Extraconnectivity of Folded Hypercubes*

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Abstract. Given a graph G and a non-negative integer g, the g-extraconnectivity of G (written $\kappa_g(G)$)is the minimum cardinality of a set of vertices of G, if any, whose deletion disconnects G, and every remaining component has more than g vertices. The usual connectivity and restricted vertex connectivity of G correspond to $\kappa_0(G)$ and $\kappa_1(G)$, respectively. In this paper, we determine $\kappa_g(FQ_n)$ for $0 \leq g \leq n-4$, $n \geq 8$, where FQ_n denotes the n-dimensional folded hypercube.

Key words: Interconnection network; Folded Hypercubes; Fault tolerant; Conditional connectivity

1 Introduction

The traditional connectivity(denote by κ the connectivity), is an important measure for the networks, which can correctly reflects the fault tolerance of systems with few processor, but it always underestimates the the resilience of large networks. The discrepancy incurred is because events whose occurrence would disrupt a large network after a few processor/link failures are highly unlikely, therefore, the disruption envisaged occurs in a worst case scenario. With the development of multiprocessor systems, improving the traditional connectivity is necessary. Motivated by the shortcomings of the traditional connectivity, Harary [5] introduced the concept of conditional connectivity. Here, we consider the extraconnectivity which was defined by Fàbrega and Fiol [3]. The extraconnectivity corresponds to a kind of conditional connectivity introduced by Harary [5].

Let G be a connected undirected graph, and $\mathcal{P}$ a graph-theoretic property. Harary [5] defined the conditional connectivity $\kappa(G;\mathcal{P})$ as the minimum cardinality of a set of vertices, if any, whose deletion disconnects G and every remaining component has property $\mathcal{P}$. Let g be a non-negative integer and $\mathcal{P}_g$ be the property of having more than g vertices. Fàbrega and Fiol [3] defined the g-

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extraconnectivity $\kappa_g(G)$ of G as $\kappa(G;\mathcal{P}_g)$. However, we have not yet known if the problem determining $\kappa_g(G)(g \geq 1)$ is NP-hard for any graph as there is no known polynomial-time algorithm to find $\kappa_g(G)$ for any graph even if $g = 1$.

An n-dimensional hypercube is an undirected graph $Q_n = (V, E)$ with $|V| = 2^n$ and $|E| = n2^{n-1}$. Each vertex can be represented by an n-bit binary string. There is an edge between two vertices whenever their binary string representation differs in only one bit position. Following Latifi [6], we express Q_n as $D_0 \bigodot D_1$, where D_0 and D_1 are the two $(n-1)$-dimensional subcube (called $(n-1)$-subcube) of Q_n induced by the vertices with the ith coordinate 0 and 1 respectively. Sometimes we use $X^{i-1}0X^{n-i}$ and $X^{i-1}1X^{n-i}$ to denote D_0 and D_1, where $X \in Z_2$. Clearly, the vertex v in one $(n-1)$-subcube has exactly one neighbor v' in the other $(n-1)$-subcube, we call v' the *out neighbor* of v. Let $u = x_1x_2\cdots x_i\cdots x_n \in V(Q_n)$, we use $u_{i_1i_2\cdots i_s}$ to denote a vertex whose $i_1th, i_2th, \cdots, i_sth$ coordinates are different from u's, and $\bar{u}$ denotes a vertex such that all coordinates of $\bar{u}$ are different from u's. We use P_n to denote a path with n vertices, $d_G(u, v)$ to denote the distance of u, v, $diam(G) = max\{d_G(u, v)|u, v \in G\}$ to denote the diameter of G. It well known that $d_{Q_n}(u, \bar{u}) = n$, see [8].

Folded hypercube FQ_n is superior to Q_n in some properties, see [2, 7]. Thus, the folded hypercube FQ_n is an enhancement on the hypercube Q_n. FQ_n is obtained by adding a prefect matching M on the hypercube where $M = \{(u, \bar{u})|\ u \in V(Q_n)\}$. In addition, denote by M_i the edgeset $\{(x_1x_2\cdots x_{i-1}x_ix_{i+1}\cdots x_n, x_1x_2\cdots x_{i-1}\bar{x}_ix_{i+1}\cdots x_n)|\bar{x}_i$ represents the complement of $x_i\}$. Fu in [4] showed that $E(Q_n) = \cup_{i=1}^{n} M_i$ and $E(FQ_n) = E(Q_n) \cup M$.

