Schultz and Zagreb indices in corona of two graphs*

Xiaoling Ma[1], Hong Bian[2]† Haizheng Yu[1]

[1] College of Mathematics and System Sciences, Xinjiang University, Urumqi, Xinjiang 830046, P.R. China

[2] School of Mathematical Science, Xinjiang Normal University, Urumqi, Xinjiang 830054, P.R. China

Abstract. The corona of two graphs G and H, written as $G \odot H$, is defined as the graph obtained by taking one copy of G and $|V(G)|$ copies of H, and joining by an edge the ith vertex of G to every vertex in the ith copy of H. In this paper, we present the explicit formulae of the (modified) Schultz and Zagreb indices in the corona of two graphs.

MSC: 05C99

Key words: corona graph; (modified) Schultz index; Zagreb index

1 Introduction

Throughout this paper we consider only finite undirected connected graphs without loops and multiples edges. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. We denote by $deg_G(x)$ and $\overline{G}$ the degree of x, and the complement of graph G, respectively. As usual, the distance between the vertices x and y of $V(G)$ is denoted by $d_G(x, y)$ and it is defined as the number of edges in a shortest path connecting the vertices x and y in G.

A topological index is a real number related to a graph; it does not depend on the labeling or the pictorial representation of a graph. There are several topological indices: the Wiener index, the Randić index, the Hosoya index, the Merrifield-Simmons index, the Szeged index and the vertex and edge Padmakar-Ivan indices PI_v and PI_e. These topological indices have found applications as means for modeling chemical, pharmaceutical and other properties of molecules. For more results on topological indices of graphs see, for example, [6, 8, 10, 11, 12, 16] and the references therein.

The Wiener index is the first and one of the most studied topological indices, both from theoretical point of view and applications. It is equal to the sum of distances between all ordered pairs of vertices of the respective graph, see for details [29].

The Wiener index of a graph G is defined by:

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u, v).$$

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†Corresponding author. Email: bh1218@163.com

Another structure-descriptor introduced long time ago is the Zagreb index M_1 [13], or more precisely, the "first Zagreb index" , because there exists also the "second Zagreb index" M_2 [28]. The indices M_1 and M_2 are defined as follows.

$$M_1(G) = \sum_{u \in V(G)} deg_G^2(u), \quad M_2(G) = \sum_{uv \in E(G)} deg_G(u) deg_G(v).$$

For more results on M_1 and M_2, one can reference on [3, 14, 22, 32, 33, 34] and the references therein.

We now define the index S introduced by Schultz [27] and its modification S^* introduced by Klavžar and Gutman [19].

The standard and modified Schultz indices for a connected graph G are

$$S(G) = \sum_{\{u,v\} \subset V(G)} (deg_G(u) + deg_G(v)) d_G(u, v);$$

$$S^*(G) = \sum_{\{u,v\} \subset V(G)} (deg_G(u) \cdot deg_G(v)) d_G(u, v).$$

Corona graphs were introduced by Roberto Frucht and Frank Harary in 1970 [9]. The corona of two graphs G and H (where G has n vertices), written as $G \odot H$, is defined as the graph obtained by taking one copy of G and n copies of H, and then joining by an edge the ith vertex of G to every vertex in the ith copy of H. In 2002, Barrientos [4] first studied the graceful labelings of the corona graphs; soon after, some results on the corona graphs are obtained in succession. Lai et al. [20] gave the exact values of the profiles of coronas $G \odot H$; Kwong and Lee [18] investigated the integer-magic spectra of the coronas of some specific graphs including paths, cycles, complete graphs and stars; the basis number of the corona of graphs is determined by Shakhatreh et al. [26]; Barik et al. [1] and Kojima [17] investigated the spectrum and the bandwidth of the corona of two graphs respectively. In 2011, Yero and Rodríguez-Velázquez [31] considered the Randić index of corona product graphs. In 2012, Hong Bian et al. [2] gave the Wiener-type indices of the corona of two graphs.

In this paper, we present the explicit formulae of the (modified) Schultz and Zagreb indices in corona of two graphs.

2 Main results

2.1 The Schultz index of the corona of graphs

The following properties of the corona of graphs follow immediately from the definition.

Lemma 2.1 *Let G and H be two graphs. Then we have:*
(a) $|V(G \odot H)| = |V(G)| \cdot |V(H)| + |V(G)|$ *and*
$|E(G \odot H)| = |E(G)| + |V(G)| \cdot |E(H)| + |V(G)| \cdot |V(H)|$;
(b) $G \odot H$ *is connected if and only if G is connected;*
(c) For any graph K, the associative law of the corona of graphs never holds: $G \odot (H \odot K)$ and $(G \odot H) \odot K$ are always different graphs.

