Doubly resolvable designs with small parameters*

Svetlana Topalova, Stela Zhelezova
Mathematical Foundations of Informatics Department
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
PO.Box 323, 5000 Veliko Tarnovo, Bulgaria
svetlana@math.bas.bg, stela@math.bas.bg

Abstract

Doubly resolvable 2-(v,k,λ) designs (DRDs) with small parameters and their resolutions which have orthogonal resolutions (RORs) are constructed and classified up to isomorphism. Exact values or lower bounds on the number of the nonisomorphic sets of m mutually orthogonal resolutions (m-MORs) are presented. The implemented algorithms and the parameter range of this method are discussed, and the correctness of the computational results is checked in several ways.

Keywords: classification; orthogonal resolution; combinatorial design.

1 Introduction

For the basic concepts and notations concerning combinatorial designs and their resolvability refer, for instance, to [3], [4], [8], [21], [39].

Let $V = \{P_i\}_{i=1}^{v}$ be a finite set of *points*, and $\mathcal{B} = \{B_j\}_{j=1}^{b}$ – a finite collection of k-element subsets of V, called *blocks*. If any 2-subset of V is contained in exactly λ blocks of $\mathcal{B}$, then $D = (V, \mathcal{B})$ is a 2-(v,k,λ) *design*. Each point of D is incident with r blocks.

Two designs are *isomorphic* if there exists a one-to-one correspondence between the point and block sets of the first design and respectively, the point and block sets of the second design, and if this one-to-one correspondence does not change the incidence. An *automorphism* is an isomorphism

*Part of the results were announced at IV International Workshop Optimal Codes and Related Topics, 2005, Bulgaria; XI International Workshop Algebraic and Combinatorial Coding Theory, 2008, Bulgaria; NATO Advanced Research Workshop, 2008, Bulgaria of the design to itself, i.e. a permutation of the points that transforms the blocks into blocks.

A 2-(v,k,$m\lambda$) design is called an *m-fold multiple* of 2-(v,k,λ) designs if there is a partition of its blocks into m subcollections $\mathcal{B}_1$, $\mathcal{B}_2$, ... $\mathcal{B}_m$, which form 2-(v,k,λ) designs $D_1, D_2, ..., D_m$.

A *resolution* of the design is a partition of the collection of blocks into *parallel classes*, such that each point is in exactly one block of each parallel class. We denote by q the number of blocks in a parallel class. We shall call two parallel classes of the resolution $\mathcal{R}$, R_1 and R_2 equal ($R_1 = R_2$) if each block of R_1 is incident with the same points as some block of R_2. The design is *resolvable* if it has at least one resolution. Two resolutions are isomorphic if there exists an automorphism of the design transforming each parallel class of the first resolution into a parallel class of the second one.

Let $Z_q = \{0, 1, ..., q-1\}$. A word of length r over Z_q is an r-tuple $x = (x_1, x_2, ..., x_r) \in Z_q^r$. The Hamming distance $d(x, y)$ between two words $x, y \in Z_q^r$ is the number of coordinates in which the words differ. An equidistant $(r, v, d)_q$ code is a set of v words of length r over Z_q, with the property that the distance between any two distinct words is d. There is a one-to-one correspondence [36] between the resolutions of 2-(qk,k,λ) designs and the $(r, qk, r-\lambda)_q$ equidistant codes, $q > 1$.

Consider two resolutions $\mathcal{R}$ and $\mathcal{T}$ of one and the same design. A parallel class T_i ($i = 1, 2, ..., r$) of $\mathcal{T}$ is *orthogonal* to $\mathcal{R}$ if the number of blocks in $T_i \cap R_j$ is either 0 or 1 for all $1 \leq j \leq r$. (blocks are labelled in this case) The resolutions $\mathcal{R}$ and $\mathcal{T}$ are orthogonal if all classes of $\mathcal{T}$ are orthogonal to $\mathcal{R}$. Orthogonal resolutions may or may not be isomorphic to each other. A *doubly resolvable design (DRD)* is a design which has at least two orthogonal resolutions. We denote by *ROR* a resolution which is orthogonal to at least one other resolution, by *m-MOR* a set of m mutually orthogonal resolutions, and by *m-MORs* sets of m mutually orthogonal resolutions. Two m-MORs are isomorphic if there is an automorphism of the design transforming the first one into the second one. The m-MOR is maximal if no more resolutions can be added to it.