Similarly, we express FQ_n as $D_0 \bigotimes D_1$, where D_0 and D_1 are the two $(n-1)$-subcubes of Q_n induced by the vertices with the ith coordinate 0 and 1 respectively. It was known that $\kappa_0(FQ_n) = n + 1$, $\kappa_1(FQ_n) = 2n$ for $n \geq 4$, and $\kappa_2(FQ_n) = 3n - 2$ for $n \geq 8$, see [10, 11, 12]. In this paper, we show that $\kappa_g(FQ_n) = (n + 1)(g + 1) - 2g - \binom{g}{2}$ for $n \geq 8$, $0 \leq g \leq n - 4$.

Let $A \subseteq G$, $v \in V(G)$. We use $N_G(v)$ to denote the set of the neighbors of v in G, $N_G(A)$ to denote the set $(\bigcup_{v \in V(A)} N_G(v)) \setminus V(A)$, $C_G(A)$ to denote the set $N_G(A) \cup V(A)$. We follow Bondy [1] for terminologies not given here.

In [9], we verified the following result.

Theorem 1.1. *If $n \geq 4$, then $\kappa_g(Q_n) = (g + 1)n - 2g - \binom{g}{2}$ for $0 \leq g \leq n - 4$.*

In this paper, we shall verify the similar result of folded hypercubes as follows.

Theorem 1.2. *If $n \geq 8$, then $\kappa_g(FQ_n) = (g + 1)(n + 1) - 2g - \binom{g}{2}$ for $0 \leq g \leq n - 4$.*

2 Preliminaries

Before discussing the $\kappa_g(FQ_n)$, we review some results of hypercubes which can be found in [9].

Lemma 2.1. Assume $n \geq 4$ and $A \subseteq Q_n$. If $|V(A)| = g+1$, then $|N_{Q_n}(A)| \geq (g+1)n - 2g - \binom{g}{2}$.

Remark 2.2. Note that $h_n(g) = (g+1)n - 2g - \binom{g}{2}$ is increasing when $g \leq n-2$, the maximum of $h_n(g)$ is $h_n(n-2) = (n-1)n - 2(n-2) - \binom{n-2}{2} = h_n(n-1) = n^2 - 2(n-1) - \binom{n-1}{2} = \frac{n(n-1)}{2} + 1$ and $h_n(n-1) = h_n(n-2) > h_n(n) > (g+1)n - 2g - \binom{g}{2}$ for $0 \leq g \leq n-4$. In particular, $h_{n-1}(g_1) + h_{n-1}(g_2) > h_n(g) + 1$ when $0 \leq g_1, g_2 \leq n-1$ and $g_1 + 1 + g_2 + 1 > g + 1$.

Lemma 2.3. Assume $n \geq 4$, $B \subseteq Q_n$ and $|V(B)| \geq n$. If $|V(Q_n) \setminus C_{Q_n}(B)| \geq n$, then $|N_{Q_n}(B)| \geq \frac{n(n-1)}{2}$.

Lemma 2.4[12]. Any two vertices in $V(Q_n)$ have exactly two common neighbors for $n \geq 3$ if they have any.

Lemma 2.5. Let $0 \leq g \leq n$ $(n \geq 3)$, $A \subseteq Q_n$ and $A \cong K_{1,g}$. Then $|N_{Q_n}(A)| = (g+1)n - 2g - \binom{g}{2}$ and $Q_n - C_{Q_n}(A)$ is a connected subgraph of Q_n with proper $\mathcal{P}_g$.

Lemma 2.6. Assume $A \subseteq Q_n$, $|V(A)| = g+1$ and $0 \leq g \leq n-4 (n \geq 4)$, then $Q_n - C_{Q_n}(A)$ is a connected subgraphs of Q_n with property $\mathcal{P}_g$.

Similarly, we give some useful results of folded hypercubes in the following arguments.

Lemma 2.7. Let $FQ_n = D_0 \bigotimes D_1$ and A be a connected subgraph of D_1. If $diam_{Q_n}(A) < n-1$, then $|N_{D_0}(A)| = 2|A|$.