Now we can compute the Schultz index of $G \odot H$ in terms of Schultz and Wiener indices of G and Zagreb index of H.

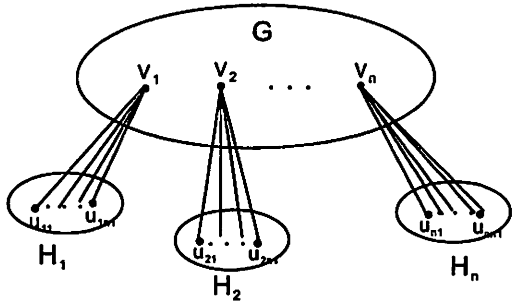

Figure 1: Corona of two graphs G and H, where H_i is the copy of H, for $i \in \{1, 2, \cdots, n\}$.

Theorem 2.1 *Let G and H be two graphs with $|V(G)| = n, |V(H)| = n_1, |E(G)| = m$ and $|E(H)| = m_1$. Then $S(G \odot H) = (n_1+1)S(G) + (4n_1^2 + 4n_1 + 4m_1n_1 + 4m_1)W(G) - nM_1(H) + n^2(3n_1^2 + n_1 + 2m_1 + 4m_1n_1) + n(6n_1m - 6m_1 - 4m_1n_1 - 2n_1)$.*

Proof. Set $V(G)=\{v_1, v_2, \cdots, v_n\}$ and $V(H_i)= \{u_{i1}, u_{i2}, \cdots, u_{in_1}\}$, $i \in \{1, 2, \cdots, n\}$ (see Figure 1). We partition the set $V(G \odot H) \times V(G \odot H)$ into four subsets: V_1, V_2, V_3, and V_4, where $V_1=\{\{u, v\} \subset V(G \odot H)| u, v \in V(H_i), i \in \{1, 2, \cdots, n\}\}$, $V_2=\{\{u, v\} \subset V(G \odot H)| v \in V(G), u \in V(H_i), i \in \{1, 2, \cdots, n\}\}$, $V_3=\{\{u, v\} \subset V(G \odot H)| u, v \in V(G)\}$, and $V_4=\{\{u, v\} \subset V(G \odot H)| u \in V(H_i), v \in V(H_j), i, j \in \{1, 2, \cdots, n\}$ and $i \neq j\}$. By definition, we have

$$S(G) = \sum_{i=1}^{4} \sum_{\{u,v\} \in V_i} (deg_{G \odot H}(u) + deg_{G \odot H}(v)) d_{G \odot H}(u, v).$$

In the following we work out each of the four terms in the above summation.

Case 1. $\{u, v\} \in V_1$. Suppose that $u_{ik}, u_{il} \in V(H_i)$, for $i \in \{1, 2, \cdots, n\}$, and $k, l \in \{1, 2, \cdots, n_1\}$, we show that the summation of some expression relating degrees and distance for vertices of V_1 is

$$\begin{aligned}
A := n & \sum_{u_{ik}, u_{il} \in V(H_i)} (deg_{G \odot H}(u_{ik}) + deg_{G \odot H}(u_{il})) d_{G \odot H}(u_{ik}, u_{il}) \\
&= n \sum_{u \in V(H_i)} deg_{H_i}(u)(deg_{H_i}(u) + 1) \\
&\quad + n \sum_{u \in V(H_i)} 2(n_1 - 1 - deg_{H_i}(u))(deg_{H_i}(u) + 1) \\
&= 4nn_1m_1 + 2nn_1^2 - 6nm_1 - 2nn_1 - nM_1(H)
\end{aligned}$$

Case 2. $\{u,v\} \in V_2$. Suppose that $v_i \in V(G), u_{jk} \in V(H_j)$, for $i,j \in \{1,2,\cdots,n\}, k \in \{1,2,\cdots,n_1\}$, then the summation of some expression relating degrees and distance for vertices of V_2 is