There are already a lot of works on the existence or classification of resolvable 2-(v,k,λ) designs with definite parameters, see for instance [11], [19],[20], [21], [34], [31], [32]. A very good recent survey of the different approaches for constructing and classifying design resolutions is contained in [22]. It can be seen from this survey that the most popular construction approach is to generate not the resolution itself, but the corresponding equidistant code, and this is usually done word by word in lexicographic order. Since rejection of equivalent partial solutions takes most of the computation time, in some works it is only used up to a certain point, and a clique search is applied next [22]. We also use the word by word orderly generation, but from some word on, we start applying an orthogonal resolution existence (ORE) test before the equivalence (E) test, and this makes it possible to prune a great number of partial solutions.

Papers on DRDs mainly deal with the existence problem – establishing existence or nonexistence of DRDs with certain parameters and setting lower bounds on m for the m-MORS with certain parameters. The *starter - adder method* [35] is the most often and very successfully used one and plenty of serious results have been obtained in this field. The newest achievements and an extended bibliography and summary of previous works can be found in [1]. For more details see for instance [9], [10], [12], [16], [25], [26], [27], [28], [42]. Another approach that has been used by some authors is to apply orthogonality tests to the resolutions of the classified designs with certain parameters and sometimes additional properties (automorphisms, etc.), see for instance, [7], [24], [37], [38]. A Room square of side n, $RS(n)$, is equivalent to a 2-MOR of a 2-(n + 1,2,1) BIBD. Full classification of Room squares with small parameters is known [13], [15], [33]. We do not know any other previous classifications of m-MORs, DRDs or RORs.

The aim of the present work is the classification up to isomorphism of 2-(v,k,λ) DRDs and their RORs and m-MORs. The results might be of interest for possible applications in cryptography, statistics, etc. For some ways, in which MORs can be used see, for instance, [2], [5], [6], [29]. Applications, however, often depend on properties of the underlying design (block intersections for instance), or of the m-MOR (critical sets, etc.), which may not follow from the design parameters. From that point of view, classification results for doubly resolvable designs and orthogonal resolutions might be very useful. We approach the problem by directly constructing RORs. We then classify the DRDs and construct m-MORs.

2 Construction details and results

The problem of classifying orthogonal resolutions can be approached in different ways. For instance, you may try and directly construct all pairs of orthogonal resolutions of designs with certain parameters, which means that you construct a structure twice bigger than the resolution itself, yet for the backtrack search "twice bigger" is too much slower. In another possible approach you may first find the DRDs, because their number is smaller or equal to that of RORs, but you will have to establish whether the design is doubly resolvable, or not, and this is more complicated than the ORE test. In our opinion the classification approach you choose is of greatest importance and it is possible that for some of the open cases, approaches different from ours might be more suitable and leading to new future results.

We construct nonisomorphic RORs first, then classify the corresponding DRDs, and find m-MORs last. This section presents a brief overview of the computation methods used, and tables of results, from which the first open cases can easily be found.

2.1 Construction of RORs and DRDs

2.1.1 Construction method

Since RORs are resolutions with some additional properties, we use the most popular way of constructing design resolutions, i.e. we construct the corresponding equidistant code by backtrack search word by word, so that each word is lexicographically greater than the previous one. Without loss of generality, we only use words whose coordinate entries are in lexicographic order, and we fix the first symbol of each word (i.e. the first parallel class of the resolution) (see [18], Section 5.2 for proof).

If the words are less than n_e, we apply E test after each word. For more than n_e words the E test already costs too much computational time, so we only apply it after constructing a whole ROR. Contrary to the E test, the ORE test works faster if the number of words is greater, and we do not apply it to less than n_o words. The efficiency of the algorithm is very sensitive to the application of these two tests. Usually n_e about 10, and n_o between $v/2$ and $2v/3$ leads to best speed.

