Decoding of Blockwise-Burst Errors in Row-Cyclic Array Codes*

Sapna Jain
Department of Mathematics
University of Delhi
Delhi 110 007
India
E-mail: sapna@vsnl.com

Abstract. In this paper, we obtain an upper bound on the order of a blockwise-burst [11] that can be detected by a row-cyclic array code [10] and obtain the fraction of blockwise-bursts of order exceeding the upper bound that go undetected. We also give a decoding algorithm for the correction of blockwise-bursts in row-cyclic array codes.

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1. Introduction

Blockwise-bursts in linear array codes equipped with m-metric have already been introduced by the author [11]. In this paper, we extend the study of blockwise-bursts to row-cyclic array codes [10]. We first obtain an upper bound on the order of a blockwise-burst that can be detected by a row-cyclic array code and then obtain the ratio of blockwise-bursts of order exceeding the upper bound that go undetected. Finally, we give a decoding algorithm for the correction of blockwise-bursts in row-cyclic array codes.

2. Definitions and Notations

Let F_q be a finite field of q elements. Let $\text{Mat}_{m\times s}(F_q)$ denote the linear space of all $m \times s$ matrices with entries from F_q. An m-metric array code is a subset of $\text{Mat}_{m\times s}(F_q)$ and a linear m-metric array code is an F_q-linear subspace of $\text{Mat}_{m\times s}(F_q)$. Note that the space $\text{Mat}_{m\times s}(F_q)$ is identifiable with the space F_q^{ms}. Every matrix in $\text{Mat}_{m\times s}(F_q)$ can be represented as a $1 \times ms$ vector by writing the first row of matrix followed by second row

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and so on. Similarly, every vector in F_q^{ms} can be represented as an $m \times s$ matrix in $\mathrm{Mat}_{m\times s}(F_q)$ by separating the co-ordinates of the vector into m groups of s-coordinates. The m-metric on $\mathrm{Mat}_{m\times s}(F_q)$ is defined as follows [13]:

Definition 2.1. Let $Y \in \mathrm{Mat}_{1\times s}(F_q)$ with $Y = (y_1, y_2, \cdots, y_s)$. Define row weight (or ρ-weight) of Y as

$$wt_\rho(Y) = \begin{cases} \max \{ i \mid y_i \neq 0\} & \text{if } Y \neq 0 \\ 0 & \text{if } Y = 0. \end{cases}$$

Extending the definitions of wt_ρ to the class of $m \times s$ matrices as

$$wt_\rho(A) = \sum_{i=1}^{m} wt_\rho(R_i)$$

where $A = \begin{bmatrix} R_1 \\ R_2 \\ \cdots \\ R_m \end{bmatrix} \in \mathrm{Mat}_{m\times s}(F_q)$ and R_i denotes the i^{th} row of A. Then wt_ρ satisfies $0 \leq wt_\rho(A) \leq n(= ms)\ \forall\ A \in \mathrm{Mat}_{m\times s}(F_q)$ and determines a metric on $\mathrm{Mat}_{m\times s}(F_q)$ known as m-metric (or ρ-metric).

Now we define blockwise-burst in linear array codes [11]:

Definition 2.2. A blockwise-burst of order pr(or $p \times r$) $(1 \leq p \leq m \quad 1 \leq r \leq s)$ in the space $\mathrm{Mat}_{m\times s}(F_q)$ is an $m \times s$ matrix A such that all the nonzero entries of matrix A are confined to a $p \times r$ submatrix B of it where each of the p rows of B forms a burst of length r in classical sense [12].

Definition 2.3. A blockwise-burst of order pr or less $(1 \leq p \leq m \quad 1 \leq r \leq s)$ in the space $\mathrm{Mat}_{m\times s}(F_q)$ is a blockwise-burst of order cd (or $c \times d$) where $1 \leq c \leq p \leq m$ and $1 \leq d \leq r \leq s$. We now give the definition of row-cyclic array codes [10].