$$\begin{aligned}
B :=& \sum_{v_i \in V(G), u_{jk} \in V(H_j)} (deg_{G\odot H}(v_i) + deg_{G\odot H}(u_{jk}))d_{G\odot H}(v_i, u_{jk}) \\
=& \sum_{v_i \in V(G), u_{jk} \in V(H_j)} (deg_G(v_i) + n_1 + deg_{H_j}(u_{jk}) + 1)(d_G(v_i, v_j) + 1) \\
=& \sum_{v_i \in V(G), u_{jk} \in V(H_j)} deg_G(v_i)d_G(v_i, v_j) + \sum_{v_i \in V(G), u_{jk} \in V(H_j)} deg_G(v_i) \\
&+ \sum_{v_i \in V(G), u_{jk} \in V(H_j)} deg_{H_j}(u_{jk})d_G(v_i, v_j) + \sum_{v_i \in V(G), u_{jk} \in V(H_j)} deg_{H_j}(u_{jk}) \\
&+ (n_1+1) \sum_{v_i \in V(G), u_{jk} \in V(H_j)} d_G(v_i, v_j) + (n_1+1) \sum_{v_i \in V(G), u_{jk} \in V(H_j)} 1 \\
=& n_1 S(G) + (2n_1^2 + 4m_1 + 2n_1)W(G) + n^2 n_1(n_1+1) + 2mnn_1 + 2n^2 m_1.
\end{aligned}$$

Case 3: $\{u,v\} \in V_3$. Suppose that $v_i, v_j \in V(G)$, for $i,j \in \{1,2,\cdots,n\}$, then the summation of some expression relating degrees and distance for vertices of V_3 is

$$\begin{aligned}
C :=& \sum_{\{v_i, v_j\} \subset V(G)} (deg_{G\odot H}(v_i) + deg_{G\odot H}(v_j))d_{G\odot H}(v_i, v_j) \\
=& \sum_{\{v_i, v_j\} \subset V(G)} (deg_G(v_i) + n_1 + deg_G(v_j) + n_1)d_G(v_i, v_j) \\
=& \sum_{\{v_i, v_j\} \subset V(G)} [(deg_G(v_i) + deg_G(v_j))d_G(v_i, v_j) + 2n_1 d_G(v_i, v_j)] \\
=& S(G) + 2n_1 W(G).
\end{aligned}$$

Case 4. $\{u,v\} \in V_4$. Suppose that $u_{ik} \in V(H_i), u_{jl} \in V(H_j)$, for $i,j \in \{1,2,\cdots,n\}(i \neq j), k,l \in \{1,2,\cdots,n_1\}$, then the summation of some expression relating degrees and distance for vertices of V_4 is

$$\begin{aligned}
D :=& \sum_{u_{ik} \in V(H_i), u_{jl} \in V(H_j)(i\neq j)} (deg_{G\odot H}(u_{ik}) + deg_{G\odot H}(u_{jl}))d_{G\odot H}(u_{ik}, u_{jl}) \\
=& \sum_{u_{ik} \in V(H_i), u_{jl} \in V(H_j)(i\neq j)} (deg_{H_i}(u_{ik}) + 1 + deg_{H_j}(u_{jl}) + 1)(d_G(v_i, v_j) + 2) \\
=& \sum_{u_{ik} \in V(H_i), u_{jl} \in V(H_j)(i\neq j)} deg_{H_i}(u_{ik})d_G(v_i, v_j) + 2 \sum_{u_{ik} \in V(H_i), u_{jl} \in V(H_j)(i\neq j)} deg_{H_i}(u_{ik}) \\
&+ \sum_{u_{ik} \in V(H_i), u_{jl} \in V(H_j)(i\neq j)} deg_{H_j}(u_{jl})d_G(v_i, v_j) + 2 \sum_{u_{ik} \in V(H_i), u_{jl} \in V(H_j)(i\neq j)} deg_{H_j}(u_{jl})
\end{aligned}$$

$$+2 \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} d_G(v_i,v_j)+4 \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} 1$$
$$= (2n_1^2+4n_1m_1)W(G)+(2n_1^2+4n_1m_1)(n^2-n).$$

Hence,

$$S(G\odot H) = A+B+C+D$$
$$= (n_1+1)S(G)+(4n_1^2+4n_1+4m_1n_1+4m_1)W(G)-M_1(H)n$$
$$+n^2(3n_1^2+n_1+2m_1+4m_1n_1)+n(6n_1m-6m_1-4m_1n_1-2n_1).$$

2.2 The modified Schultz index of the corona of graphs

Theorem 2.2 *Let G and H be two graphs with $|V(G)| = n, |V(H)| = n_1, |E(G)| = m$ and $|E(H)| = m_1$. Then $S^*(G\odot H)= (n_1+1)S(G)+(4n_1^2+4n_1+4m_1n_1+4m_1)W(G)-M_1(H)n+n^2(3n_1^2+n_1+2m_1+4m_1n_1)+n(6n_1m-6m_1-4m_1n_1-2n_1)$.*