Having constructed all nonisomorphic RORs, we consider them as designs and take away the isomorphic ones by a design isomorphism test. Thus only nonisomorphic DRDs remain.

2.1.2 Equivalence (E) test

Equivalent codes are obtained by permuting words, coordinates and symbols coordinatewise. We check if there exists a partial solution equivalent to the current one and lexicographically smaller than it. If we find such a solution, we skip the current one. The details of such a technique are well described, for instance, in [21], Section 7.1.2 and in [18].

2.1.3 Orthogonal Resolution Existence (ORE) test

If the whole resolution $\mathcal{R}$ is a ROR, a partial solution (of only n points) obviously has an orthogonal partial solution too. Experimenting how the ORE test works on partial solutions, we found out that if a resolution is not a ROR, and we take partial solutions on $n < 2v/3$ points, they usually have orthogonal partial solutions, and thus no pruning can be done by the ORE test for a small number of codewords (points). The reason for this is that some of the blocks of partial solutions may contain less points, and even no points at all and therefore there are much more ways for these blocks to participate in different orthogonal parallel classes.

We have implemented two algorithms for the ORE test. They both use backtrack search to partition the blocks of the partial resolution into orthogonal parallel classes. The search stops if one such partition is constructed, or if all possibilities have been tested and no partition can be found. In both algorithms we sort the blocks of the design in lexicographic order. The first point is in the first r blocks. Thus without loss of generality we assume that for $i = 1, 2, ..., r$ the i−th block is in the i−th parallel class of the orthogonal resolution. The first algorithm tries to construct an orthogonal mate block by block (BB), while the second one class by class (CC).

In construction BB we add the missing blocks to the first class of the orthogonal resolution, then to the second,..., and finally to the r−th one. Since an orthogonal parallel class should contain all points, at each step we try to add only blocks containing the first missing point in the class, and we check that the blocks in each orthogonal class are disjoint and from different classes of the resolution.

In construction CC at the beginning we generate for each of the first r blocks all possibilities for an orthogonal class containing this block. Then we choose the i−th class of the orthogonal resolution among the classes containing block $i \leq r$,

Since we only check for the existence of one orthogonal mate, the speed of the RORs generation using the BB or the CC construction does not differ much, CC being a little faster if the number r of the parallel classes is relatively small (some comparison statistics can be found in [40]).

2.1.4 Parameter specific restrictions

If $q = 2$, then a resolution is a ROR iff for each of its parallel classes, there exists at least one more parallel class equal to it. We use this simple observation to fix the first and second parallel classes equal to one another. This does not change much the RORs generation software, but cuts off the generation of a lot of resolutions that cannot be RORs and makes it possible to classify RORs with parameters 2-(12,6,10), 2-(14,7,12), 2-(16,8,14), 2-(12,6,15) and 2-(18,9,16).

We have tried to find and use parameter specific restrictions which double resolvability imposes on the intersection possibilities of the parallel classes (see [43] for more details). This speeds up the computation for some of the already covered by the general approach parameters, and makes it possible to classify by this method the RORs of 2-(20,10,18) designs.

2.1.5 Classification results

The results are summarized in Table 1 where "Nr" is the number of nonisomorphic resolutions known by now. The value is presented without any comments if it is taken from [30]. It is followed by $\surd$ if we have independently obtained the same number by our programme too, and by $*$ if it is obtained by our programme and we do not know a better bound calculated by other authors. In the column "No" the number of the design in the tables of [30] is given.

Table 1 presents a classification of RORs with $q = 2, 3, 4$ and 5, and for each q the list starts with resolvable designs with $k = 3$ and goes on with ascending values of k. We do not include designs that have no resolutions according to [30]. For each pair q and k we cover the smallest possible values of λ, cases with bigger λ are open. If for some q we have not classified the RORs for some value(s) of k, but have accomplished the classification for a bigger k, an empty row indicates the break in the eligible parameters. Thus it is clear what the first not classified parameters are. We should remark here that the tables in the present paper include only full classification results. As it was already said in the introduction, there are a lot of existence results due to plenty of other authors. These results are not included in the following tables.