Definition 2.4. An $[m \times s, k]$ linear array codes $C \subseteq \mathrm{Mat}_{m\times s}(F_q)$ is said to be row-cyclic if

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{ms} \end{pmatrix} \in \mathbf{C}$$

$$\Longrightarrow \begin{pmatrix} a_{1s} & a_{11} & a_{12} & \cdots a_{1,s-1} \\ a_{2s} & a_{21} & a_{22} & \cdots a_{2,s-1} \\ \vdots & \vdots & \vdots & \vdots \\ a_{ms} & a_{m1} & a_{m2} & \cdots a_{m,s-1} \end{pmatrix} \in \mathbf{C}$$

i.e. the array obtained by shifting the columns of a code array cyclically by one position of the right and the last column occupying the first place is also a code array. In fact, a row-cyclic array code C of order $m \times s$ turns out to be $C = \bigoplus_{i=1}^{m} C_i$ where each C_i is a classical cyclic code of length s. Also, every matrix/array in $\text{Mat}_{m\times s}(F_q)$ can be identified with an m-tuple in $A_s^{(m)}$ where $A_s^{(m)}$ is the direct product of algebra A_s taken m times and A_s is the algebra of all polynomials over F_q modulo the polynomial $x^s - 1$ and this identification is given by

$$\theta : \text{Mat}_{m\times s}(F_q) \rightarrow A_s^{(m)}$$

$$\theta(A) = \theta \begin{pmatrix} R_1 \\ \vdots \\ R_m \end{pmatrix} = \begin{pmatrix} \theta' R_1 \\ \theta' R_2 \\ \vdots \\ \theta' R_m \end{pmatrix} = (\theta' R_1, \theta' R_2, \cdots, \theta' R_m) \qquad (1)$$

where $R_i (i = 1 \text{ to } m)$ denotes the i^{th} row of A and $\theta' : F_q^s \longrightarrow A_s$ is given by

$$\theta'(a_0, a_1, \cdots, a_{s-1}) = a_0 + a_1 x + \cdots + a_{s-1} x^{s-1}.$$

An equivalent definition of row-cyclic array code is given by [10]:

Definition 2.5. An $m \times s$ linear array codes $C \subseteq \text{Mat}_{m\times s}(F_q)$ is said to be row-cyclic if

$$C = \bigoplus_{i=1}^{m} C_i$$

where each C_i is an $[s, k_i, d_i]$ classical cyclic code equipped with m-metric. The parameters of row-cyclic array code C are given by $[m\times s, \sum_{i=1}^{m} k_i, \min_{i=1}^{m} d_i]$. If $g_i(x)$ is the generator polynomial of classical cyclic code C_i, then the m-tuple $(g_i(x) \cdots, g_m(x))$ is called the generator m-tuple of row cyclic code C.

3. Detection of Blockwise-Bursts in Row-Cyclic Array Codes

In this section, we first obtain an upper bound on the order of blockwise-bursts that can be detected in a row-cyclic array code and then obtain the ratio of blockwise-bursts (of order exceeding the upper bound) that can go undetected. The upper bound on the order of blockwise-bursts that can be detected in a row-cyclic array codes is obtained in the following theorem:

Theorem 3.1 *Let* $C = \bigoplus_{i=1}^{m} C_i$ *be an* $[m \times s, \sum_{i=1}^{m} k_i, \min_{i=1}^{m} d_i]$ *row-cyclic array code. Then no code array is a blockwise-burst of order* $m \times r$ *or less where* $r = \max_{i=1}^{m}\{s - k_i\}$. *Therefore, every* $[m \times s, \sum_{i=1}^{m} k_i, \min_{i=1}^{m} d_i]$ *row-cyclic array code detects any blockwise-burst of order* $m \times \max_{i=1}^{m}\{s - k_i\}$ *or less.*