Proof. Let $A \subset D_1$ and $u, v \in V(A)$. Clearly, if u, v have common neighbors in D_0, then $v = \bar{u}'$. For any $u \in V(A)$, since $diam_{Q_n}(A) < n-1$ and A is connected, $\bar{u}'$ is not in $V(A)$. That is, any two distinct vertices of $V(A)$ have no common neighbors in D_0. Therefore, $|N_{D_0}(A)| = 2|V(A)|$. □

Lemma 2.8. Let $FQ_n = D_0 \bigotimes D_1$, $0 \leq g \leq n-4 (n \geq 8)$, $A \subseteq D_1$, and $A \cong K_{1,g}$, then $|N_{FQ_n}(A)| = (g+1)(n+1) - 2g - \binom{g}{2}$ and $FQ_n - C_{FQ_n}(A)$ is connected with property $\mathcal{P}_g$.

Proof. By Lemmas 2.5 and 2.7, we have $|N_{FQ_n}(A)| = |N_{D_0}(A)| + |N_{D_1}(A)| = 2|V(A)| + (g+1)(n-1) - 2g - \binom{g}{2} = (g+1)(n+1) - 2g - \binom{g}{2}$.

By Lemmas 2.5 and 2.6, we have that $D_0 - N_{D_0}(A)$ and $D_1 - C_{D_1}(A)$ are connected. Note that $|V(D_1) - C_{D_1}(A)| = 2^{n-1} - ((g+1)(n-1) - 2g - \binom{g}{2}) - (g+1) > 2(g+1) = |N_{D_0}(A)|$ for $n \geq 8$, we have that $D_0 - N_{D_0}(A)$ connects to $D_1 - C_{D_1}(A)$. Since $2^n - |C_{FQ_n}(A)| = 2^n - ((g+1)(n+1) - 2g - \binom{g}{2}) > g+1$ for $n \geq 8$, we have that $|V(FQ_n - C_{FQ_n}(A))| > g+1$ for $n \geq 8$. Thus $FQ_n - C_{FQ_n}(A)$ is connected with property $\mathcal{P}_g$. □

Lemma 2.9. Let A be a connected subgraph of Q_n with $|V(A)| \leq n-1$. Then there exists a decomposition $Q_n = D_0 \bigodot D_1$ such that $V(A) \subset V(D_1)$ or $V(D_0)$.

Proof. Let T_A be a spanning tree of A. Clearly, $|E(T_A)| \leq n-2$. Thus there exists an integer i such that $M_i \cap E(T_A) = \emptyset$. Let $D_0 = X^{i-1}0X^{n-i}, D_1 = X^{i-1}1X^{n-i}$. Then $V(A) \subset V(D_1)$ or $V(D_0)$. □

Lemma 2.10. Let A be a connected subgraph of FQ_n with $|V(A)| \leq n-1$ and T_A be a spanning tree of A. Then there exists an integer i such that $T_A \subset FQ_n - M_i$.

Proof. Clearly, $|E(T_A)| \leq n-2$. Thus there exists an integer i such that $M_i \cap E(T_A) = \emptyset$. The result is clearly true. □

Lemma 2.11[4]. $FQ_n - M_i$ is isomorphic to Q_n for all i.

Lemma 2.12. Let $FQ_n = Q_n + M$ and A be a connected subgraph of Q_n. If $diam_{Q_n}(A) \leq n-2$, then $|N_{FQ_n}(A)| = |V(A)| + |N_{Q_n}(A)|$.

Proof. For any $v \in V(A)$. Note that $diam_{Q_n}(v, \bar{v}) = n$ and $A \subset Q_n, diam_{Q_n}(A)$ $n-2$, we have $\bar{v} \notin C_{Q_n}(A)$. We thus have $|N_{FQ_n}(A)| = |N_{Q_n}(A) \cup N_M(A)| = |V(A)| + |N_{Q_n}(A)|$. □

Corollary 2.13. Let A be a connected subgraph of FQ_n with $|V(A)| = g+1 \leq n-1$. Then $|N_{FQ_n}(A)| \geq (g+1)(n+1) - 2g - \binom{g}{2}$.

Proof. This result follows immediately from Lemmas 2.10, 2.11, 2.12 and Lemma 2.1. □

lemma 2.14. Let $FQ_n = D_0 \bigotimes D_1$, $n \geq 8$, A be a connected subgraph of D_1. If $|V(A)| \geq n-1$, then $|N_{FQ_n}(A)| > \frac{n(n+1)}{2} - 2$.