Let λ_B be the greatest number of points which are the same for two arbitrary blocks of the design. Obviously $0 \leq \lambda_B \leq k$. If $\lambda = 1$, then $\lambda_B = 1$. Since doubly resolvable designs with smaller λ_B might be of interest for possible applications, we present in Table 2 classifications for some parameters. In some cases this is just an addition to the classification of RORs with these parameters given in Table 1, in other cases we have no classification results for the maximal possible λ_B, but only for some smaller values. We present nonexistence results too. In Table 2 "Nr" is the number of all resolutions with this λ_B. It is usually smaller than the number of all resolutions of designs with these parameters, and if given, it is always obtained by our programme.

2.2 Classification of m-MORs

We start with a DRD and construct its resolutions block by block. For each resolution $\mathcal{R}_1$ we check if it is isomorphic to a lexicographically smaller one, and if not, we try to construct another resolution $\mathcal{R}_2$, which is lexicographically greater than $\mathcal{R}_1$ and orthogonal to it, then $\mathcal{R}_3$ orthogonal to both $\mathcal{R}_1$ and $\mathcal{R}_2$, etc. We apply isomorphism test after having constructed a class of the resolution $\mathcal{R}_1$, or a whole resolution $\mathcal{R}_i$, $i > 1$. We output a new m-MOR if it is maximal.

Table 1: Computational results for RORs and DRDs

q	v	k	λ	N_r		RORs	DRDs	No
2	6	3	4	1	√	0	0	43
2	6	3	$4n$	1	√	1	1	$2 \leq n \leq 13$
2	8	4	3	1	√	0	0	15
2	8	4	6	4	√	1	1	101
2	8	4	9	10	√	1	1	278
2	8	4	12	31	√	4	4	524
2	8	4	15	82	√	4	4	819
2	8	4	18	240	*	13	13	-
2	8	4	21	650	*	16	16	-
2	8	4	24	1803	*	44	44	-
2	8	4	27	4763	*	70	70	-
2	10	5	8	5	√	0	0	195
2	10	5	16	27121734		5	5	891
2	10	5	24	≥73534	*	6	6	-
2	12	6	5	1	√	0	0	58
2	12	6	10	545	√	1	1	319
2	12	6	15	≥128284	*	1	1	743
2	12	6	20	≥546	*	546	546	-
2	14	7	12	1363486		0	0	451
2	16	8	7	5	√	0	0	130
2	16	8	14	≥1895	*	5	5	618
2	16	8	21	≥5	*	5	5	-
2	18	9	16	≥ 1		0	0	791
2	20	10	9	3	√	0	0	224
2	20	10	18	≥ 4		3	3	1007
2	24	12	11	130		0	0	346
2	28	14	13	7570		0	0	499
2	32	16	15	≥1		0	0	668
2	36	18	17	≥91		0	0	855
3	9	3	1	1	√	0	0	2
3	9	3	2	9	√	0	0	21
3	9	3	3	426	√	5	3	66
3	9	3	4	149041	√	83	38	145
3	9	3	5	203047732		≥76992	≥27269	235
3	12	4	3	5	√	0	0	56
3	27	9	4	68		0	0	90
4	12	3	2	74700		70	20	55
4	16	4	1	1	√	0	0	5
4	16	4	2	339592		1	1	44
5	15	3	1	7	√	0	0	14
5	20	4	3	≥ 204				220
5	25	5	1	1		0	0	11

Table 2: Computational results for RORs and DRDs with $\lambda_B < k$

q	v	k	λ	λ_B	N_r	RORs	DRDs	No
3	9	3	3	2	22	2	1	66
3	9	3	4	2	287	9	3	145
3	9	3	5	2	3960	3944	13	235
3	9	3	6	2	$\geq$ 3543	$\geq$3543	$\geq$1	356
3	12	4	6	2	0	0	0	316
3	15	5	6	2	0	0	0	280
3	18	6	5	2	0	0	0	176
3	21	7	6	2	0	0	0	250
3	21	7	6	3	0	0	0	250
3	24	8	7	2	0	0	0	343
3	27	9	8	2	0	0	0	453
3	30	10	9	2	0	0	0	576
3	33	11	10	2	0	0	0	704
4	12	3	2	2		32	9	55

The results are presented in Table 3 and some results for smaller λ_B in Table 4. The first value is the number of maximal m-MORs, and the second one all m-MORs for $m = 2, 3, 4$.