Proof. Consider a blockwise-burst A of order $m \times r$ or less where $r = \max_{i=1}^{m}\{s - k_i\}$. We can write A as

$$A = \begin{pmatrix} & 0 & \\ 0 & B & 0 \\ & 0 & \end{pmatrix}$$

where B is a $p \times t$ submatrix of $A (1 \le p \le m, 1 \le t \le r)$ given by

$$B = \begin{pmatrix} b_{l_1} \\ b_{l_2} \\ \vdots \\ b_{l_p} \end{pmatrix} = \begin{pmatrix} b_{l_1}(x) \\ b_{l_2}(x) \\ \vdots \\ b_{l_p}(x) \end{pmatrix} \quad \text{(under the identification } \theta)$$

where each b_{l_j} is a $1 \times t$ row vector having first and last (if $t \ge 2$) component nonzero. Let $(g_1(x), \cdots, g_m(x))$ be the generator m-tuple of row-cyclic array code C. Then $deg(g_i(x)) = s - k_i$ $(i = 1, 2, \cdots, m)$. Now, the blockwise-burst A will be detected if $g_i(x)$ does not divide $b_i(x)$ for some i where

$$i \in \{l_1, l_2, \cdots, l_p\} \quad \text{and } \{l_1, l_2, \cdots, l_p\} \subseteq \{1, 2, \cdots, m\}.$$

Now, from the theory of classical codes, we know that every component code C_i detects every classical burst of length $s - k_i$ or less, or in other words

$g_i(x)$ does not divide $b_i(x)$ where $b_i(x)$ is a burst of length $s - k_i$ or less $(i = 1 \text{ to } m)$. Thus we get the blockwise-burst A of order $m \times \max_{i=1}^{m}\{s - k_i\}$ or less will be detected by the row-cyclic array code. □

Now we obtain the ratio of blockwise bursts of order $m \times r$ where $r > \max_{i=1}^{m}\{s - k_i\}$ that go undetected in a row-cyclic array code.

Theorem 3.2. *Let $N = \{1, 2, \cdots, m\}$. Let $C = \bigoplus_{i=1}^{m} C_i$ be a row-cyclic array code over F_q where each C_i is a $[s, k_i, d_i]$ classical cyclic code equipped with m-metric and having generator polynomial $g_i(x)$. Then the ratio of blockwise-bursts of order $m \times r$ (where $r > \max_{i=1}^{m}\{s - k_i\}$) that go undetected in a row-cyclic array code is given by*

$$\frac{(q)^{|J|(s-r+2)-ms+\sum_{i \in N/J} k_i}}{(q-1)^{|J|}} \tag{2}$$

where J is a subset of N such that

$$i \in J \Rightarrow r - 1 = s - k_i$$

and

$$i \notin J \Rightarrow r - 1 > s - k_i.$$

Proof. Consider a blockwise-burst A of order $m \times r$ where $r > \max_{i=1}^{m}\{s - k_i\}$. We can write A as

$$A = (0 \quad B \quad 0)$$

where B is an $m \times r$ submatrix of A and is given by

$$B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} = \begin{pmatrix} b_1(x) \\ b_2(x) \\ \vdots \\ b_m(x) \end{pmatrix} \quad \text{(under the identification } \theta\text{)},$$

where each b_i is a classical burst of length r. We observe that the blockwise-burst A will go undetected if $g_i(x)|b_i(x) \forall\, i = 1 \text{ to } m$. Without any loss of generality, we may assume that $\gcd(x^j, b_i(x)) = 1\ \forall\ j$ and $\deg b_i(x) = r - 1 \forall i = 1 \text{ to } m$. Now $g_i(x)|b_i(x)$ iff

$$b_i(x) = g_i(x)q_i(x) \qquad \text{for some } q_i(x).$$

Since deg $b_i(x) = r - 1$ and deg $g_i(x) = s - k_i$ implies that deg $q_i(x) = (r-1) - (s-k_i)\ \forall\ i \in N$.

There are two cases to consider:

Case 1. When $i \in J$.

In this case, $r - 1 = s - k_i$. Therefore deg $q_i(x) = (r-1) - (s-k_i) = 0$ which implies that the possibilities for $q_i(x) = q - 1$. Thus possibilities for $q_i(x)$ for all $i \in J$ taken together is equal $to(q-1)^{|J|}$.