Proof. Let T_A be a connected subgraph of A and $|V(T_A)| = n-1$. If $|V(D_1) \backslash C_{D_1}(A)| \geq n-1$, by Lemma 2.3 and 2.7, we have $|N_{D_1}(A)| \geq \frac{(n-1)(n-2)}{2}$ and $|N_{D_0}(T_A)| = 2(n-1)$. Thus $|N_{FQ_n}(A)| = |N_{D_1}(A)| + |N_{D_0}(A)| \geq |N_{D_1}(A)| + |N_{D_0}(T_A)| \geq \frac{(n-1)(n-2)}{2} + 2(n-1) = \frac{n(n+1)}{2} - 1$. If $|V(D_1) \setminus C_{D_1}(A)| < n-1$, then $|C_{D_1}(A)| > 2^{n-1} - (n-1) > \frac{n(n+1)}{2} - 2$ for $n \geq 8$. For any $x \in C_{D_1}(A)$, at last one of $\{x, x\prime\}$ in $N_{FQ_n}(A)$, then $|N_{FQ_n}(A)| > \frac{n(n+1)}{2} - 2$. □

3 Main result

In the following, we shall determine $\kappa_g(FQ_n)$.

Theorem 3.1 If $n \geq 8$, then $\kappa_g(FQ_n) = (g+1)(n+1) - 2g - \binom{g}{2}$ for $0 \leq g \leq n-4$.

Proof. We first show that $\kappa_g(FQ_n) \leq (g+1)(n+1) - 2g - \binom{g}{2}$. Let $FQ_n = D_0 \bigotimes D_1$, $u \in V(D_1)$, $u_i \in N_{D_1}(u), 0 \leq i \leq g$, $A = FQ_n[u, u_1, \cdots, u_g]$ and $F = N_{FQ_n}(A)$. By Lemma 2.8, we have $\kappa_g(FQ_n) \leq (g+1)(n+1) - 2g - \binom{g}{2}$.

Next we verify that $\kappa_g(FQ_n) \geq (g+1)(n+1) - 2g - \binom{g}{2}$. By contradiction. Suppose that F is a vertex cutset such that every component of $FQ_n - F$ has property $\mathcal{P}_g$ and $|F| \leq (g+1)(n+1) - 2g - \binom{g}{2} - 1$. Assume that A is the smallest component of $FQ_n - F$.

If $|V(A)| \leq n-1$, the theorem follows immediately from Corollary 2.13. Next we assume that $|V(A)| \geq n$.

We can decompose $FQ_n = D_0 \bigotimes D_1$ such that $|V(A) \cap V(D_1)| \geq n-1$ (or $|V(A) \cap V(D_0)| \geq n-1$) since Lemma 2.9, 2.10, 2.11. Let $F_0 = F \cap V(D_0)$ and $F_1 = F \cap V(D_1)$, then we have either $|F_0| \leq \frac{(g+1)(n+1)-2g-\binom{g}{2}-1}{2}$ or $|F_1| \leq \frac{(g+1)(n+1)-2g-\binom{g}{2}-1}{2}$.

Case 1. $|F_0| \leq \frac{(g+1)(n+1)-2g-\binom{g}{2}-1}{2}$.

Assume that $G_1, G_2, \cdots, G_s$ are all components of $D_0 - F_0$ such that $|V(G_i)| < \frac{n}{2}$ and G^* denotes $D_0 - F_0 \cup V(G_1 \cup \cdots \cup G_s)$. We obtain two claims as follows.

Claim 1. $\sum_{i=1}^{s} |V(G_i)| < \frac{n}{2}$

Clearly, $F_0 \leq \frac{(g+1)(n+1)-2g-\binom{g}{2}-1}{2} < \frac{n}{2}(n-1) - 2(\frac{n}{2}-1) - \binom{\frac{n}{2}-1}{2}$ for $n \geq 8$. By Lemma 2.1, we have that $D_0 - F_0$ has no component C such that $\frac{n}{2} \leq |V(C)| \leq n$.

If $\frac{n}{2} \leq \sum_{i=1}^{s} |V(G_i)| \leq n$, by Lemma 2.1, we have $|N_{D_0}(G_1 \cup \cdots \cup G_s)| > |F_0|$, a contradiction.

If $\sum_{i=1}^{s} |V(G_i)| > n$, we can find a subgraph S consisting of G_i such that $\frac{n}{2} \leq |V(S)| \leq n$. By Lemma 2.1, we have $|N_{D_0}(S)| > |F_0|$, a contradiction. Thus $\sum_{i=1}^{s} |V(G_i)| < \frac{n}{2}$.

