Correction of 2-dimensional occasional and cluster errors in LRTJ-spaces

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Abstract. Two-dimensional codes in LRTJ spaces are subspaces of the space $\mathrm{Mat}_{m\times s}(\mathbf{Z}_q)$, the linear space of all $m \times s$-matrices (or arrays) with entries from a finite ring $\mathbf{Z}_q$ endowed with the LRTJ-metric [3,9]. Also, the error correcting capability of a linear code depends upon the number of parity check symbols. In this paper, we obtain a lower bound over the number of parity checks of two-dimensional codes in LRTJ-spaces correcting both independent as well as cluster array errors.

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1. Introduction

Burst (or cluster) error correcting array codes are developed [5] to protect clustered errors over a particular subarray part of the transmitted array message. These types of errors occur in many practical situations e.g. due to lightening and thunder in deep space and satellite communication. In fact, burst-error correcting codes are suitable for correcting errors which do not occur independently but are clustered over a given subarray in the transmitted array. In many practical situations, the weights of the burst array errors are not large. In [10], the author considered the problem of burst error correction in LRTJ-spaces. However, burst error correcting codes fail to correct even a few independent array errors when these are not with in bursts of specified order. Therefore, in actual communication, while it is important to consider correction of low-weight bursts, care must be taken to correct array errors of upto a specificed weight, no matter where they occur. Keeping this in view, in this paper, we obtain a lower bound on the number of parity check digits for 2-dimensional array codes in LRTJ-spaces correcting simultaneously cluster as well as independent array errors.

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2. Definitions and notations

Let $\mathbf{Z}_q$ be the ring of integers modulo q. Let $Mat_{m\times s}(\mathbf{Z}_q)$ be the set of all $m \times s$ matrices with entries from $\mathbf{Z}_q$. Then $Mat_{m\times s}(\mathbf{Z}_q)$ is a module over $\mathbf{Z}_q$. Let V be a $\mathbf{Z}_q$-submodule of the module $Mat_{m\times s}(\mathbf{Z}_q)$. Then V is called an array code (In fact, linear array code). For q prime, $\mathbf{Z}_q$ becomes a field and correspondingly $Mat_{m\times s}(\mathbf{Z}_q)$ and V become the vector space and a sub space respectively over the field $\mathbf{Z}_q$. We note that the space $Mat_{m\times s}(\mathbf{Z}_q)$ is identifiable with the space $\mathbf{Z}_q^{ms}$. Every matrix in $Mat_{m\times s}(\mathbf{Z}_q)$ can be represented as an $1\times ms$ vector by writing the first row of matrix followed by second row and so forth. Similarly, every vector in $\mathbf{Z}_q^{ms}$ can be represented as an $m \times s$ matrix in $Mat_{m\times s}(\mathbf{Z}_q)$ by separating the co-ordinates of the vector into m groups of s-coordinates. Also, we define the modular value $|a|$ of an element $a \in \mathbf{Z}_q$ by

$$|a| = \begin{cases} a & \text{if} \quad 0 \leq a \leq q/2 \\ q-a & \text{if} \quad q/2 < a \leq q-1. \end{cases}$$

We note that the non-zero modular value $|a|$ can be obtained by two different elements a and $q-a$ of $\mathbf{Z}_q$ provided $\{q$ is odd$\}$ or $\{q$ is even and $a \neq [q/2]\}$, i.e.

$$|a| = |q-a| \quad \text{if} \quad \begin{cases} q \quad \text{is odd} \\ \text{or} \\ q \text{ is even and} \quad a \neq q/2. \end{cases}$$

If q is even and $a = [q/2]$ or if $a = 0$, then $|a|$ is obtained in only one way viz., $|a| = a$.