The DRDs, RORs and m-MORs themselves can be downloaded from the first author's web page (presently http://www.moi.math.bas.bg/~svetlana).

2.3 Parameter range

The number of RORs is often much smaller than the number of all resolutions, and that is why early pruning of partial solutions is very important. The ORE test, however, becomes applicable and efficient after about $2v/3$ of the resolution points have been added to the partial solution. That is why this construction method easily covers most parameters for which all the resolutions have been classified, but it does not go much further.

The classification time for the results we present varies from several minutes to several days, while classification of all the resolutions for some of these designs has only been accomplished by means of parameter specific approaches and/or within a lot of computer time [18], [19], [22], [31], [32]. There are only four parameter sets, for which full classification or enumeration of the resolutions is known, but we cannot construct the RORs by our general programme, namely 2-(9,3,5) [20], 2-(12,2,1) [14], 2-(14,2,1) [23] and 2-(28,4,1) designs [22]. For the first three cases the expected number of RORs is very big, while in the last case we find that there are no RORs by applying ORE test to the resolutions of 2-(28,4,1) designs.

Table 3: Classification results for m-MORs by computer search

q	v	k	λ	RORs	DRDs	2-MORs		3-MORs		4-MORs		No
2	6	3	8	1	1	1	1	-	-	-	-	236
2	6	3	12	1	1	0	1	1	1	-	-	596
2	6	3	16	1	1	0	$\geq$15	0	$\geq$485	$\geq$485	$\geq$485	1078
2	8	4	6	1	1	1	1	-	-	-	-	101
2	8	4	9	1	1	0	1	1	1	-	-	278
2	8	4	12	4	4	7	17	0	60	60	60	524
2	10	5	16	5	5	5	5	-	-	-	-	891
2	10	5	24	6	6	2	7	5	5	-	-	-
2	12	6	10	1	1	1	1	-	-	-	-	319
2	12	6	15	1	1	0	1	1	1	-	-	743
2	12	6	20	546	546	691	$\geq$ 701	0	$\geq$ 223	$\geq$ 223	$\geq$ 223	-
2	16	8	14	5	5	5	5	-	-	-	-	618
2	16	8	21	5	5	0	5	5	5	-	-	-
2	20	10	18	3	3	3	3	-	-	-	-	1007
3	9	3	3	5	3	2	7	5	5	-	-	66
3	9	3	4	83	38	351	449	284	285	1	1	145
4	12	3	2	70	20	252	254	1	2	1	1	55
4	16	4	2	1	1	0	1	1	1	-	-	44

Table 4: Classification results for m-MORs by computer search, $\lambda_B < k$

q	v	k	λ	λ_B	RORs	DRDs	2-MORs		3-MORs		4-MORs		No
3	9	3	3	2	2	1	2	2	-	-	-	-	66
3	9	3	4	2	9	3	11	11	-	-	-	-	145
3	9	3	5	2	3944	13	24668	27322	1178	1396	135	135	235
4	12	3	2	2	32	9	30	31	1	1	-	-	55

For $q = 2$ thanks to additional double resolvability restrictions (see Section 2.1.4) we cover parameters, for which all the resolutions have not been classified yet (in these cases only lower bounds for Nr are presented in Table 1). Classification of the m-MORs in the way we do it, is not possible for some of the parameters, for which we have classified the RORs. One of the reasons for this is that due to multiple designs the number of m-MORs grows very rapidly for higher values of λ. A lower bound on their number can be computed if the number of inequivalent sets of $q-1$ mutually orthogonal latin squares of side m is known [41]. For instance, by this bound the 5-MORs of 2-(6,3,20) designs are at least 11, the 6-MORs of 2-(6,3,24) at least 352716, the 7-MORs of 2-(6,3,28) at least $2 \cdot 10^{15}$, and the 8-MORs of 2-(6,3,32) at least $3 \cdot 10^{42}$. The big number of m-MORs also shows that our approach to classify RORs and DRDs first is quite suitable.