Proof. Similar to the proof of Theorem 2.1, set $V(G)=\{v_1,v_2,\cdots,v_n\}$ and $V(H_i)=\{u_{i1},u_{i2},\cdots,u_{in_1}\}$, $i\in\{1,2,\cdots,n\}$. We partition the set $V(G\odot H)\times V(G\odot H)$ into four subsets: V_1, V_2, V_3, and V_4, where $V_1=\{\{u,v\}\subset V(G\odot H)|\ u,v\in V(H_i),\ i\in\{1,2,\cdots,n\}\}$, $V_2=\{\{u,v\}\subset V(G\odot H)|\ v\in V(G), u\in V(H_i), i\in\{1,2,\cdots,n\}\}$, $V_3=\{\{u,v\}\subset V(G\odot H)|\ u,v\in V(G)\}$, and $V_4=\{\{u,v\}\subset V(G\odot H)|\ u\in V(H_i), v\in V(H_j)$, $i,j\in\{1,2,\cdots,n\}$ and $i\neq j\}$. By definition, we have

$$S^*(G)=\sum_{i=1}^{4}\sum_{\{u,v\}\in V_i}(deg_{G\odot H}(u)\cdot deg_{G\odot H}(v))d_{G\odot H}(u,v).$$

In the following we work out each of the four terms in the above summation.

Case 1. $\{u,v\}\in V_1$. Suppose that $u_{ik},u_{il}\in V(H_i)$, for $i\in\{1,2,\cdots,n\}$, and $k,l\in\{1,2,\cdots,n_1\}$, we show that the summation of some expression relating degrees and distance for vertices of V_1 is

$$\alpha := n\sum_{u_{ik},u_{il}\in V(H_i)}(deg_{G\odot H}(u_{ik})\cdot deg_{G\odot H}(u_{il}))d_{G\odot H}(u_{ik},u_{il})$$
$$= n\sum_{u_{ik},u_{il}\in V(H_i)}(deg_{H_i}(u_{il})+1)(deg_{H_i}(u_{ik})+1)d_{G\odot H}(u_{ik},u_{il})$$
$$= n\sum_{u_{ik}u_{il}\in E(H_i)}(deg_{H_i}(u_{ik})deg_{H_i}(u_{il})+deg_{H_i}(u_{ik})+deg_{H_i}(u_{il})+1)$$
$$+n\sum_{u_{ik}u_{il}\notin E(H_1)}2(deg_{H_i}(u_{ik})deg_{H_i}(u_{il})+deg_{H_i}(u_{ik})+deg_{H_i}(u_{il})+1)$$
$$= nM_2(H_i)-nM_1(H_i)-n(n_1-1)M_1(\overline{H_i})+nM_2(\overline{H_i})+nn_1(n_1-1)^3$$
$$-2nm_1(n_1-1)^2+(4m_1+n_1)n(n_1-1)-nm_1.$$

Case 2. $\{u,v\} \in V_2$. Suppose that $v_i \in V(G), u_{jk} \in V(H_j)$, for $i,j \in \{1,2,\cdots,n\}, k \in \{1,2,\cdots,n_1\}$, then the summation of some expression relating degrees and distance for vertices of V_2 is

$$\begin{aligned}
\beta = & \sum_{v_i\in V(G),u_{jk}\in V(H_j)} (deg_{G\odot H}(v_i)\cdot deg_{G\odot H}(u_{jk}))d_{G\odot H}(v_i,u_{jk}) \\
= & \sum_{v_i\in V(G),u_{jk}\in V(H_j)} (deg_G(v_i)+n_1)(deg_{H_j}(u_{jk})+1)(d_G(v_i,v_j)+1) \\
= & \sum_{v_i\in V(G),u_{jk}\in V(H_j)} deg_G(v_i)deg_{H_j}(u_{jk})d_G(v_i,v_j) + \sum_{v_i\in V(G),u_{jk}\in V(H_j)} deg_G(v_i)deg_{H_j}(u_{jk}) \\
& + \sum_{v_i\in V(G),u_{jk}\in V(H_j)} deg_G(v_i)d_G(v_i,v_j) + \sum_{v_i\in V(G),u_{jk}\in V(H_j)} deg_G(v_i) \\
& + n_1 \sum_{v_i\in V(G),u_{jk}\in V(H_j)} deg_{H_j}(u_{jk})d_G(v_i,v_j) + n_1 \sum_{v_i\in V(G),u_{jk}\in V(H_j)} deg_{H_j}(u_{jk}) \\
& + n_1 \sum_{v_i\in V(G),u_{jk}\in V(H_j)} d_G(v_i,v_j) + n_1 \sum_{v_i\in V(G),u_{jk}\in V(H_j)} 1 \\
= & (2m_1+n_1)S(G)+(4m_1n_1+2n_1^2)W(G)+n^2n_1(n_1+2m_1)+2mn(n_1+2m_1).
\end{aligned}$$