Thus, there may be one or two equivalent values of $|a|$ which we shall refer to as repetitive equivalent values of a. The number of repetitive equivalent values of a will be denoted by e_a, where

$$e_a = \begin{cases} 1 & \text{if } \{\, q \text{ is even and } a = q/2\} \text{ or } \{a = 0\} \\ 2 & \text{if } \{\, q \text{ is odd and } a \neq 0\} \text{ or } \{q \text{ is even, } a \neq 0 \text{ and } a \neq q/2\}. \end{cases}$$

We now define the LRTJ-metric as follows [3,9]:

Let $Y \in Mat_{1\times s}(\mathbf{Z}_q)$ with $Y = (y_1, y_2, \cdots, y_s)$.

Define the row-weight of Y as

$$wt_\rho(Y) = \begin{cases} \max\limits_{j=1}^{s}|y_j| + \max\limits_{j=1}^{s}\{j-1 \mid y_j \neq 0\} & \text{if } Y \neq 0 \\ 0 & \text{if } Y = 0. \end{cases}$$

Then $0 \leq wt_\rho(Y) \leq [q/2]+s-1$. Extending the definition of the row-weight to the class of all $m \times s$ matrices as

$$wt\rho(A) = \sum_{i=1}^{m} wt\rho(R_i)$$

where $A = \begin{bmatrix} R_1 \\ R_2 \\ \cdots \\ R_m \end{bmatrix} \in Mat_{m\times s}(\mathbf{Z}_q)$ and R_i denotes the i^{th} row of A.

Then wt_ρ satisfies $0 \leq wt_\rho(A) \leq m([q/2]+s-1)\ \forall\ A \in Mat_{m\times s}(\mathbf{Z}_q)$ and determines a pseudo-metric on $Mat_{m\times s}(\mathbf{Z}_q)$ if we set $d(A, A') = wt_\rho(A - A')\ \forall\ A, A' \in Mat_{m\times s}(\mathbf{Z}_q)$ known as LRTJ-metric. We beserve the following facts about LRTJ-metric:

1. For $s = 1$, LRTJ-metric is just the classical Lee metric [13].
2. For $q = 2, 3$, the LRTJ-metric reduces to the RT metric [15].

Remarks.:

1. For $q > 3$,

$$\text{LRTJ-wt } (A) > \text{RT-wt } (A)\ \forall\ A \in Mat_{m\times s}(\mathbf{Z}_q)$$

2. For $s = 1$ and $q = 2, 3$, the LRTJ-metric reduces to the Hamming metric [14].

Also, we shall use the following notations:

1. $[x]$ = The largest integer less than or equal to x.
2. Q_i will denote the sum of repetitive equivalent values up to i i.e.,

$$Q_i = e_0 + e_1 + \cdots + e_i$$

where e_i denotes the repetitive equivalent value of i.

Note. If $i < 0$ say $i = -j$ where $j > 0$ then $Q_i = e_0 + e_{-1} + e_{-2} + \cdots + e_{-j} = e_0 = -1$.

3. Lower bound for codes in LRTJ-spaces correcting independent and burst errors simultaneously

We start with the definition of bursts in m-metric array codes [5].

Definition 3.1. A burst of order pr(or $p \times r$)$(1 \leq p \leq m, 1 \leq r \leq s)$ in the space $\mathrm{Mat}_{m\times s}(F_q)$ is an $m \times s$ matrix in which all the nonzero entries are confined to some $p \times r$ submatrix which has non-zero first and last rows as well as non-zero first and last columns.

Note. For $p = 1$, Definition 3.1 reduces to the definition of burst for classical codes [4].

Definition 3.2. A burst of order pr or less $(1 \leq p \leq m, 1 \leq r \leq s)$ in the space $\mathrm{Mat}_{m\times s}(F_q)$ is a burst of order cd(or $c \times d$) where $1 \leq c \leq p \leq m$ and $1 \leq d \leq r \leq s$.

To obtain the desired bound, we need to find all $m \times s$ arrays of LRTJ-weght t or less and additional arrays of LRTJ-weight w or less which are bursts of order $p \times r$ or less. We obtain in the next two lemmas, the number of these array patterns separately.