2.4 Correctness of the computational results

We checked the work of the resolution generating part of our software by switching off the ORE test and obtaining all the resolutions of the design for parameters, for which this was possible. We obtained the same number of nonisomorphic resolutions as indicated in [30]. For 2-(8,2,1) and 2-(10,2,1) designs we obtained the known number of nonisomorphic one-factorizations of the complete graph on 8 and 10 vertices respectively [17].

In some cases, in which all nonisomorphic resolutions of the designs are available, we obtained the number of RORs in two different ways, namely on the one hand we directly generated them, and on the other hand we applied the ORE test to all the nonisomorphic resolutions of designs with these parameters.

For most design parameters we obtained the results for RORs twice using first the BB and then the CC ORE test.

We ran the RORs generation software with parameters of designs that cannot be doubly resolvable, and no RORs were generated. Affine designs are such an example. A design is affine if it admits a resolution and a positive integer μ such that any two blocks from different parallel classes are incident to precisely μ common points. An affine design has a unique resolution and can not be doubly resolvable.

Designs with parameters 2-(6,3,4n) are unique and have a unique resolution ([21], Theorem 6.30). We ran our software for $n = 2, ..., 13$ and obtained a unique ROR.

We examined the structure of the constructed RORs of some multiple designs with $q = 2$, and verified the number of DRDs in another way. Consider as an example 2-(8,4,3n) designs. The unique up to isomorphism affine 2-(8,4,3) design has a unique resolution and is not doubly resolvable. Denote by $A_1, A_2, ..., A_{10}$ isomorphic 2-(8,4,3) designs. There are four nonisomorphic resolvable 2-(8,4,6) designs, namely A_1A_1, A_1A_2, A_1A_3 and A_1A_4, each of them having a unique up to isomorphism resolution. Only A_1A_1 is doubly resolvable because for each parallel class there is a class equal to it. The 2-(8,4,9) design has 10 nonisomorphic resolutions, namely $A_1A_1A_1$, $A_1A_1A_2$, $A_1A_1A_3$, $A_1A_1A_4$ and six resolutions of the type $A_1A_iA_j$, where $1 < i < j \leq 10$. The four 2-(8,4,12) DRDs are $A_1A_1A_1A_1$, $A_1A_2A_1A_2$, $A_1A_3A_1A_3$ and $A_1A_4A_1A_4$, and the four 2-(8,4,15) DRDs - $A_1A_1A_1A_1A_1$, $A_1A_1A_2A_1A_2$, $A_1A_1A_3A_1A_3$ and $A_1A_1A_4A_1A_4$. Ten 2-(8,4,18) DRDs are obtained as true doubles of the 10 resolvable 2-(8,4,9) designs, and four as true triples of the four resolvable 2-(8,4,6) designs. One of these DRDs, namely $A_1A_1A_1A_1A_1A_1$, is obtained in both ways, and, of course as a 6-fold multiple of the 2-(8,4,3) too. That is why the whole number of 2-(8,4,18) DRDs is 13.

For some parameters we obtained in parallel the maximum m for which m-MORs exist. For that purpose we generated all orthogonal mates of a ROR by the CC algorithm, and then examined them for mutual orthogonality.

A Room square of side n, $RS(n)$ is equivalent to a 2-MOR of a 2-$(n+1,2,1)$ BIBD. The number of inequivalent Room squares $IR(n)$ is known for $n \leq 9$, i.e. $IR(3) = IR(5) = 0$, $IR(7) = 6$, $IR(9) = 257630$ [13]. Applied to 2-$(n + 1,2,1)$ designs for $n = 3, 5, 7$ our m-MORs generating software gives the same number of 2-MORs.

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