Case 3. $\{u,v\} \in V_3$. Suppose that $v_i, v_j \in V(G)$, for $i,j \in \{1,2,\cdots,n\}$, then the summation of some expression relating degrees and distance for vertices of V_3 is

$$\begin{aligned}
\gamma := & \sum_{\{v_i,v_j\}\subset V(G)} (deg_{G\odot H}(v_i)\cdot deg_{G\odot H}(v_j))d_{G\odot H}(v_i,v_j) \\
= & \sum_{\{v_i,v_j\}\subset V(G)} (deg_G(v_i)+n_1)(deg_G(v_j)+n_1)d_G(v_i,v_j) \\
= & S^*(G)+n_1S(G)+n_1^2W(G).
\end{aligned}$$

Case 4. $\{u,v\} \in V_4$. Suppose that $u_{ik} \in V(H_i), u_{jl} \in V(H_j)$, for $i,j \in \{1,2,\cdots,n\}(i \neq j)$, and $k,l \in \{1,2,\cdots,n_1\}$, then the summation of some expression relating degrees and distance for vertices of V_4 is

$$\begin{aligned}
\delta := & \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} (deg_{G\odot H}(u_{ik})\cdot deg_{G\odot H}(u_{jl}))d_{G\odot H}(u_{ik},u_{jl}) \\
= & \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} (deg_{H_i}(u_{ik})+1)(deg_{H_j}(u_{jl})+1)(d_G(v_i,v_j)+2) \\
= & \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} deg_{H_i}(u_{ik})deg_{H_j}(u_{jl})d_G(v_i,v_j) \\
& +2 \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} deg_{H_i}(u_{ik})deg_{H_j}(u_{jl}) + \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} deg_{H_i}(u_{ik})d_G(v_i,v_j)
\end{aligned}$$

$$+2 \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} deg_{H_i}(u_{ik}) + \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} deg_{H_j}(u_{jl})d_G(v_i,v_j)$$
$$+2 \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} deg_{H_j}(u_{jl}) + \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} d_G(v_i,v_j)$$
$$+2 \sum_{u_{ik}\in V(H_i),u_{jl}\in V(H_j)(i\neq j)} 1$$
$$= (4m_1^2 + n_1^2 + 4n_1m_1)W(G) + (n_1^2 + 4n_1m_1 + 4m_1^2)(n^2 - n).$$

Hence,

$$\begin{aligned} S^*(G \odot H) =&\alpha + \beta + \gamma + \delta \\ =&S^*(G) + (2m_1 + 2n_1)S(G) + (2n_1 + 2m_1)^2 W(G) \\ &+ n(M_2(H) - M_1(H) + M_2(\overline{H}) - (n_1 - 1)M_1(\overline{H})) + nn_1(n_1 - 1)^3 \\ &- 2nm_1(n_1 - 1)^2 + 2n^2(n_1^2 + 2n_1m_1 + 2m_1^2 + n_1^2m_1) \\ &+ n(4mm_1 + 2mn_1 - 4m_1^2 - 5m_1 - n_1). \end{aligned}$$

2.3 The Zagreb indices of the corona of graphs

In this subsection, some exact formulae for the first and second Zagreb indices of $G \odot H$ are presented. We begin with the following result related to the first Zagreb index.