Lemma 3.1. *If G_t denote the all $m \times s$ arrays in $Mat_{m\times s}(F_q)$ having LRTJ-weight t or less then*

$$G_t = \sum_{r_{ij}} \frac{m!}{\prod_{i=1}^{[q/2]} \prod_{j=1}^{s} r_{ij}! \left(m - \sum_{i=1}^{[q/2]} \sum_{j=1}^{s} r_{ij}\right)!} \times \prod_{i=1}^{[q/2]} \prod_{j=1}^{s} \left(e_i (Q_i)^{j-2} (Q_i + (j-1)(Q_{i-1} - 1))\right)^{r_{ij}}, \quad (1)$$

where

$$r_{ij} \geq 0 \quad \forall \quad 1 \leq i \leq [q/2], \quad 1 \leq j \leq s,$$
$$\sum_{i=1}^{[q/2]} \sum_{j=1}^{s} (i + (j-1)) r_{ij} \leq d,$$
$$\sum_{i=1}^{[q/2]} \sum_{j=1}^{s} r_{ij} \leq m. \quad (2)$$

Proof. Let $A \in Mat_{m\times s}(\mathbf{Z}_q)$ be an $m \times s$ matrix having the LRTJ-weight t or less. Out of m rows of A, let $r_{ij} (\geq 0)$ denotes the number of rows having the LRTJ-weight $i+(j-1)$ where $1 \leq i \leq [q/2]$, $1 \leq j \leq s$. Now, there are following two mutually exclusive ways of obtaining the LRTJ-weight r_{ij} of a row vector:

Case (i): The j^{th} position which is the maximum non-zero position contains the entry with modular value i. In this case, the remaining previous $(j-1)$ entries can be filled in $(e_0+e_1+\cdots+e_i)^{j-1}$ ways. Therefore, the number of ways falling in this case by which we can obtain the LRTJ-weight of a row vector as r_{ij} is given by

$$e_i(e_0+e_1+\cdots+e_i)^{j-1} = e_i(Q_i)^{j-1} \tag{3}$$

Case (ii): The j^{th} position which is the maximum non-zero position does not contain the entry with modular value i. In this case, number of choices for the j^{th} position is $e_1+e_2+\cdots+e_{i-1}$. Out of the remaining $(j-1)$ positions, choose one position, fill it with entry having modular value e_i and the remaining $(j-2)$ positions with any of the $e_0+e_1+\cdots+e_i$ entries from $\mathbf{Z}_q$. The number of ways falling in this case to obtain the LRTJ-weight of a row vector as r_{ij} is given by

$$\begin{aligned} & (e_1+e_2\cdots+e_{i-1})\binom{j-1}{1}e_i(e_0+e_1+\cdots+e_i)^{j-2} \\ = & (Q_{i-1}-1)\binom{j-1}{1}e_i(Q_i)^{j-2} \\ = & (j-1)e_i(Q_{i-1}-1)(Q_i)^{j-2}. \end{aligned} \tag{4}$$

From (3) and (4), we get the total number of ways in which we can obtain $r_{ij}(1 \leq i \leq [q/2],\ 1 \leq j \leq s)$ as LRTJ- weight of a row vector and is given by

$$\begin{aligned} & e_i(Q_i)^{j-1}+(j-1)e_i(Q_{i-1}-1)(Q_i)^{j-2} \\ = & e_i(Q_i)^{j-2}(Q_i+(j-1)(Q_{i-1}-1)). \end{aligned} \tag{5}$$

Using (5), the equation (1) satisfying constraint (2) directly follows as the number of rows of an $m \times s$ matrix is m. □