Claim 2. G^* is connected.

Since $|V(D_0) \setminus (F_0 \cup (V(G_1 \cup \cdots \cup G_s))| > 2^{n-1} - |F_0| - \frac{n}{2} > 0$ for $n \geq 8$, thus G^* is not an empty graph.

Suppose that G^* is disconnected, then every component of G^* has order at least n. By Lemma 2.3, we have $|F_0 \cup (V(G_1 \cup \cdots \cup G_s))| \geq \frac{(n-1)(n-2)}{2}$. However, $|F_0 \cup (V(G_1 \cup \cdots \cup G_s))| < |F_0| + \frac{n}{2} \leq \frac{(g+1)(n+1)-2g-\binom{g}{2}-1}{2} + \frac{n}{2} < \frac{(n-1)(n-2)}{2}$ for $n \geq 8$, a contradiction. Thus G^* is connected.

Let $\sum_{i=1}^{s} |V(G_i)| = N$ and $C_1, C_2, \cdots, C_m$ are all components of $D_1 - F_1$ such that $|V(C_i)| < n-1, 1 \leq i \leq m$.

Clearly, if $N = 0$, then $G^* = D_0 - F_0$ is connected, that is, $A \subset D_1$. By Lemma 2.14, we have $|F| \geq \frac{n(n+1)}{2} - 2 > (g+1)(n+1) - 2g - \binom{g}{2}$ for $0 \leq g \leq n-4$, a contradiction.

Next we derive the contradictions when $N \geq 1$ by considering two cases.

Subcase 1.1. $\sum_{i=1}^{m} |V(C_i)| \geq n-1$.

We claim that $|F_1| \geq \frac{(n-1)(n-2)}{2}$. In fact, if $|V(D_1)| - \sum_{i=1}^{m} |V(C_i)| - |F_1| \geq n-1$, by Lemma 2.3, we have the result immediately; if not, and we suppose $|F_1| < \frac{(n-1)(n-2)}{2}$, then $\sum_{i=1}^{m} |V(C_i)| = |V(D_1)| - |F_1| - (|V(D_1)| - \sum_{i=1}^{m} |V(C_i)| - |F_1|) > 2^{n-1} - |F_1| - (n-1) > 4n$ for $n \geq 8$. Note that $|V(C_i)| < n-1$, there exists an integer j such that $\sum_{i=1}^{j}|V(C_i)| \geq n-1$ and $\sum_{i=j+1}^{m}|V(C_i)| \geq n-1$. By Lemma 2.3, we have $|N_{D_1}(\cup_{i=1}^{j}(C_i))| \geq \frac{(n-1)(n-2)}{2}$, that is $|F_1| \geq \frac{(n-1)(n-2)}{2}$.

It is not difficult to see that $|F| = |F_0|+|F_1| \geq N(n-1)-2(N-1)-\binom{N-1}{2}+\frac{(n-1)(n-2)}{2} \geq \frac{n(n+1)}{2}-3 > (g+1)(n+1)-2g-\binom{g}{2}-1$ for $N \geq 2, g \leq n-4$, a contradiction.

Next we verify the results when $N=1$.

If $N=1$, then $D_0 - F_0$ has a isolated vertex v. Assume that $\bar{G}_1, \cdots, \bar{G}_t$ are all the components of $D_1 - F_1$ such that $|V(\bar{G}_i)| \geq n-1$. By Lemma 2.14, we have that $\bar{G}_i$ connects to G^*(otherwise $|N_{FQ_n}(\bar{G}_i) \setminus v| > |F|$). Clearly, v disconnects to $\bar{G}_i$(otherwise, v connects to G^*, that is, $A \subset D_1 - F_1$. But $|V(A)| \geq n$, By lemma 2.14, we have that $|N_{FQ_n}(A)| > |F|$, a contradiction). Since $|V(A)| \geq n$ and $|V(C_i)| < n-1$, we have that v has two neighbors in $D_1 - F_1$, that is $A = FQ_n[\{v\} \cup V(C_k) \cup V(C_l)]$ where C_k, C_l are two components of $D_1 - F_1$ such that $v\prime \in V(C_k), \bar{v} \in V(C_l)$. Clearly, C_k and C_l disconnects to G^*(in fact, $D_1 - F_1$ has exactly two components with order less than $n-1$ and disconnect to G^*, that is, $\{k,l\} = \{1,2\}$).