Case 2. When $i \notin J$.

In this case $r - 1 > s - k_i$. Therefore deg $g_i(x) > 0$. The number of possibilities for $q_i(x) = (q-1)^2 q^{(r-1)-(s-k_i)-1}$.

Thus, total number of possibilities for $q_i(x)$ for all $i \notin J$ taken together is given by

$$(q-1)^{2(m-|J|)} \times (q)^{(m-|J|)(r-2-s)+\sum\limits_{i\in N/J} k_i}.$$

Combining the two cases, we get the number of blockwise-bursts of order $m \times r$ where $r > \max\limits_{i=1}^{m}\{s - k_i\}$ (with a fixed starting position) that go undetected is given by

$$(q-1)^{|J|}(q-1)^{2(m-|J|)}(q)^{(m-|J|)(r-2-s)+\sum\limits_{i\in N/J} k_i}.$$

Also, total number of blockwise-bursts of order $m \times r$ where $r > \max\limits_{i=1}^{m}\{s-k_i\}$ (with a fixed starting position) that is given by

$$(q-1)^{2m}(q)^{m(r-2)}.$$

Therefore, the ratio of blockwise-bursts of order $m \times r (r > \max\limits_{i=1}^{m}\{s - k_i\})$ that go undetected is given by

$$\frac{(q-1)^{|J|}(q-1)^{2(m-|J|)}(q)^{(m-|J|)(r-2-s)+\sum\limits_{i\in N/J} k_i}}{(q-1)^{2m}q^{m(r-2)}}$$

$$= \frac{(q)^{|J|(s-r+2)-ms+\sum\limits_{i\in N/J} k_i}}{(q-1)^{|J|}}.$$

□

Example 3.1. Let C be the binary $[2 \times 3, 3]$ row-cyclic array code of order 2×3 generated by $(g_1(x), g_2(x)) = (1 + x + x^2, 1 + x)$. Then $C = C_1 \oplus C_2$ where C_1 and C_2 are classical cyclic codes of length 3 generated by $1+x+x^2$ and $1 + x$ respectively.

Here $k_1 = 1, \quad k_2 = 2$ and $s = 3$.

Therefore, $s - k_1 = 2$, $s - k_2 = 1$ and $\max_{i=1}^{2}\{s - k_i\} = 2$. We consider blockwise-bursts of order 2×3, i.e. let $r = 3$. Then $r > \max_{i=1}^{2}\{s - k_i\}$.

Also $r - 1 = 2 = s - k_1$ and $r - 1 = 2 > s - k_2$.

$\Rightarrow \quad J = \{1\} \subseteq \{1, 2\} = N$ and $N/J = \{2\}$.

The ratio computed in (2) for the code considered in this example becomes

$$2^{2+2-6} = 2^{-2} = 1/4.$$

The ratio is justified by the fact that there are 4 blockwise-bursts of order 2×3 in $\mathrm{Mat}_{2\times3}$ over F_2 given by

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

and out of these 4 blockwise-bursts, only one blockwise-burst viz. $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ is undetected by the row-cyclic array code C.

4. Decoding Algorithm for Blockwise-Burst Error Correction

Let $C = \bigoplus_{i=1}^{m} C_i$ be a q-ary $[m \times s, \sum_{i=1}^{m} k_i, \min_{i=1}^{m} d_i]$ row-cyclic array code having generator m-tuple of polynomials $(g_1(x), g_2(x), \cdots, g_m(x))$ and correcting all blockwise-burst errors of order $mr (1 \leq r \leq s)$. Let $w(x) = (w_1(x), w_2(x), \cdots, w_m(x))$ be a received array with an error pattern $(e_1(x), e_2(x), \cdots, e_m(x))$ that is a blockwise-burst of order $mr (1 \leq r \leq s)$. The goal is to determine $(e_1(x), e_2(x), \cdots, e_m(x))$.