Lemma 3.2. *If $B^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w)$ is the total number of arrays of order $m \times s$ in $Mat_{m\times s}(\mathbf{Z}_q)$ which are bursts of order $p \times r$ having LRTJ-weight* *between* $t+1$ *and* w, *then*

$$B^{p\times r}_{m\times s}(\mathbf{Z}_q,t+1,w)=\begin{cases} m\sum\limits_{j=1}^{min(w,s)}(Q_{w-(j-1)}-Q_{t-j-1}) & \text{if } p=r=1,\\ m\sum\limits_{j=1}^{min(w-r+1,s-r+1)}(Q_{w-(j+r-2)})-Q_{t-(j+r-2)})^2\times \\ \times(Q_{w-(j+r-2)}-Q_{t-(j+r-2)}+1)^{r-2} & \text{if } p=1,r\geq 2,\\ (m-p+1)\times \\ \times\sum\limits_{j=1}^{min(w-r+1,s-r+1)}(L^p_{j,r}-2L^{p-1}_{j,r}+L^{p-2}_{j,r}) & \text{if } p\geq 2,r\geq 1,\end{cases} \quad (6)$$

where

$$L^p_{j,r} = \sum_{r_{lf}}\frac{p!}{\prod\limits_{l=1}^{[q/2]}\prod\limits_{f=j}^{j+r-1}r_{lf}!\Big(p-\sum\limits_{l=1}^{[q/2]}\sum\limits_{f=j}^{j+r-1}r_{lf}\Big)!}\times$$
$$\times\prod_{l=1}^{[q/2]}\prod_{f=j}^{j+r-1}\Big(e_l(Q_l)^{f-j-1}(Q_l+(f-j)(Q_{l-1}-1))\Big)^{r_{lf}}, \quad (7)$$

and r_{lf} $(1\leq l\leq [q/2], j\leq f\leq j+r-1$ *in the expression for* $L^p_{j,r}$ *are non-negative integers satisfying the following constraints:*

at least one of $r_{lj}>0$ $(1\leq l\leq [q/2]$, j *fixed occuring in the expression for* $L^p_{j,r}$),
at least one of $r_{l,j+r-1}>0(1\leq l\leq [q/2],\ j+r-1$ *fixed*),

$$t+1\leq\sum_{l=1}^{[q/2]}\sum_{f=j}^{j+r-1}(l+(f-1))r_{lf}\leq w, \quad (8)$$

$$\sum_{l=1}^{[q/2]}\sum_{f=j}^{j+r-1}r_{lf}\leq p.$$

Proof. Consider a burst $A=\begin{bmatrix}A_1\\A_2\\\cdots\\A_m\end{bmatrix}$ where $A_i=(a_{i_1},a_{i_2},\cdots,a_{i_s})$, of order $pr(1 \leq p \leq m, 1 \leq r \leq s)$ having LRTJ-weight between $t+1$ and w. Let B be the $p \times r$ nonzero submatrix of A such that all the nonzero entries of A are confined to B with first and last rows as well as first and last columns nonzero. There are three cases depending upon the values of p and r.

Case 1. When $p = 1,\ r = 1$.

In this case, the 1×1 nonzero submatrix B can have (i, j) as its starting position in $m \times s$ matrix A where j can take values from 1 to min (w, s). With (i, j) as the starting position of 1×1 nonzero submatrix B, entry in B can be filled in $e_{t-j} + e_{t-j+1} + \cdots + e_{w-(j-1)} = Q_{w-(j-1)} - Q_{t-j-1}$ ways as any nonzero element of $\mathbf{Z}_q$ having modular value between $t - j$ and $w - (j-1)$ can be filled in that position. Therefore, number of bursts of order 1×1 having LRTJ-weight between $t+1$ and w in $Mat_{m \times s}(\mathbf{Z}_q)$ is given by

$$B^{1\times 1}_{m\times s}(\mathbf{Z}_q, t+1, w) = m \sum_{j=1}^{min(w,s)} (Q_{w-(j-1)} - Q_{t-j-1}).$$

Case 2. When $p = 1,\ r \geq 2$.