If $C_{D_1}(C_1) \cap C_{D_1}(C_2) = \emptyset$. Assume $|V(C_1)| = N_1$, $|V(C_2)| = N_2$. Clearly, $N_1 + N_2 \geq n-1$. By Remark 2.2, we have $|N_{D_1}(C_1)| + |N_{D_1}(C_2)| > n(n-2) - 2(n-3) - \binom{n-3}{2}$. Thus $|F| \geq |N_{D_0}(v)| + |N_{D_1}(C_1)| + |N_{D_1}(C_2)| > n-1+n(n-2)-2(n-3)-\binom{n-3}{2} = (n-2)(n+1)-2(n-3)-\binom{n-3}{2}+1 > (g+1)(n+1)-2g-\binom{g}{2}-1$ for $n \geq 8$, a contradiction.

If $C_{D_1}(C_1) \cup C_{D_1}(C_2) \neq \emptyset$. Let $x \in C_{D_1}(C_1) \cap C_{D_1}(C_2)$ and $S = D_1[V(C_1) \cup V(C_2) \cup x]$(induced connected subgraph). Let P be the shortest $(v\prime, \bar{v})$-path of S. Clearly, $|V(P)| \geq n$. By lemma 2.7, we have $|N_{D_0}(P - v\prime)| = 2|V(P - v\prime)| \geq 2(n-1)$, that is, $|N_{D_0}(P - v\prime - x)| = 2|V(P - v\prime)| - 2$. We have that $|F \cup \{v\}| \geq |F_1| + |N_{D_0}(P - v\prime - x)| \geq \frac{(n-1)(n-2)}{2} + 2(n-1) - 2 \geq \frac{n(n+1)}{2} - 3$, but $|F| \leq \frac{n(n+1)}{2} - 6$, a contradiction.

Subcase 1.2. $|\sum_{i=1}^{m} V(C_i)| < n-1$.

We claim that $A \subset (\cup G_i) \cup (\cup C_i)$.

We first assume that $\bar{G}^* = D_1 - F_1 \cup V(C_1 \cup \cdots \cup C_m)$ is connected. We shall show that $\bar{G}^*$ connects to G^*. In fact, it is sufficient to show that $|N_{D_0}(\bar{G}^*)|(\geq |D_1 - F_1 \cup V(\cup C_i)|) > |F_0 \cup V(\cup G_i)|$. Since $|D_1| - |F_1| - |F_0| - |V(\cup C_i)| - |F_0 \cup V(\cup G_i)| > 2^{n-1} - |F| - (n-1) - \frac{n}{2} \geq 2^{n-1} - \frac{n(n-1)}{2} + 6 - (n-1) - \frac{n}{2} > 0$ for $n \geq 8$, we have that $\bar{G}^*$ connects to G^*. A similar count as above clearly implies that $|V(\bar{G}^* \cup G^*)| > \frac{|V(FQ_n)|}{2}$, thus $A \subset (\cup G_i) \cup (\cup C_i)$ since A is the smallest component of $FQ_n - F$.

If $\bar{G}^*$ is disconnected. Clearly, each component of $\bar{G}^*$ has order at least $n-1$, then we have $|F_1| \geq \frac{(n-1)(n-2)}{2}$ by Lemma 2.3. Assume that $\bar{G}_1^*, \cdots, \bar{G}_{t_1}^*$ are all the components of $D_1 - F_1$ such that $|V(\bar{G}_i^*)| \geq n-1$. By the argument of Subcase 2.1, we have that $N = |\cup(V(G_i))| = 1$ and $\bar{G}_i^*, i = 1, 2, \cdots, t_1$, connect to G^*. Clearly, $|V(\bar{G}^* \cup G^*)| > \frac{|V(FQ_n)|}{2}$, thus we have $A \subset (\cup G_i) \cup (\cup C_i)$.

Note that $|V(A) \cap V(D_1)| \leq |V(\cup C_i)| < n-1$ and $|V(A) \cap V(D_1)| \geq n-1$, a contradiction.

Case 2. $|F_1| \leq \frac{(g+1)(n+1)-2g-\binom{g}{2}-1}{2}$.

By a similar argument of Case 1, It is not difficult to this case is imposable since A is the smallest component.

Thus $\kappa_g(FQ_n) = (g+1)(n+1) - 2g - \binom{g}{2}$. □

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