Algorithm.

Step 1. Compute the syndrome m-tuple $(S_j^{(1)}(x), S_j^{(2)}(x), \cdots,$ $\cdots, S_j^{(m)}(x))$ for $j = 0, 1, 2, \cdots$ where for all $i = i$ *to* m, $S_j^{(i)}(x)$ is given by

$$S_j^{(i)}(x) = \text{syndrome of } x^j w_i(x).$$

Step 2. Find the nonnegative integer l such that syndrome for $x^l w_i(x)(1 \leq i \leq m)$ is a classical burst of length r.

Step 3. Compute the remainder m-tuple $e = (e_1(x), \cdots, e_m(x))$ where for all $i = i$ *to* $m, e_i(x)$ is given by

$$e_i(x) = x^{s-l} S_l^{(i)}(x) (\text{mod } (x^s - 1)).$$

Step 4. Decode $(w_1(x), \cdots, w_m(x))$ to $(w_1(x) - e_1(x), \cdots, w_m(x) - e_m(x))$.

Proof of Algorithm. First of all, we show the existence of nonnegative integer l is step 2. By the assumption, there exists an error pattern $(e_1(x), \cdots, e_m(x))$ such that each $e_i(x)(1 \leq i \leq m)$ has a cyclic run of zeros of length $s - r$ starting from the same position. (A cyclic run of zeros of length p in an s-tuple is a succession of p cyclically consecutive zero components). Thus there exists a nonnegative integer l such that shifting the error pattern $e = (e_1(x), \cdots, e_m(x))$ cyclically through l columns will bring all the nonzero components to the first r columns of e. The cyclic shift of error $e_i(x)(1 \leq i \leq m)$ through l positions is in fact the remainder of $x^l w_i(x)(\text{mod } (x^s - 1))$ divided by $g_i(x)$.

Also, for all $i = 1$ *to* m

$$\begin{aligned} S_l^{(i)}(x) &= (x^l w_i(x)(\text{mod } (x^s - 1))(\text{mod } g_i(x)) \\ &= (x^l w_i(x)(\text{mod } g_i(x)). \end{aligned}$$

Therefore, each $S_l^{(i)}(x)(1 \leq i \leq m)$ is a classical burst of length r. Now, for all $i = 1$ *to* m, the word

$$t_i(x) = (x^{s-l} S_l^{(i)}(x)(\text{mod } (x^s - 1)))$$

is a cyclic shift of $(S_l^{(i)}, 0)$ through $s - l$ positions, where $S_l^{(i)}$ is a vector in $F_q^{s-k_i}$ corresponding to the polynomial $S_l^{(i)}$. It is clear that each $t_i(x)$ is a classical burst of order r. Also, for all $i = 1$ *to* m, we have

$$\begin{aligned} x^l(w_i(x) - t_i(x)) &= x^l(w_i(x) - x^{s-l}S_l^{(i)}(x)) \\ &= x^l w_i(x) - x^s S_l^{(i)}(x) \\ &= S_l^{(i)}(x) - x^s S_l^{(i)}(x) \\ &= (1 - x^s)S_l^{(i)}(x) \\ &\equiv 0(\text{mod } (g_i(x))). \end{aligned} \quad (3)$$

Now, from equation (3), since $g_i(x)$ and x^l are coprime to each other, we get

$$g_i(x)|(w_i(x) - t_i(x)) \quad \forall i = 1, 2, \cdots, m$$

$$\Rightarrow w_i(x) - t_i(x) \in C_i \quad i = 1 \text{ to } m.$$

Also $w_i(x) - e_i(x) \in C_i \quad \forall\ i = 1$ *to* m implies $e_i(x)$ and $t_i(x)$ belong to the same coset $(\text{mod} g_i(x))$. Since both $e_i(x)$ and $t_i(x)$ are classical burst of length r and each C_i is r-burst error correcting classical cyclic code (since $C = \bigoplus_{i=1}^{m} C_i$ corrects all blockwise bursts of order $m \times r$), we get