In this case, for the number of starting positions (i, j) of the $1 \times r$ nonzero submatrix B in $m \times s$ matrix A, i can take values from 1 to m and j can take values from 1 to $\min(w - r + 1, s - r + 1)$. Also, with (i, j) as the starting position of a single rowed submatrix B of the $m \times s$ matrix A, entries in B can be selected in $(Q_{w-(j+r-2)}) - Q_{t-(j+r-2)})^2 \times (Q_{w-(j+r-2)} - Q_{t-(j+r-2)} + 1)^{r-2}$ ways as the first and last components of the submatrix B can be chosen in $(Q_{w-(j+r-2)}) - Q_{t-(j+r-2)})^2$ ways and intermediate $(r-2)$ components can be chosen in $(Q_{w-(j+r-2)} - Q_{t-(j+r-2)} + 1)^{r-2}$ ways. Therefore, the number of bursts of order $1 \times r$ having LRTJ-weight between $t+1$ and w in $Mat_{m\times s}(\mathbf{Z}_q)$ is given by

$$B^{1\times r}_{m\times s}(\mathbf{Z}_q, t+1, w) = m \sum_{j=1}^{\min(w-r+1, s-r+1)} (Q_{w-(j+r-2)}) - Q_{t-(j+r-2)})^2 \times \times (Q_{w-(j+r-2)} - Q_{t-(j+r-2)} + 1)^{r-2}.$$

Case 3. When $p \geq 2,\ r \geq 1$.

In this case, let the $p \times r$ nonzero submatrix B starts at the $(i, j)^{th}$ position in A. Out of p rows of B, let r_{lf} $(1 \leq l \leq [q/2], j \leq f \leq j + r - 1)$ be the number of rows of B (and hence that of matrix A) having LRTJ-weight as $l + (f-1)$. Then $r_{lf} \geq 0 \quad \forall\ 1 \leq l \leq [q/2], j \leq f \leq j + r - 1$.

The number of ways in which p rows of B can be selected is given by

$$L^{p}_{j,r} - 2L^{p-1}_{j,r} + L^{p-2}_{j,r} \tag{9}$$

where $L^{p}_{j,r}$ is given by (7) and r_{lf} $(1 \le l \le [q/2], j \le f \le j+r-1)$ being nonnegative integers satisfying (8). Since in the starting position (i,j) of the submatrix B, i can take values from 1 to $(m-p+1)$ and j can take values from 1 to $\min(w-r+1, s-r+1)$, therefore, summing (9) over i and j, we get number of bursts of order pr (or $p \times r$)$(2 \le p \le m, 1 \le r \le s)$ having LRTJ-weight between $t+1$ and w and is given by

$$\begin{aligned} B^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w) &= \sum_{i=1}^{m-p+1} \sum_{j=1}^{\min(w-r+1,s-r+1)} (L^{p}_{j,r} - 2L^{p-1}_{j,r} + L^{p-2}_{j,r}) \\ &= (m-p+1) \sum_{j=1}^{min(w-r+1,s-r+1)} (L^{p}_{j,r} - 2L^{p-1}_{j,r} + L^{p-2}_{j,r}), \end{aligned}$$

where $L^{p}_{j,r}$ is given by (7) satisfying the constraints (8). □

Remark 3.1. The number of bursts of order $p \times r$ or less having LRTJ-weight lying between $t+1$ and w is given by

$$R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w) = \sum_{c=1}^{p} \sum_{d=1}^{r} B^{c\times d}_{m\times s}(\mathbf{Z}_q, t+1, w) \tag{10}$$

Now, we obtain the desired bound.

Theorem 3.1. *A linear $[m \times s, k]$ code of order $m \times s$ in LRTJ-spaces that simultaneously corrects arbitrary errors of LRTJ-weight t or less and bursts of order $p \times r$ or less with LRTJ-weight w or less should have at least*

$$log_q(G_t + R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w)) \tag{11}$$

parity checks where G_t and $R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w)$ are given by (1) and (10) respectively.

