(F|_B, B)$ is an FPFS filter of L.

Definition 3.7 Let L be an lattice implication algebra, E be a parameter and μ be a fuzzy set over U and $\gamma(e)$ be a fuzzy set over L for all $e \in E$. Then an FPFS-set $\Gamma = (\frac{\mu(e)}{e}, \gamma(e))$ over L is called an FPFS implicative filter of L if for all $e \in E$ and $x, y, z \in L$, it satisfies (FP1) and

(FP3) $\gamma(e)(x \to z) \geq \gamma(e)(x \to (y \to z)) \wedge \gamma(e)(x \to y)$.

From the above definitions, we can get the following:

Remark 3.8 Every FPFS implicative filter of lattice implication algebras is an FPFS filter.

Example 3.9 Let $L = \{0, a, b, 1\}$ be a set with Caylay tables as follows:

x	x'
0	1
a	b
b	a
1	0

$\rightarrow$	0	a	b	1
0	1	1	1	1
a	a	1	1	1
b	0	a	1	1
1	0	a	b	1

Define $\vee-$ and $\wedge-$ operations on L as follows $x \vee y = (x \rightarrow y) \rightarrow y$, $\quad x \wedge y = ((x' \rightarrow y') \rightarrow y')'$. Then it can be verified that L is a lattice implication algebra.

Let $E = \{e_1, e_2\}$ and $\mu = \frac{0.5}{e_1} + \frac{0.3}{e_2}$. Define a fuzzy set γ over L by $\gamma(e_1) = \frac{1}{1} + \frac{0.6}{a} + \frac{0.6}{b} + \frac{0.2}{0}$ and $\gamma(e_2) = \frac{1}{1} + \frac{0.4}{a} + \frac{0.4}{b} + \frac{0.4}{0}$.

One can easily check that $\Gamma = (\frac{\mu(e)}{e}, \gamma(e))$ over L is an FPFS implicative filter of L.

Theorem 3.10 Let μ be a fuzzy set over U and $\gamma(e)$ be a fuzzy set over L for all $e \in E$. Then an FPFS-set $\Gamma = (\frac{\mu(e)}{e}, \gamma(e))$ over L is an FPFS implicative filter of L if and only the level subset Γ_t is an implicative filter of L for all $t \in [0, 1]$.

Proof. Let $\Gamma = (\frac{\mu(e)}{e}, \gamma(e))$ be an FPFS implicative filter of L and $t \in [0, 1]$. If $x \in \Gamma_t$ for all $e \in E$ and $x \in L$, then $\gamma(e)(x) \geq t$, and so $\gamma(e)(1) \geq \gamma(e)(x) \geq t$, which implies, $1 \in \Gamma_t$. Let $x, y, z \in L$ be such that $x \rightarrow (y \rightarrow z), x \rightarrow y \in \Gamma_t$, then $\gamma(e)(x \rightarrow (y \rightarrow z)) \geq t$ and $\gamma(e)(x \rightarrow y) \geq t$ Thus, $\gamma(e)(x \rightarrow z) \geq \gamma(e)(x \rightarrow y) \wedge \gamma(e)(x \rightarrow (y \rightarrow z)) \geq t$ and so $x \rightarrow z \in \Gamma_t$. Γ_t is an implicative filter of L for all $t \in [0, 1]$.

Conversely, assume that Γ_t is an implicative filter of L for all $t \in [0, 1]$. If $\gamma(e)(1) < t < \gamma(e)(x)$, then $1 \notin \Gamma_t$, but $x \in \Gamma_t$, contradiction. If there exist $x \rightarrow (y \rightarrow z), x \rightarrow y \in \Gamma_t$ such that $\gamma(e)(x \rightarrow z) < t = \gamma(e)(x \rightarrow y) \wedge \gamma(e)(x \rightarrow (y \rightarrow z))$, then $x \rightarrow z \notin \Gamma_t$, but $x \rightarrow y \in \Gamma_t$ and $x \rightarrow (y \rightarrow z) \in \Gamma_t$, and so $x \rightarrow z \in \Gamma_t$, contradiction. Thus Γ is an FPFS implicative filter of L. □

In [1], Cagman defined the concept of FPFS-aggregation operators, we apply this concept to lattice implication algebras as follows.

Definition 3.11 Let Γ be a FPFS-set. Then FPFS-aggregation operator, denoted by $FPFS_{agg}$ is defined by

$FPFS_{agg} : F(E) \times FPFS(L) \rightarrow F(L)$,

$FPFS_{agg}(A, \Gamma) = \Gamma^*$,

where $\Gamma^* = \{\frac{\mu_{T^*}(a)}{a} | a \in L\}$ is a fuzzy filter of L.
The value Γ^* is called fuzzy set of Γ.
Here, the membership $\mu_{T^*}(a)$ of a is defined as follows:
where $\mu_{T^*}(a) = \frac{1}{|E|} \sum_{e \in E} \mu(e)\mu_{\Gamma^*(e)}(a)$ is the cardinality of $|E|$.

By the FPFS-aggregation operators of lattice implication algebras, we can construct an FPFS-decision making method by the following algorithm.

Step1 Construct an FPFS-set Γ over L,

Step2 Find the aggregate set Γ^* over Γ,

Step3 Find the largest membership grade $\max \mu_{T^*}(a)$, where $a \in L$.

4 Conclusion

In this paper, we apply fuzzy parameterized fuzzy soft set theory to lattice implication algebras. In the future study of soft lattice implication algebras, we will put forward some examples on FPFS-aggregation operators of lattice implication algebras and apply this kind of new soft lattice implication algebras to some applied fields, such as decision making, data analysis and forecasting and so on.

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