$$e_i(x) = t_i(x) = (x^{s-l}S_l^{(i)}(x)(\text{mod } (x^s - 1)))$$

□

Example 4.1. Consider the binary row-cyclic array code $C = \oplus_{i=1}^{2} C_i$ where C_1 and C_2 are [7,4,4] classical cyclic codes in F_2 equipped with m-metric and generated by $g_1(x) = 1 + x^2 + x^3$ and $g_2(x) = 1 + x + x^3$ respectively. Then parameters of row cyclic code C are $[2 \times 7, 4+4, 4]$. The row-cyclic array code C corrects all blockwise-bursts of order 2×2 (taken cyclically) as seen from the following table which shows that syndrome 2-tuple of all blockwise-bursts of order 2×2 (taken cyclically) are all distinct.

Table 4.1

Blockwise-bursts of order 2×2 in $\text{Mat}_{2\times 7}(F_2)$	Syndrome 2-tuple
$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$	(110, 110)
$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$	(011, 011)
$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$	(100, 111)
$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$	(010, 101)
$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$	(001, 100)
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$	(101, 010)
$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$	(111, 001)

The syndrome 2-tuple $S = (S_1, S_2)$ for a blockwise-burst $b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ of order 2×2 for the code C have been found by using the relation $S = bH^T$ where H is the parity check matrix of the code C and is given by

$$H = \begin{pmatrix} H_1 & 0 \\ 0 & H_2 \end{pmatrix},$$

where

$$H_1 = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

and

$$H_2 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

Now, consider the received array

$$w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \in \text{Mat}_{2\times 7}(F_2).$$

Under the identification $\theta : \mathrm{Mat}_{m\times s}(F_2) \longleftrightarrow A_s^{(m)}$, w can be identified as

$$w = \begin{pmatrix} 1 + x^2 + x^3 + x^4 \\ x^2 \end{pmatrix} = \begin{pmatrix} w_1(x) \\ w_2(x) \end{pmatrix}.$$

Compute the syndrome $S_j^{(i)}(x)$ of $x^j w_i(x)(1 \le i \le 2)$ until $S_j^{(i)}$ is a classical burst of length 2.

Table 4.2

j	$S_j^{(1)}(x)$	$S_j^{(2)}(x)$
0	$1+x+x^2$	x^2
1	$1+x$	$1+x$

Therefore, $l = 1$.

Decode $w_1(x) = (1011100)$ to $w_1(x) - t_1(x)$ where

$$\begin{aligned} t_1(x) = e_1(x) &= x^{s-l} S_l^{(1)}(x) (\mathrm{mod}\ (x^s - 1)) \\ &= x^{7-1} S_1^{(1)}(x) (\mathrm{mod}\ (x^7 - 1)) \\ &= x^6(1+x)(\mathrm{mod}\ (x^7 - 1)) \\ &= x^6 + x^7 (\mathrm{mod}\ (x^7 - 1)) \\ &= x^6 + 1 \end{aligned}$$

Thus $w_1(x)$ is decoded to

$$w_1(x) - t_1(x) = 1 + x^2 + x^3 + x^4 - x^6 - 1 = x^2 + x^3 + x^4 + x^6 = 0011101$$

Similarly decode $w_2(x) = 0010000 = x^2$ to $w_2(x) - t_2(x)$ where

$$\begin{aligned} t_2(x) = e_2(x) &= x^{s-l} S_l^{(2)}(x) (\mathrm{mod}\ (x^s - 1)) \\ &= x^6 S_1^{(2)}(x) (\mathrm{mod}\ (x^7 - 1)) \\ &= x^6(1+x)(\mathrm{mod}\ (x^7 - 1)) \\ &= x^6 + 1. \end{aligned}$$

Therfore, $w_2(x)$ is decoded to

$$w_2(x) - t_2(x) = x^2 - x^6 - 1 = 1 + x^2 + x^6 = 1010001.$$

Hence

$$w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

is decoded to $\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$.

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