Proof. The total number of correctable error patterns for an m-code correcting simultaneous arbitrary errors of ρ-weight t or less and bursts of order $p \times r$ or less with ρ-weight w_ρ or less is given by

$$G_t + R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w).$$

Also, the number of available cosets is q^{ms-k} and since a linear $(m \times s, k)$ code should have at least as many cosets as the number of correctable error patterns, we have

$$q^{ms-k} \ge G_t + R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w)$$

i.e.

$$ms - k \geq log_q(G_t + R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w)).$$

□

Remark 3.2. The weight constraint over the burst can be removed by taking w to be the maximum possible weight for a burst of order $p \times r$. This requires that we take $w = p([q/2] + s - 1)$. The result in that case reduces to the one given by the following corollary:

Corollary 3.1. *A linear $[m \times s, k]$ code of order $m \times s$ in LRTJ-spaces that simultaneously corrects independent errors of LRTJ-weight t or less and any burst of order pr or less should have at least*

$$log_q(G_t + R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, p([q/2] + s - 1)). \tag{12}$$

parity checks. □

On the other hand, we can derive results for burst correction only. This requires the dropping of the independent error correction constraint. Taking $t = 0$, the corresponding result which is obtained, can be given in the following corollary.

Corollary 3.2 [10]. *A linear $[m \times s, k]$ code of order $m \times s$ that corrects all bursts of order $p \times r$ or less with LRTJ-weight w or less should have at least*

$$log_q(G_t + R^{p\times r}_{m\times s}(\mathbf{Z}_q, 1, w)) \tag{13}$$

parity checks. □

A bound for independent error correction can also be deduced from the result obtained in Theorem 3.1. This requires the dropping of the burst correction constraint. On taking $w = t$, it gives $B^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w)$ and hence $R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w) = 0$. The result so obtained can be given in the following corollary.

Corollary 3.3 [9]. *A linear $[m \times s, k]$ code of order $m \times s$ in LRTJ-spaces that corrects independent errors of LRTJ-weight t or less should have at least*

$$log_q(G_t) \tag{14}$$

parity checks. □

Remark 3.3. For $q = 2, 3$, the LRTJ-metric coincides with the RT-metric and the bounds obtained in (11), (12), (13) and (14) coincides with the corresponding bounds for RT-metric codes obtained in [12].

4. Bound for codes in LRTJ-spaces correcting independent errors and burst errors with limited intensity

Here, we have the situation in which the effect of the noise on a single position is no greater than an intensity $a(< [q/2])$ and the errors occur in the form of independent errors as well as bursts. In other words, our error patterns are independent errors and burst errors with specified LRTJ-weight and no nonzero entry has an equivalent value greater than a.

To obtain the required bound, we count the number of independent errors and burst errors of order $p \times r$ or less with no entry exceeding $a(< [q/2])$ in equivalent value such the LRTJ-weight of independent errors is atmost t and LRTJ-weight of clustered errors lies between $t+1$ and w. If $G_{t,a}$ denotes the restriction of G_t and $B^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w, a)$ (or $R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w, a)$) denotes the restriction of $B^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w)$ (resp. $R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w)$) for entries not exceeding a in equivalent values, then the total number of correctable error patterns with bounded intensity a is given by

$$G_{t,a} + R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w, a).$$

Having calculated the total number of correctable patterns, we now have the following theorem.

Theorem 4.1. *A linear $[m \times s, k]$ code of order $m \times s$ in LRTJ-spaces that corrects all independent errors of LRTJ-weight t or less and all bursts of order $p \times r$ or less having LRTJ-weight w or less with no entry exceeding a should have at least*

$$log_q(G_{t,a} + R^{p\times r}_{m\times s}(\mathbf{Z}_q, t+1, w, a).$$

parity checks. □

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