Theorem 2.3 *Let G and H be two graphs with $|V(G)| = n, |V(H)| = n_1, |E(G)| = m$ and $|E(H)| = m_1$. Then Then $M_1(G \odot H) = M_1(G) + nM_1(H) + 4n_1m + n_1^2 n + 4nm_1 + nn_1$.*

Proof. By the definition of the first Zagreb index, we have

$$\begin{aligned} M_1(G \odot H) = &\sum_{u\in V(G\odot H)} [deg_{G\odot H}(u)]^2 = \sum_{u\in V(G)} [deg_G(u) + n_1]^2 \\ &+ \sum_{u\in V(H_1)} [deg_{H_1}(u) + 1]^2 + \cdots + \sum_{u\in V(H_n)} [deg_{H_n}(u) + 1]^2 \\ = &\sum_{u\in V(G)} deg_G^2(u) + 2n_1 \sum_{u\in V(G)} deg_G(u) + n_1^2 \sum_{u\in V(G)} 1 \\ &+ \sum_{i=1}^{n} \sum_{u\in V(H_1)} [deg_{H_i}^2(u) + 2deg_{H_i}(u) + 1] \\ = &M_1(G) + nM_1(H) + 4n_1m + n_1^2 n + 4nm_1 + nn_1. \end{aligned}$$

The line graph of a graph G, denoted by $L(G)$, has $E(G)$ as its vertex set, where two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G have at least one vertex in common. Similarly, we can obtain the second Zagreb index of the corona of graphs G and H in terms of the line graph of G and H.

Theorem 2.4 *Let G and H be two graphs with $|V(G)| = n, |V(H)| = n_1, |E(G)| = m$ and $|E(H)| = m_1$. Then $M_2(G \odot H) = M_2(G) + nM_2(H) + 2n_1E(L(G)) + 4n_1m + n_1^2m + 2nE(L(H)) + 3nm_1 + 4m_1m + 2nm_1n_1 + n_1^2n$.*

Proof. By the definition of the second Zagreb index, we have

$$\begin{aligned}
M_2(G \odot H) = & \sum_{uv\in E(G\odot H)} deg_{G\odot H}(u)deg_{G\odot H}(v) \\
= & \sum_{uv\in E(G)} [deg_G(u) + n_1][deg_G(v) + n_1] \\
& + n \sum_{uv\in E(H_i)} [deg_{H_i}(u) + 1][deg_{H_i}(v) + 1] \\
& + \sum_{v_i\in V(G), u\in V(H_i)} [deg_G(v_i) + n_1][deg_{H_i}(u) + 1] \\
= & M_2(G) + nM_2(H) + n_1 \sum_{uv\in E(G)} [deg_G(u) + deg_G(v)] + n_1^2m \\
& + n\sum_{uv\in E(H)} [deg_H(u) + deg_H(v)] + nm_1 + \sum_{\substack{v\in V(G), u\in V(H)\\ uv\in E(G\odot H)}} deg_G(v)deg_G(u) \\
& + \sum_{v\in V(G), u\in V(H), uv\in E(G\odot H)} [deg_G(v) + n_1deg_H(u)] + n_1^2n.
\end{aligned}$$

The proof of the theorem is also based on the following observation: $deg_{L(G)}(e) + 2 = deg_G(u) + deg_G(v)$. Hence

$$\sum_{uv\in E(G)} [deg_G(u) + deg_G(v)] = \sum_{e\in V(L(G))} [deg_{L(G)}(e) + 2] = 2E(L(G)) + 2m.$$

Similarly, we have

$$\sum_{uv\in E(H)} [deg_H(u) + deg_H(v)] = \sum_{e\in V(L(H))} [deg_{L(H)}(e) + 2] = 2E(L(H)) + 2m_1;$$

$$\begin{aligned}
\sum_{\substack{v\in V(G), u\in V(H)\\ uv\in E(G\odot H)}} deg_G(v)deg_H(u) = & deg_G(v_1)\sum_{i=1}^{n_1} deg_H(u_i) + deg_G(v_2)\sum_{i=1}^{n_1} deg_H(u_i) \\
& + \cdots + deg_G(v_n)\sum_{i=1}^{n_1} deg_H(u_i) = 2m_1\left[\sum_{i=1}^{n} deg_G(v_i)\right] = 4m_1m;
\end{aligned}$$

$$\begin{aligned}
\sum_{\substack{v\in V(G), u\in V(H)\\ uv\in E(G\odot H)}} [deg_G(v) + n_1deg_H(u)] = & n_1\sum_{i=1}^{n} deg_G(v_i) + n_1 \sum_{\substack{v\in V(G), u\in V(H)\\ uv\in E(G\odot H)}} deg_H(u) \\
= & 2mn_1 + 2nm_1n_1.
\end{aligned}$$

Now by easy calculation we obtain $M_2(G \odot H) = M_2(G) + nM_2(H) + 2n_1E(L(G)) + 4n_1m + n_1^2m + 2nE(L(H)) + 3nm_1 + 4m_1m + 2nm_1n_1 + n_1^2n$, and this completes the proof.

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